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# ON RADICAL AND ZERO DIVISORS IN SUPERTOPOLOGICAL RINGS

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**Abstract.** The concept of supertopological rings is introduced and results related to the structure of their radical are proved. Further, we have defined D-compactness and total D-disconnectedness and proved their extension from the set of right zero divisors to the whole ring.

## 1. Introduction

The study of topological groups and topological rings in mathematics is of much importance which interconnects the aspects of algebra and topology. Topological groups and topological rings have been studied extensively by several mathematicians such as L.S. Pontryagin [9], Irving Kaplansky [4, 5] , S. Warner [12] , V. Arnautov [1] etc. Since then many authors have found usefulness of the properties of these spaces in other branches of mathematics like mathematical analysis, complex analysis and functional analysis. As a result, many generalizations of these spaces such as semi-topological groups [2], semi-topological rings [11], irresolute topological rings [10] etc have appeared in literature and became an interesting topic for exploration.

In [4], Kaplansky studied how compactness affects the structure of radical in a topological ring and also determined the structure of compact semisimple topological rings to be isomorphic and homeomorphic to a Cartesian direct product of finite simple rings. Kohli and Singh [7], defined D-supercontinuity and studied their properties.

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In [4], Kaplansky studied how compactness affects the structure of radical in a topological ring and also determined the structure of compact semisimple topological rings to be isomorphic and homeomorphic to a Cartesian direct product of finite simple rings. Kohli and Singh [7], defined D-supercontinuity and studied their properties. We have used D-supercontinuity to introduce the notion of supertopological rings. Every supertopological ring is a topological ring but not conversely.

In section 2, we define $D_r$-rings (a particular type of supertopological rings) and in theorem 29 it is proved that the radical of a $D_r$-ring is always d-closed and hence closed. Further, theorem 38 shows that, in a supertopological ring that may not be a $D_r$-ring, imposing d-compactness implies d-closedness of the radical.

For both topological and supertopological rings, it is an interesting problem to check which properties can be extended from the set of zero divisors to the whole ring under various conditions. In last section of the article, we have introduced D-compact spaces, D-connectedness and total D-disconnectedness. In theorem 42 we have proved the extension of D-compactness from the set of right zero divisors in supertopological rings to the whole ring. In theorem 54 total D-disconnectedness is extended from the set of right zero divisors to the topological ring.

Throughout this paper, a ring will mean an assosiative topological ring which is also a Hausdorff space and may not contain unity, unless otherwise mentioned and will be denoted by A. Rings without any topological structure, will be mentioned as algebraic rings.

We begin with definitions and remarks which will be used throughout the article.

**Definition 1.** [7] *A function $f : X \to Y$ from topological space $X$ to topological space $Y$ is said to be D-supercontinuous if for each $x \in X$ and each open set $U \subset Y$ containing $f(x)$ there exists an open $F_\sigma$-set $V \subset X$ containing $x$ such that $f(V) \subset U$.*

**Remark 2.** *A D-supercontinuous function is also continuous but the converse need not be true. A counterexample is given in* [7].

**Remark 3.** *In metric spaces, every open set is an $F_\sigma$-set and hence D-supercontinuity and continuity are equivalent in metrizable spaces.*

For more theory and equivalent definitions of D-supercontinuous functions, see [7].

**Definition 4.** *A topological space $G$ that is also a group, where $G \times G$ carries product topology, is a supertopological group if the mappings*

$$g_1 : G \times G \to G \text{ such that } (x, y) \to xy$$

*and*

$$g_2 : G \to G \text{ such that } x \to x^{-1}$$

*are D-supercontinuous.*

**Example 5.** *Obviously every metric group is supertopological, but for non-metrizable example of a supertopological group, consider $\mathbb{R}^\infty$ with weak topology (which is not even first countable) where $\mathbb{R}^\infty$ is the set of finite-support sequences of real numbers, with the topology that a set is open if and only if its intersection with $\mathbb{R}^n$ is open for each $n$. This is a topological group with respect to coordinate-wise addition in which every open subset is also $F_\sigma$ because an open set is the union of its intersections with $\mathbb{R}^n$ for each $n$, and the intersection with $\mathbb{R}^n$ is an open subset of $\mathbb{R}^n$ and thus a countable union of closed subsets of $\mathbb{R}^n$, which are then also closed in $\mathbb{R}^\infty$.*

