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ALPHA GENERALIZED PRE CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract. The concept of intuitionistic fuzzy set was introduced by Atanassov as a generalization of fuzzy sets . In 1997 Coker introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. Several Authors studied in various forms of intuitionistic fuzzy g-closed set and related topological properties. The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy αgp closed sets in intuitionistic fuzzy topological space. The class of all intuitionistic fuzzy αgp- closed sets lies between the class of all intuitionistic fuzzy α-closed sets and class of all intuitionistic fuzzy gspr-closed sets. We also introduce the concepts of intuitionistic fuzzy αgp open sets, intuitionistic fuzzy αgp- continuous mappings in intuitionistic fuzzy topological spaces.

Keywords and phrases: Intuitionistic fuzzy αgp-closed sets, Intuitionistic fuzzy αgp-open sets Intuitionistic fuzzy αgp continuous mappings, intuitionistic fuzzy αgp-irresolute mappings and intuitionistic fuzzy T*α space.
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1. Introduction

After the introduction of fuzzy sets by Zadeh [23] in 1965 and fuzzy topology by Chang [5] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [8] introduced the concept of intuitionistic fuzzy topological spaces. In 2008 Thakur introduced the concepts of intuitionistic fuzzy generalized closed sets [15] in intuitionistic fuzzy topology. After that many weak and strong forms of intuitionistic fuzzy g-closed sets such as intuitionistic fuzzy rg-closed sets[16], intuitionistic fuzzy sg-closed sets[17], intuitionistic fuzzy g*-closed sets[6], intuitionistic fuzzy gα-closed sets[11], intuitionistic fuzzy w-closed sets[18], intuitionistic fuzzy rw-closed sets[19], intuitionistic fuzzy gpr-closed sets[20], intuitionistic fuzzy rgα-closed sets[21], intuitionistic fuzzy gsp-[14]closed sets, intuitionistic fuzzy gp-closed set[12], intuitionistic fuzzy strongly g*-closed sets [2], intuitionistic fuzzy sgp-closed sets[3] and intuitionistic fuzzy rgw-closed sets[4] have been appeared in the literature. In the present paper we introduce the concepts of αgp-closed sets in intuitionistic fuzzy topological spaces. The class of intuitionistic fuzzy α-closed sets is properly placed between the class of intuitionistic fuzzy α- closed sets and the class of intuitionistic fuzzy gspr-closed sets. We also introduced the concepts of intuitionistic fuzzy αgp-open sets, and obtain some of their characterization and properties. As an application we introduce intuitionistic fuzzy alpha pre $T_{1/2}$-space and intuitionistic fuzzy alpha pre $T{*}_{1/2}$ –space Further, we introduce intuitionistic fuzzy αgp-continuous mappings.

2. Preliminaries

Let X be a nonempty fixed set. An intuitionistic fuzzy set[1] A in X is an object having the form A = $\{$<x,μ_A(x), γ_A(x)>: x $\in$ X $\}$, where the functions μ_A :X $\to$ [0,1] and γ_A:X $\to$ [0,1] denotes the degree of membership μ_A(x) and the degree of non membership γ_A(x) of each element x$\in$X to the set A respectively and $0 \leq \mu_A$ (x)+ γ_A(x) ≤ 1 for each x$\in$X. The intuitionistic fuzzy sets 0 = $\{$<x, 0, 1 >: x $\in$ X$\}$ and 1 = $\{$<x, 1, 0>: x $\in$ X$\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set A = $\{$<x,μ_A(x) γ_A(x)>: x $\in$ X$\}$ is called a subset of an intuitionistic fuzzy set B = $\{$<x,μ_B (x), γ_B(x) >: x $\in$ X$\}$ (for short A $\subseteq$ B) if μ_A(x) $\leq$ μ_B (x) and γ_A(x)$\geq \gamma_B$(x) for each x $\in$ X. The complement of an intuitionistic fuzzy set A = $\{<$ x,μ_A(x), γ_A(x)$>$: x $\in$X$\}$ is the intuitionistic fuzzy set A^c = $\{$x, γ_A(x), μ_A(x) : x $\in$ X $\}$.The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets A_i = $\{<$ x, μ_{A_i} (x) , γ_{A_i}(x) $>$: x $\in$ X , (i$\in \wedge$) $\}$ of X be the intuitionistic fuzzy set $\{$ $\cap A_i$ = $\{<$ x , $\wedge$ μ_{A_i} (x) , $\vee$ γ_{A_i} (x) $>$: x $\in$ X$\}$ $\}$ (resp. $\cup$ A_i = $\{<$ x, $\vee$ μ_{A_i} (x) , $\wedge$ γ_{A_i} (x) $>$: x $\in$ X $\}$). Two intuitionistic fuzzy sets A = $\{<$ x μ_A (x), γ_A (x) $>$: x $\in$ X $\}$ and B = $\{<$ x μ_B (x), γ_B (x) $>$: x $\in$ X $\}$ are said be q-coincident (A_qB for short) if and only if $\exists$ an element x$\in$ X such that $\{\mu_A$ (x) $>$ γ_B (x) or γ_A (x) $<$ μ_B (x)$\}$. A family τ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [3]on X if the intuitionistic fuzzy sets 0 ,1 $\in$ τ, and τ is closed under arbitrary union and finite intersection. The ordered pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [8].