**Remark 6.** *Every supertopological group is a topological group but the converse need not be true.*

**Example 7.** *Let $H = \{0, 1\}$ be discrete topological group. Consider the uncountable product of copies of $H$ under product topology, i.e. $G = H^I$ where $I$ is an uncountable index set. Then $G$ is also a topological group and the function $g_2 : G \to G$ which takes $g \to g^{-1}$ is a homeomorphism which is not D-supercontinuous. Hence $G$ is an example of a topological group which is not supertopological.*

**Remark 8.** *Any topological group $G$ that is not perfectly normal i.e. there is some open set $U$ that is not an $F_\sigma$-set will serve as an example of topological group which is not supertopological as $g_2 : G \to G$ being a homeomorphism, $g_2^{-1}(U)$ is also not an $F_\sigma$-set.*

**Definition 9.** *A topological space $A$ that is also a ring with operations '+' and '.' where $A \times A$ carries product topology, is a supertopological* *ring if the mappings*

$$a_1 : A \times A \to A \text{ such that } (x, y) \to x + y$$

,

$$a_2 : A \to A \text{ such that } x \to -x$$

*and*

$$a_3 : A \to A \text{ such that } (x, y) \to x.y$$

*are D-supercontinuous.*

**Example 10.** *Any metric ring is a supertopological ring, but for non-metrizable supertopological ring we again consider the group* $\mathbb{R}^\infty$ *with weak topology and coordinate-wise addition along with trivial multiplication. This ring lacks unity element.*

**Remark 11.** *Every supertopological ring is a topological ring but the converse need not be true.*

**Example 12.** *Consider the discrete topological ring* $S = \{0, 1\}$ *and the uncountable product of copies of* $S$ *under product topology, i.e.* $A = S^I$ *where* $I$ *is an uncountable index set. Then* $A$ *is an example of a topological ring which is not supertopological.*

**Definition 13.** [7] *A set* $U$ *in a topological space* $X$ *is said to be d-open if for each* $x \in U$*, there exists an open* $F_\sigma$*-set* $H$ *such that* $x \in H \subset U$*. Complement of a d-open set is called d-closed.*

**Remark 14.** *Every d-open set is an open set but the converse need not be true in general topological spaces. Although in metric spaces, every open set is d-open.*

**Example 15.** *For* $X = \{1, 2, 3, 4\}$ *with topology* $\tau = \{\phi, X, \{4\}, \{1, 2\}, \{1, 2, 4\}\}$*, then* $\{1, 2, 4\}$ *is an open set that is not d-open.*

Throughout this paper a *d-neighborhood* of a point $x$ will mean an open $F_\sigma$-set containing $x$.

**Definition 16.** [8] *A topological space* $X$ *is said to be a d-compact space if every cover of* $X$ *by open* $F_\sigma$*-sets has a finite subcover.*

**Remark 17.** [8] *Compactness implies d-compactness but not conversely, as the set* $X = \mathbb{Z}^+$ *with particular point topology is d-compact but not compact.*

## 2. D-Rings and Structure of Radical

In his paper [4], Kaplansky used right quasiregular elements and r.q.r. ideal to study the structure of radical in topological rings. In this section we will study how d-compactness helps us to determine the structure of the radical in a supertopological ring. An element $x$ of an algebraic ring $S$ is said to be right quasiregular if there exists another element $y \in S$ such that $x \circ y = x + y + xy = 0$. A right ideal of $S$ is said to be an r.q.r. ideal if all of its elements are right quasiregular.

We mention a lemma proved by Kaplansky in [4] without proof, that will be useful to us in proving some results.

**Lemma 18.** *If $a$ is an element of an r.q.r. ideal and $x$ is an r.q.r. element, then $a + x$ is a right quasiregular element.*

*Radical* of an algebraic ring $S$, denoted by $Rad(S)$ or $R$, is defined to be the sum of all r.q.r. ideals in $S$. It can be shown that $Rad(S)$ itself is an ideal. In [4], Kaplansky defined a $Q_r$-ring to be a topological ring where its r.q.r. elements form an open set and showed that the radical of a $Q_r$-ring is always closed.