Lemma 2.1. *[8]*
Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space (X,τ). Then:
(a) (A_q B) if and only if $A \subset B^c$.
(b) A is an intuitionistic fuzzy closed set in X if and only if cl (A) = A
(c) A is an intuitionistic fuzzy open set in X if and only if int (A) = A.
(d) cl (A^C) = $(int(A))^C$.
(e) int (A^C) = $(cl(A))^C$.

Definition 2.2 (9)**.** *Let X be a nonempty set and c$\in$X a fixed element in X. If α $\in$(0, 1] and β $\in$[0, 1) are two real numbers such that α + β $\leq$ 1 then:*
(a) c(α,β) = $<$ x,c_α, $c_{1-\beta}$ $>$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of c(α,β) and β denotes the degree of non membership of c(α,β).
(b) c(β) = $<$ x,0, 1-$c_{1-\beta}$ $>$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of c(β).

Definition 2.3 (10). *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called:*

(a) Intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that O$\subset$ A$\subset$ cl(O) .

(b) Intuitionistic fuzzy semi closed if the complement of A is an intuitionistic fuzzy semi open set.

(c) Intuitionistic fuzzy regular open of X if int(cl(A)) = A .

(d) Intuitionistic fuzzy regular closed of X if cl(int(A))= A.

(e) Intuitionistic fuzzy pre open if A $\subset$ int (cl(A)) .

(f)Intuitionistic fuzzy pre closed if cl(int(A)) $\subset$ A

(g)Intuitionistic fuzzy α-open A $\subset$ int (cl(int(A)))

(h) Intuitionistic fuzzy α- closed if cl(int(cl(A))) $\subset$ A

Definition 2.4 (10). *If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space(X,τ) then*

(a) scl (A) = $\cap$ { F : A $\subseteq$ F, F is intuitionistic fuzzy semi closed}

(b) pcl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy pre closed}

(c) αcl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy α closed}

(d) spcl(A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy semi pre-closed}

Definition 2.5. *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called:*

(a) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[15]

(b) Intuitionistic fuzzy rg-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[16]

(c) Intuitionistic fuzzy sg-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[17]

(d) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open.[6]*

(e) Intuitionistic fuzzy gα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy α- open.[11]

(f) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[18]

(g) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular semi open.[19]

(h) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[20]

(i) Intuitionistic fuzzy rgα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular α-open.[21]

(j) Intuitionistic fuzzy gsp-closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy -open.[14]

(k) Intuitionistic fuzzy gp-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[12]

(l) Intuitionistic fuzzy strongly g-closed set if cl (int(A)) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open in X.[2].*

(m) Intuitionistic fuzzy gspr-closed set if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – regular open.[13]

(n) Intuitionistic fuzzy sgp-closed set if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – semi open.[3]

(o) Intuitionistic fuzzy rgw-closed set if cl (int(A)) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – regular semi open.[4] The complements of the above mentioned closed set are their respective open sets.

Remark 2.6. *(a) Every intuitionistic fuzzy g-closed set is intuitionistic fuzzy rg-closed.[16]*

(b) Every intuitionistic fuzzy g-closed set is intuitionistic fuzzy gp-closed [12]

(c) Every intuitionistic fuzzy gp-closed set is intuitionistic fuzzy gsp-closed 12]

(d) Every intuitionistic fuzzy gpr- closed set is intuitionistic fuzzy gspr closed.[13]

(e) Every intuitionistic fuzzy gsp- closed set is intuitionistic fuzzy gspr-closed.[13]

(f) Every intuitionistic fuzzy gα- closed set is intuitionistic fuzzy αg-closed. [11]

(g) Every intuitionistic fuzzy αg- closed set is intuitionistic fuzzy rgα-closed. [21]

(h) Every intuitionistic fuzzy rgα- closed set is intuitionistic fuzzy gpr –closed [21]

Definition 2.7 (10)**.** *Let X and Y be two nonempty sets and f: X $\to$ Y is a function :*

(a) If B = $\{< y, \mu_B(y), \gamma_B(y) > : y \in Y\}$is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B) = \{ < x, f^{-1}\ (\mu_B)\ (x), f^{-1}\ (\gamma_B)\ (x) >: x \in X\}$.