**Definition 19.** *A supertopological ring is said to be a $D_r$-ring if its right quasiregular elements form a d-open set. $D_l$-ring is defined similarly by taking left quasiregular elements. A supertopological ring that is both $D_r$-ring and $D_l$-ring is called a D-ring*

**Example 20.** *Rationals under usual topology is a D-ring.*

**Theorem 21.** *A supertopological ring $A$ which has a d-neighborhood $U_0$ of 0 consisting of right qausiregular elements is a $D_r$-ring.*

*Proof.* Let $x$ be any r.q.r. element in the ring $A$ such that $x \circ y = 0$. Choose a sufficiently small element $a$ such that it lies inside the d-neighborhood $U_0$ which consists of right quasiregular elements and $ay$ lies in the right ideal generated by $a$, which is an r.q.r. ideal. By lemma 2.1, $a + ay$ is right quasiregular and there exists a $z \in A$ such that $(a + ay) \circ z = 0$. Hence $a + x$ is also right quasiregular as $(a+x) \circ y \circ z = (a+ay) \circ z + (x \circ y)(1+z) = 0$. Therefore $x + U_0$ is a d-neighborhood around $x$ which consists entirely of right quasiregular elements and thus $A$ is a $D_r$-ring.

□

**Remark 22.** *For a $D_r$-ring, there always exists a d-neighborhood around* 0 *that consists of right quasiregular elements.*

Before we proceed further into the structure of radical, we mention a lemma on idempotents which will be useful in proving that maximal ideals of a unital supertopological ring are d-closed.

**Lemma 23.** *If $e$ is an idempotent in a unital $D_r$-ring $A$ and $I$ is a right ideal dense in $eA$, then $B = eA$, where $eA$ is the right ideal generated by $e$.*

*Proof.* Follows by lemma 7 of [4]. □

**Definition 24.** *For a set $M \subset X$, the intersection of all the d-closed sets in $X$ containing $M$ is called the d-closure of $M$ which is denoted by $[M]_d$.*

**Remark 25.** *$M$ is d-closed if and only if $M = [M]_d$.*

**Lemma 26.** *In a supertopological ring, d-closure of an ideal is also an ideal.*

**Theorem 27.** *In a $D_r$-ring with unity element, the maximal right ideals are d-closed.*

*Proof.* Let $M$ be the maximal right ideal in a unital $D_r$-ring $A$. Then, by definition, $M$ is dense in the d-closure $[M]_d$. As $M$ is maximal and $M \subset [M]_d$, implies that $[M]_d = M$ or $[M]_d = A$. But if $[M]_d = 1.A$, and 1 being idempotent, by lemma 23, $M = A$ and hence $M = [M]_d$. Thus, $M$ is d-closed. □

The above theorem is also true for maximal left ideals.

**Lemma 28.** *If $X$ is a topological space and $M \subset X$, then $x \in [M]_d$ if and only if every d-open set $U$ containing $x$ intersects $X$.*

**Theorem 29.** *In a $D_r$-ring, the radical $R$ is d-closed.*

*Proof.* Let $y \in [R]_d$. Then any d-neighborhood of $y$ must contain an element of $R$. As the set of all right quasiregular elements form a d-open set in a $D_r$-ring and 0 itself is right quasiregular, there exists an open $F_\sigma$-set around 0, say $U_0$, consisting of r.q.r. elements. Consider the d-neighborhood $y - U_0$ of $y$, which must contain some $a \in R$. Therefore, $a = y - x$ for some right quasiregular element $x$, i.e. $y = a + x$, where $a \in R$ and $x$ is right quasiregular. By lemma 18, $y$ is right quasiregular, thus, all elements of $[R_d]$ are right quasiregular. Therefore, $[R]_d$ is an ideal consisting of r.q.r. elements. Hence $[R]_d \subset R$ and $R = [R]_d$. $\square$

**Definition 30.** *A subset $S$ of a supertopological ring $A$ is right d-bounded if for any neighborhood $U$ of $0$, there exists a d-neighborhood $V$ such that $V.S \subset U$, where $V.S$ is the set of all products of elements in $V$ and $S$.*

Left d-bounded is defined similarly and a ring that is both left and right d-bounded, is called a d-bounded ring.

**Lemma 31.** *In a supertopological group, if there exists a fundamental system of d-neighborhoods of $e$, then there exists a fundamental system of symmetric d-neighborhoods of $e$.*

*Proof.* Let $\{V\}$ be a fundamental system of d-neighborhoods of $e$. Since $e = e^{-1}$, by translation homeomorphism, for each $V \in \{V\}$, $V^{-1}$ is a d-neighborhood of $e$. But $U = V \cap V^{-1}$ is a symmetric d-neighborhood of identity. Therefore, each $V$ contains some symmetric d-neighborhood $U$. Hence, $\{U\}$ is a fundamental system of symmetric d-neighborhoods of $e$. $\square$

**Theorem 32.** *For each d-neighborhood $W$ of $e$ in a supertopological group (whenever it exists), there exists a symmetric d-neighborhood $U$ of $e$ such that $\prod_{i=1}^{n} U^{\delta_i} \subset W$, where each $\delta_i = \pm 1$.*

*Proof.* Follows from the repeated use of D-supercontinuity of the map $(x, y) \to xy^{-1}$. $\square$

**Lemma 33.** *In a right d-bounded supertopological ring, if there exists a fundamental system of d-neighborhoods of $0$, then it has a system of right ideal d-neighborhoods of $0$.*

*Proof.* Suppose that $A$ is right d-bounded with $U$ a d-neighborhood of 0. By lemma 31, there exists a system of symmetric d-neighborhoods of 0. Let $V$ be a symmetric d-neighborhood with $V + V \subset U$ and $W$ be a symmetric d-neighborhood with $W \subset V$, $WA \subset V$. Then $W + WA$ is d-open right ideal contained in $U$ and thus makes a system of right ideal d-neighborhoods of 0. $\square$

**Theorem 34.** *Any d-compact set in a supertopological ring is d-bounded.*

*Proof.* Given a d-neighborhood $U$ of 0 and a point $x$ in a d-compact set $D$, by D-supercontinuity of $(x, 0) \to 0$, we may find d-neighborhoods $V_x$ and $W_0$ of $x$ and 0 respectively such that $V_x.W_0 \subset U$. Because of d-compactness of $D$, $\{V_x\}$ has a finite subcover. Let $T = \cap W_{0_i}$, where each $W_{0_i}$ corresponds to an element of finite subcover of $\{V_x\}$, and $T$ being open $F_\sigma$, we get $D.T \subset U$, which proves $D$ to be left d-bounded. Similarly it can be proved right d-bounded by using D-supercontinuity of $(0, x) \to 0$. □

**Theorem 35.** *In a right d-bounded $D_r$-ring $A$, if there exists a fundamental system of d-neighborhoods of* 0*, then its radical is d-open and hence open.*

*Proof.* As the ring is $D_r$, there exists a d-neighborhood around 0 consisting of right quasiregular elements and by right d-boundedness, there exists a d-neighborhood $V'$ such that $V'.A \subset U$. By lemma 33, there exists a fundamental system of right ideal d-neighborhoods $\{V\}$ of 0 such that $V \subset V'$. Then $V.A \subset U$ and for any $v \in V$ and $a \in A$, $va \in U$ and is an r.q.r. element. Hence every element of $V$ is right quasiregular. Thus $V$ is contained in the radical of $A$, and the radical can be realized as union of r.q.r. ideals, which are d-neighborhoods and thus the radical of $A$ is d-open. □