(b) If A = $\{< x, \lambda_A(x), \upsilon_A(x)> : x \in X\}$ is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by $f\ (A) = \{< y, f\ (\lambda_A)\ (y), f(\upsilon_A)\ (y)>: y \in Y\}$ Where $f\ (\upsilon_A) = 1 - f\ (1- \upsilon_A)$.

Definition 2.8 (10). *Let (X,τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X \rightarrow Y$ be a function. Then f is said to be:*

(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

(b) Intuitionistic fuzzy α-continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy α-open set in X.

Definition 2.9. *Let (X,τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X \rightarrow Y$ be a function. Then f is said to be:*

(a) Intuitionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.[15]

(b) Intuitionistic fuzzy gp-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gp –closed in X.[12]

(c) Intutionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr –closed in X. [20]

(d) Intutionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rg –closed in X.[16]

(e) Intutionistic fuzzy rgα-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rgα –closed in X.[21] (f) Intutionistic fuzzy αg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy αg –closed in X.[11]

Remark 2.10. *(a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, [7].*

(b) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy gp-continuous[12].

(c) Every intuitionistic fuzzy gp- continuous mapping is intuitionistic fuzzy gpr-continuous [20].

(d) Every intuitionistic fuzzy rαg- continuous mapping is intuitionistic fuzzy gpr-continuous [16]

(e) Every intuitionistic fuzzy α-continuous mapping is ntuitionistic fuzzy αg-continuous .[21]

(f) Every intuitionistic fuzzy αg- continuous mapping is intuitionistic fuzzy rgα-continuous[11]

Definition 2.11. *An intuitionistic fuzzy topological space (X,τ) is said to be :*

(a) Intuitionistic fuzzy $T_{1/2}$ space if every intuitionistic fuzzy g-closed set is closed in (X,τ).[15]

(b) Intuitionistic fuzzy $pT_{1/2}$ space if every intuitionistic fuzzy gp-closed set is closed in (X,τ).[12]

(c) Intuitionistic fuzzy pre regular $T_{1/2}$ space if every intuitionistic fuzzy gpr-closed set is closed in (X,τ).[20]

(d) Intuitionistic fuzzy semi pre regular -$T_{1/2}$ space if every intuitionistic fuzzy gspr-closed set is closed in (X,τ).[13]

(e) Intuitionistic fuzzy rgα- $T_{1/2}$ if every intuitionistic fuzzy rgα-closed set in X is intuitionistic fuzzy closed in X. [21]

3. III. INTUITIONISTIC FUZZY αGP -CLOSED SET

Definition 3.1. *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called an intuitionistic fuzzy αgp –closed set if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy pre-open in X.*

First we prove that the class of intuitionistic fuzzy αgp- closed sets properly lies between the class of intuitionistic fuzzy α- closed sets and the class of intuitionistic fuzzy gspr-closed sets.

Theorem 3.2. *Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy αgp-closed.*

Proof: Let A be an intuitionistic fuzzy α- closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy pre-open sets in X. Since A is intuitionistic fuzzy α- closed set we have A= αcl(A). Hence αcl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is intuitionistic fuzzy pre-open in X. Therefore A is intuitionistic fuzzy αgp-closed set.

Remark 3.3. *The converse of above theorem is not true as we see the following example.*

Example 3.4. *Let $X = \{a,\ b,\ c\}$ and intuitionistic fuzzy sets O and U be defined as follows*

$O = \{ < a,\ 0.9,\ 0.1 > ,< b,\ 0,\ 1 > ,< c,\ 0,\ 1 >\}$

$U = \{ < a,\ 0,\ 1 > ,< b,\ 0.8,\ 0.1>,< c,\ 0.6,\ 0.3 >\}$

Let $\tau = \{0,\ O,\ U,\ 1\}$be an intuitionistic fuzzy topology on X.

Then the intuitionistic fuzzy set $A = \{< a,\ 0.7,\ 0.2>,< b\ ,0.6,\ 0.3> <c,\ 0,\ 1>\}$ is intuitionistic fuzzy αgp -closed but it is not intuitionistic fuzzy α-closed .

Theorem 3.5. *: Every intuitionistic fuzzy closed set is intuitionistic fuzzy αgp-closed.*

Proof: Let A be intuitionistic fuzzy closed set. Let $A \subseteq U$ and U is intuitionistic fuzzy pre-open sets in X. Since A is intuitionistic fuzzy -closed set we have A = cl(A). But $\alpha cl(A) \subset cl(A)$, therefore $\alpha cl(A) \subset U$. whenever $A \subseteq U$ and U is intuitionistic fuzzy pre-open in X. Hence A is intuitionistic fuzzy αgp-closed set.

Remark 3.6. *The converse of above theorem is not true as we see the following example.*

Example 3.7. *Let $X = \{a,\ b\}$ and intuitionistic fuzzy sets O and U be defined as follows:*

$O = \{ <a,0.2,0.8.> ,< b,0.1,0.9 >\}$

$U = \{ <a,0.5,0.4,> ,< b,0.6,0.4 >\}$

Let $\tau = \{0,\ O,\ U,\ 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a,\ 0.9,\ 0.1 >,< b\ ,0.6,\ 0.3> \}$ is intuitionistic fuzzy αgp -closed but it is not intuitionistic fuzzy closed .

Theorem 3.8. *Every intuitionistic fuzzy regular -closed set is intuitionistic fuzzy αgp-closed.*

Proof: It follows from the fact that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set and 3.5.

Remark 3.9. *The converse of above theorem is not true as we see the following example.*

Example 3.10. *Let $X = \{a,\ b,\ c\}$ and intuitionistic fuzzy sets O and U be defined as follows*

$O = \{ <a,0.2,0.8> ,< b,0.1,0.9>, < c,0.1 >\}$

$U = \{ <a,0.5,0.4,> ,< b,0, 1>,< c,0.4,0.3 > \}$

Let $\tau = \{0,\ O,\ U,\ 1\}$be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a,\ 0.9,\ 0.1 >,< b\ ,0.6,\ 0.3 > ,< c,0.1 >\}$ is intuitionistic fuzzy αgp -closed but it is not intuitionistic fuzzy regular-closed .

Theorem 3.11. *Every intuitionistic fuzzy g-closed set is intuitionistic fuzzy αgp-closed.*

Proof: Let A be an intuitionistic fuzzy g–closed set. Assume that $A \subseteq U$ and U is an intuitionistic fuzzy -open set in X. By the definition of intuitionistic fuzzy g-closed set, $cl(A) \subset U$. Note that $\alpha cl(A) \subset cl(A)$ is always true and every intuitionistic fuzzy open set is intuitionistic fuzzy pre open set . Therefore $\alpha cl(A) \subset U$ whenever $A \subseteq U$ and U is *intuitionistic fuzzy pre-open sets in X Hence A is intuitionistic fuzzy αgp-closed set.*

Remark 3.12. *The converse of above theorem is not true as we see the following example.*

Example 3.13. *Let $X = \{a, b, c\}$ and intuitionistic fuzzy sets O and U be defined as follows $O = \{< a , 0.9, 0.1.> ,< b, 0,1>,< c, 0, 1>\}$ $U = \{< a, 0.8, 0.1,> ,< b, 0.7, 0,2 >,< c, 0, 1 >\}$ Let $\tau = \{0, O, U, 1\}$be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a ,0, 1>,< b, 0.8, 0.1 > < c, 0, 1 >\}$ is intuitionistic fuzzy αgp -closed but it is not intuitionistic fuzzy g-closed .*

Theorem 3.14. *Every intuitionistic fuzzy αg-closed set is intuitionistic fuzzy αgp-closed.*

Proof: Let A is intuitionistic fuzzy αg-closed set. Assume that $A \subseteq U$ and U be an intuitionistic fuzzy -open sets in X. By definition of intuitionistic fuzzy αg-closed set, $\alpha cl(A) \subset U$.. Therefore $\alpha cl(A) \subset U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy pre-open sets in X Hence A is intuitionistic fuzzy αgp-closed set.