**Theorem 36.** *In a d-compact supertopological ring, the set of right quasiregular elements is d-closed.*

*Proof.* Ler $Q$ be the set of all right quasiregular elements. Let $x \notin Q$, i.e. there is no $y \in A$ such that $x \circ y = 0$. Using D-supercontinuity of $x \circ y = 0$, we can find two d-neighborhoods around $x$ and $y$, say $U_x$ and $U_y$ such that $U_x \circ U_y$ does not contain 0, for all $y \in A$. As the set $\{U_y\}$ is a cover of d-neighborhoods, there exists a finite subcover, say $U_{y_1}, \ldots, U_{y_n}$. If we take the corresponding $U_x's$ and take their intersection, then it is a d-neighborhood around $x$, which contains no right quasiregular elements and the set of non right quasiregular elements is d-open. Therefore, $Q$ is d-closed. □

**Theorem 37.** *In a supertopological ring, if the set of all r.q.r elements $Q$ is d-closed, then the radical $R$ is d-closed.*

*Proof.* Let $y \in [R]_d$ be arbitrary, whereas $[R]_d$ is the intersection of all d-closed sets containing the radical $R$ including $Q$. Thus $y \in [R]_d \subset Q$. As $[R]_d$ is also an ideal, $[R]_d \subset R$ and thus $R = [R]_d$ is d-closed. □

**Theorem 38.** *The radical of a d-compact supertopological ring is d-closed.*

*Proof.* Follows from theorem 36 and theorem 37. □

## 3. Zero Divisors and Total D-disconnectedness

**Definition 39.** *A topological space $X$ is said to be a D-compact if every cover of $X$ by d-open sets has a finite subcover.*

Each compact set is D-compact but converse need not be true.

**Example 40.** *The cocountable topology or countable complement topology on $X = \mathbb{R}$ is an example of D-compact space that is not compact.*

**Lemma 41.** *A d-closed subset of a D-compact topological space is D-compact.*

*Proof.* Let $Y$ be a d-closed subspace of the D-compact space $X$. Given a covering $\mathcal{C}$ of $Y$ by d-open sets in $X$, we form a covering $\mathcal{D}$ of $X$ by adjoining to $\mathcal{C}$ the single d-open set $X - Y$, i.e.

$$\mathcal{D} = \mathcal{C} \cup \{X - Y\}.$$

Some finite subcollection of $\mathcal{D}$ covers $X$. If this subcollection contains the set $X - Y$, discard $X - Y$. The resulting collection is a finite subcollection of $\mathcal{C}$ that covers Y. □

Before we proceed towards our next result, it is worth noting that a similar result, for topological rings, was proved by Kwangil Koh in his paper [6]. We prove a version for supertopological rings in which D-compactness of the set of zero divisors extends to the whole ring.

**Theorem 42.** *Let $A$ be a Hausdorff supertopological ring in which $xA$ is a d-closed subset of $A$ for any $x \in A$. If there exists non-trivial zero divisors in the ring and the set of all right zero divisors is D-compact, then $A$ is a D-compact ring.*

*Proof.* Let $Z$ be the non-empty set of all non-trivial right zero divisors i.e. $Z = \{0 \neq a \in A \mid ba = 0 \text{ for some } b \neq 0\}$ in $A$. Then for any $a \in Z, aA \subset Z \cup \{0\}$. As $Z \cup \{0\}$ is D-compact and $aA$ is a d-closed subset of $Z \cup \{0\}$, by lemma 41, $aA$ is D-compact. Let $B_a = \{b \in A \mid ab = 0\}$. As $\{0\}$ is closed and left multiplication by $a$ is D-supercontinuous, its inverse image $B_a$ is d-closed. As $B_a \subset Z \cup \{0\}$

and it is d-closed, $B_a$ is D-compact.
Since $aA$ is homeomorphic to $A/B_a$ , the quotient space $A/B_a$ is also D-compact. D-compactness implies d-compactness which further implies qusicompactness (see [8]) and for Hausdorff spaces quasicompactness is equivalent to compactness. As the subgroup $B_a$ and quotient group $A/B_a$ both are compact, $A$ is also compact by 5.25 in [3]. Hence, $A$ is D-compact. □

**Remark 43.** *The above theorem is also true if $Ax$ is a d-closed subset of $A$ for any $x \in A$ instead of $xA$. But the Hausdorff condition cannot be dropped.*