Remark 3.15. *The converse of above theorem is not true as we see the following example.*

Example 3.16. *Let $X = \{a, b, c, d, e\}$ and intuitionistic fuzzy sets P, Q and R defined as follows*

$P = \{ < a,0.9,0.1> ,< b,0,1>,< c,0,1>,< d,0,1> < e,0,1>\}$

$Q = \{ < a,0,1,> ,< b,0,1>,< c,0,1>,< d,0.9,0.1 > < e,0,1>\}$

$R = \{ < a,0,1> ,< b,0,1>,< c,0,1>, < d,0,1>,< e,0.8,0.2>\}$

Let $\tau = \{0, P, Q, R\ 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a, 0, 1 >,< b, 0.9, 0.1 >,< c, 0, 1 >,< d,0, 1 >,< e,0,1 >\}$ is intuitionistic fuzzy αgp -closed but it is not intuitionistic fuzzy αg-closed.

Theorem 3.17. *Every intuitionistic fuzzy $g\alpha$-closed set is intuitionistic fuzzy αgp-closed.*

Proof: Let A is intuitionistic fuzzy $g\alpha$-closed set. Assume that A $\subseteq$ U and U be an intuitionistic fuzzy α-open sets in X. By the definition of intuitionistic fuzzy $g\alpha$-closed set, $\alpha cl(A) \subset U$. Note that every intuitionistic fuzzy α-open set is intuitionistic fuzzy pre open set . Therefore $\alpha cl(A) \subset U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy pre-open sets in X Hence A is intuitionistic fuzzy αgp-closed set.

Remark 3.18. *The converse of the above theorem need not be true as from the following example.*

Example 3.19. *Example 3.6: Let $X = \{a, b\}$ and intuitionistic fuzzy sets O and U are defined as follows*

$O = \{ < a,0..2,0.8.> ,< b,0.1,0.9> \}$

$U = \{ < a,0.5,0.4 > ,< b,0, 1> \}$

Let $\tau = \{0, O, U, 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a, 0.9, 0.1>,< b ,0.6, 0.3>\}$ is intuitionistic fuzzy αgp-closed but it is not intuitionistic fuzzy gα closed.

Theorem 3.20. *Every intuitionistic fuzzy rg-closed set is intuitionistic fuzzy αgp-closed. Proof: Let A is intuitionistic fuzzy rg-closed set. Assume that $A \subseteq U$ and U is intuitionistic fuzzy regular-open sets in X. By the definition of intuitionistic fuzzy rg-closed set, cl(A) $\subset$ U. Note that αcl(A) $\subset$ cl(A) is always true and every intuitionistic fuzzy regular open set is intuitionistic fuzzy pre open set Therefore αcl(A) $\subset$ U whenever $A \subseteq U$ and U is intuitionistic fuzzy pre-open sets in X Hence A is intuitionistic fuzzy αgp-closed set*

Remark 3.21. *The converse of above theorem is not true as we see the following example.*

Example 3.22. *Example 3.7: Let $X = \{a, b, c, d, e\}$ and intuitionistic fuzzy sets P, Q, R, S, T, U and V defined as follows:*

$P = \{< a,0.9,0.1>, < b,0,1>, < c,0,1> ,< d,0,1> , < e,0,1> \}$

$Q = \{ < a, 0, 1> ,< b,0, 1 >,< c,0,1 >,< d, 0.9,0.1 > < e,0,1 >\}$

$R = \{< a, 0,1 >,< b,0, 1> ,< c, 0, 1 >,< d, 0, 1 >,< e,0.8, 0.2 >\}$

$S = \{< a, 0.9, 0.1> ,< b, 0,1 >,< c, 0, 1 > ,< d, 0.9,0.1 >, < e, 0,1 >\}$

$T = \{< a, 0.9, 0.1 > ,< b, 0, 1 >,< c,0,1>,< d, 0, 1 > < e, 0.8, 0.2>\}$

$U = \{< a, 0, 1> ,< b,0, 1>,< c, 0, 1 >, < d, 0.9,0.1 >,< e, 0.8, 0.2 >\}$

$V =\{< a, 0.9, 0.1 >,< b, 0, 1 >,< c,0,1 >, < d, 0.9,0.1 >, < e,0.8,0.2 >\}$

Let $\tau = \{0 , P, Q, R, S, T, U, V, 1 \}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a,0,1>,< ,0.9,0.1>,< c,0,1>,< d,0,1>,< e,0,1>\}$ is intuitionistic fuzzy αgp-closed sets but not intuitionistic fuzzy rg-closed sets.

Theorem 3.23. *Every intuitionistic fuzzy αgp-closed set is intuitionistic fuzzy gp-closed. Proof: Let A is intuitionistic fuzzy αgp-closed set. Assume that $A \subseteq U$ and U is an intuitionistic fuzzy pre open sets in X. By definition of intuitionistic fuzzy αgp-closed set, αcl(A)*

$\subset$ *U. Note that pcl(A)* $\subset \alpha$*cl(A) is always true. Therefore, pcl(A)* $\subset$ *U whenever A* $\subseteq$ *U and U is intuitionistic fuzzy pre-open sets in X Hence A is intuitionistic fuzzy gp- closed set.*

Remark 3.24. *The converse of above theorem is not true as we see the following example.*

Example 3.25. *Example 3.8: Let* $X = \{a,\ b,\ c\}$ *and intuitionistic fuzzy sets O be defined as follows* $O = \{ < a,\ 0.6,\ 0.2 > ,< b,\ 0,\ 1 > ,< c,\ 0,\ 1 >\}$ *Let* $\tau = \{0,\ O,\ 1\}$ *be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set* $A = \{< a,\ 0.7,\ 0.2 >,\ < b\ ,0.7,\ 0.3>,\ < c,\ 0,\ 1 >\}$ *is intuitionistic fuzzy gp-closed but it is not intuitionistic fuzzy* α*gp-closed*

Theorem 3.26. *Every intuitionistic fuzzy* α*gp-closed set is intuitionistic fuzzy gpr-closed.*

Proof: Let A be an intuitionistic fuzzy α*gp-closed set. Assume that A* $\subseteq$ *U and U is intuitionistic fuzzy regular open sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy pre open . U is intuitionistic fuzzy pre open set in X such that A* $\subseteq$ *U and A is intuitionistic fuzzy* α*gp-closed. By the definition of intuitionistic fuzzy* α*gp-closed set,* α*cl(A)* $\subset$ *U. Note that pcl(A)* $\subset \alpha$*cl(A) is always true. Therefore, pcl(A)* $\subset$ *U whenever A* $\subseteq$ *U and U is intuitionistic fuzzy pre-open sets in X Hence A is intuitionistic fuzzy gpr-closed set.*

Remark 3.27. *The converse of above theorem is not true as we see the following example.*

Example 3.28. *Let* $X = \{a,\ b,\ c,\ d\}$ *and intuitionistic fuzzy sets O, U, V, W defined as follows*

$O = \{ < a,0.9\ ,0.i > ,< b,0,\ 1 >,< c,\ 0,\ 1 >,< d,\ 0,\ 1 >\}$

$U = \{ < a\ ,0,\ 1 > ,< b,\ 0.8,\ 0.1 >,\ < c,\ 0,\ 1 >,< d\ ,0,\ 1>\}$

$V = \{ < a,\ 0.9,\ 0.1> ,< b,\ 0.8\ ,0.1>,< c,\ 0,\ 1>,< d,\ 0,\ 1>\}$

$W = \{ < a,\ 0.9,\ 0.1> ,< b,\ 0.8\ ,0.1>,< c\ ,0.7,\ 0.2 >,< d,\ 0,\ 1>\}$

Let $\tau = \{0,\ O,\ U,\ V,\ W,\ 1\}$ *be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set* $A = \{< a\ ,0,\ 1>,< b,\ 0,\ 1> < c,\ 0.7,\ 0.2>,< d\ ,0,\ 1>\}$ *is intuitionistic fuzzy gpr-closed set but it is not intuitionistic fuzzy* α*gp-closed.*

Theorem 3.29. *Every intuitionistic fuzzy* α*gp-closed set is intuitionistic fuzzy gsp-closed.*

Proof: Let A is intuitionistic fuzzy α*gp-closed set. Assume that A* $\subseteq$ *U and U is intuitionistic fuzzy open sets in X. Since every open set is intuitionistic fuzzy pre-open sets. Now U is intuitionistic fuzzy* *preopen set such that $A \subseteq U$ and A is intuitionistic fuzzy αgp-closed sets. By definition of intuitionistic fuzzy αgp-closed set, $\alpha cl(A) \subset U$. Since every intuitionistic fuzzy α-closed set is intuitionistic fuzzy semi pre-closed, so that spcl(A) $\subset \alpha cl(A)$ is always true. Therefore, spcl(A) $\subset U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy -open sets in X Hence A is intuitionistic fuzzy gsp- closed set.*

Remark 3.30. *The converse of above theorem is not true as we see the following example.*

Example 3.31. *Let $X = \{a, b\}$ and $\tau = \{0, U, 1\}$ be an intuitionistic fuzzy topology on X, where U=$\{< a, 0.5, 0.3>, < b, 0.2, 0.3>\}$. Then the intuitionistic fuzzy set $A = \{< a, 0.2, 0.4>, < b, 0.6, 0.1>\}$ is intuitionistic fuzzy gsp-closed but it is not intuitionistic fuzzy αgp-closed.*