**Definition 44.** *A topological space $X$ is D-disconnected if it can be expressed as a union of two disjoint non-empty open $F_\sigma$-sets. Otherwise, $X$ is said to be D-connected.*

**Example 45.** *The set $X = \mathbb{R}^\infty$ with weak topology is D-connected as every open set in $\mathbb{R}^\infty$ is also $F_\sigma$.*

**Example 46.** *The set $X = \{0, 1, 2, 3, 4\}$ with discrete topology is D-disconnected.*

**Remark 47.** *D-disconnectedness implies disconnectedness and connectedness implies D-connectedness.*

**Definition 48.** *A topological space $X$ is totally D-disconnected or Hereditarily D-disconnected if $D_x = \{x\}$ for each $x \in X$, where $D_x$ is the maximal D-connected set containing $x$, also known as D-component of $x$.*

**Example 49.** *Any countable or finite set with discrete topology is totally D-disconnected.*

**Example 50.** *$\mathbb{R}$ in the lower limit topology, i.e. Sorgenfrey line (which has the basis of all half-open intervals $[a, b), a < b$ and $a, b$ are real numbers) is a non-metrizable example of totally D-disconnected as well as D-disconnected space.*

**Remark 51.** *Total D-disconnectedness implies total disconnectedness.*

Next we prove a theorem that will later on help us to extend total D-disconnectedness from set of zero divisors in topological ring to the whole ring.

**Theorem 52.** *If $G$ is a topological group of order greater than 2 and $G \backslash \{e\}$ is totally D-disconnected, then $G$ is totally D-disconnected.*

*Proof.* Let $G$ be such that $G \backslash \{e\}$ is totally D-disconnected but not $G$ itself. Any proper subset $S$ of $G$ has a translate $g^{-1}S$ with $e \notin g^{-1}S$, where $g^{-1}S$ being a subset of $G \backslash \{e\}$ is also totally D-disconnected. As inversion map is a homeomorphism in topological groups, $S$ is also totally D-disconnected. If $G$ is not D-connected, this applies to the D-connected component of $\{e\}$. Thus, we may assume that $G$ is D-connected.
As $|G| \geq 3$, the cardinality of totally D-disconnected set $G \backslash \{e\}$ is $\geq 2$ and can be partitioned into two open $F_\sigma$-subsets $A$ and $B$. Since $B$ is d-open, its complement $B^c = A \cup \{e\}$ is d-closed. It is easily seen that both $A^c$ and $B^c$ are D-connected. As $A^c$ and $B^c$ are proper subsets of $G$, they are totally D-disconnected too, hence they must be singletons, which is a contradiction to the order of the group. Hence $G$ must be totally D-disconnected.

□

**Lemma 53.** *Let $G$ be a topological group and $N$ be a normal subgroup, then if $N$ and $G/N$ are both totally D-disconnected then so is $G$.*

*Proof.* Let $B$ is a D-connected set of $G$. Consider the projection mapping $p : G \to G/N$, which being continuous takes $B$ to a singleton D-connected set $p(B)$. Hence $p(B)$ is contained in some coset $gN$. Since $N$ is totally D-disconnected, so is $gN$. As $B \subset gN$, $B$ is also singleton. □

**Theorem 54.** *Let $A$ be a topological ring of order greater than 2, for which set $Z$ of all non-zero right zero divisors is non-empty and $Z$ is totally D-disconnected, then $A$ is totally D-disconnected and hence totally disconnected.*

*Proof.* Let $z \in Z$ and consider the left translation $L_z : A \to A$. Clearly $zA \subset Z \cup \{0\}$ and $zA \backslash \{0\} \subset Z$, thus $zA \backslash \{0\}$ is totally D-disconnected and by theorem 52, so is $zA$. Since subgroup $KerL_z$ is a subset of $Z \cup \{0\}$ implies $KerL_z$ is totally D-disconnected. There exists a homeomorphism from $A/KerL_z$ onto $zA$ which implies that $A/KerL_z$ is totally D-disconnected. By lemma 53, total D-disconnectedness of $KerL_z$ and $A/KerL_z$ implies that $A$ is totally D-disconnected and hence totally disconnected as well. □

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