Theorem 3.32. *Every intuitionistic fuzzy αgp-closed set is intuitionistic fuzzy gspr-closed.*

Proof: Let A be an intuitionistic fuzzy αgp-closed set. Assume that $A \subseteq U$ and U is intuitionistic fuzzy regular open sets in X. Since every regular open set is intuitionistic fuzzy pre-open sets. Now U is intuitionistic fuzzy preopen set such that $A \subseteq U$ and A is intuitionistic fuzzy αgp-closed sets. By definition of intuitionistic fuzzy αgp-closed set, $\alpha cl(A) \subset U$. Since every intuitionistic fuzzy α-closed set is intuitionistic fuzzy semi pre-closed, so that spcl(A) $\subset \alpha cl(A)$ is always true. Therefore, spcl(A) $\subset U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy regular-open sets in X Hence A is intuitionistic fuzzy gspr- closed set.

Remark 3.33. *The converse of above theorem is not true as we see the following example.*

Example 3.34. *Let $X = \{a, b, c, d\}$ and intuitionistic fuzzy sets O and U be defined as follows*

$O = \{ < a, 0.7, 0.1> ,< b, 0.6, 0.2 >,< c ,0, 1>,< d, 0, 1 > \}$

$U = \{ < a, 0, 1,> ,< b, 0, 1>,< c, 0.7, 0.2>,< d, 0.6, 0.1 > \}$

Let $\tau = \{0, O, U, 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A =$\{< a, 0.7, 0.1 >,< b, 0, 1 >,< c, 0, 1 >,< d, 0, 1>\}$ is intuitionistic fuzzy gspr -closed but it is not intuitionistic fuzzy αgp-closed.

Theorem 3.35. *Every intuitionistic fuzzy αgp-closed set is intuitionistic fuzzy $rg\alpha$-closed.*

Proof: Let A be an intuitionistic fuzzy αgp-closed set. Assume that $A \subseteq U$ and U is intuitionistic fuzzy regular α-open sets in X. Since every regular α-open set is intuitionistic fuzzy pre-open sets. Now U is intuitionistic fuzzy preopen set such that $A \subseteq U$ and A is intuitionistic fuzzy αgp-closed sets. By the definition of intuitionistic fuzzy αgp-closed set, $\alpha cl(A) \subset U$. Hence A is intuitionistic fuzzy $rg\alpha$- closed set.

Remark 3.36. *The converse of above theorem is not true as we see the following example.*

Example 3.37. *Let $X = \{a,\ b,\ c,\ d\ \}$ and intuitionistic fuzzy sets $O, U,\ V, W$ defined as follows*

$O = \{ < a,\ 0.9,0.1 > ,< b,0,1 >,< c,\ 0,1 >,< d,\ 0,1 >\}$

$U = \{ < a,\ 0,1 > ,< b,0.8,\ 0.1 >,< c,\ 0,\ 1>,< d,\ 0,1 >\}$

$V = \{ < a,0.9,0.1> ,< b,0.8,0.1>,< c,\ 0,1 >,< d,\ 0,\ 1>\}$

$W = \{ < a,\ 0.9,0.1 > ,< b,0.8,\ 0.1 >,< c,0.7,0.2 >,< d,\ 0,\ 1 >\}$

Let $\tau = \{0,\ O,\ U,\ V,\ W,\ 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a,0,\ 1 >,< b,\ 0,\ 1 > < c,\ 0.7,\ 0.2 >,< d,\ 0,\ 1 >\}$ is intuitionistic fuzzy $rg\alpha$-closed but it is not intuitionistic fuzzy αgp-closed.

Remark 3.38. *From the above discussion and known results we have the following diagram of implications:*

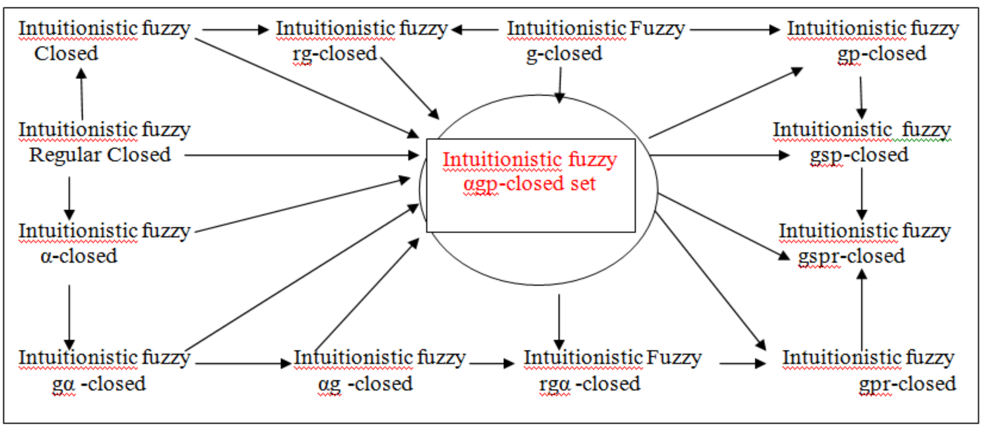

Fig.1 Relations between intuitionistic fuzzy αgp -closed set and other existing intuitionistic fuzzy closed sets

Theorem 3.39. *Let (X,τ) be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy*

αgp-closed if and only if $\rceil(AqF)$ then $\rceil(\alpha cl(A)q\, F)$ for every intuitionistic fuzzy pre-closed set F of X.

Proof: Necessity: Let F be an intuitionistic fuzzy pre-closed set of X and $\rceil(AqF)$. Then by Lemma(a) 2.1, $A \subseteq F^c$ and F^c is intuitionistic fuzzy pre-open in X. Therefore $\alpha cl(A\ ) \subseteq F^c$ by Def 3.1, because A is intuitionistic fuzzy αgp-closed. Hence by lemma 2.1(a), $\rceil\ (\alpha cl(A)qF)$

Sufficiency: Let O be an intuitionistic fuzzy pre-open set of X such that $A \subseteq O$ i.e. $A \subseteq (O)^c)^c$ Then by Lemma2.1(a), $\rceil\ (AqO^c)$ and O^c is an intuitionistic fuzzy pre closed set in X. Hence by hypothesis $\rceil\ (\alpha cl(A)q\ O^c)$. Therefore by Lemma 2.1(a), $\alpha cl\ (A) \subseteq ((O)^c)^c$ i .e. $\alpha cl\ (A) \subseteq O$ Hence A is intuitionistic fuzzy αgp-closed in X.

Remark 3.40. *The intersection of two intuitionistic fuzzy αgp-closed sets in an intuitionistic fuzzy topological space (X,τ) may not be intuitionistic fuzzy αgp-closed. For,*

Example 3.41. *Let $X = \{a,\ b,\ c\}$ and intuitionistic fuzzy sets O and U be defined as follows*

$O = \{ < a\ ,0.9,\ 0.1 > ,< b,\ 0\ ,1>,< c,\ 0,\ 1 >\}$

$U = \{ < a,\ 0,\ 1,> ,< b\ ,0.8\ ,0.1>,< c\ ,0,\ 1>\}$

Let $\tau = \{0,\ O,\ U,\ 1\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set $A = \{< a,\ 0.9,\ 0.1 >,< b,\ 0.8,\ 0.1 >, < c,\ 0\ ,\ 1 >\}$and $B = \{< a,\ 0.9,\ 0.1 >, < b,\ 0,\ 1>, < c,\ 0.7,\ 0.2 > \}$ are intuitionistic fuzzy αgp-closed in $(X,\ \tau)$ but $A \cap B$ is not intuitionistic fuzzy αgp-closed.

Theorem 3.42. *Let A be an intuitionistic fuzzy αgp-closed set in an intuitionistic fuzzy topological space $(X,\ \tau)$ and $A \subseteq B \subseteq \alpha cl\ (A)$ .Then B is intuitionistic fuzzy αgp-closed in X.*

Proof: Let O be an intuitionistic fuzzy pre-open set in X such that $B \subseteq O$. Then $A \subseteq O$ and since A is intuitionistic fuzzy αgp-closed, $\alpha cl(A) \subseteq O$. Now $B \subseteq \alpha cl\ (A)$, then $cl\ (B) \subseteq \alpha cl\ (A) \subseteq O$. Consequently B is intuitionistic fuzzy αgp-closed.

Definition 3.43. *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called intuitionistic fuzzy αgp-open if and only if its complement A^c is intuitionistic fuzzy αgp-closed.*

Remark 3.44. *Every intuitionistic fuzzy -open set is intuitionistic fuzzy αgp-open but its converse may not be true.*

Example 3.45. *Let $X = \{a,\ b\}$ and $\tau = \{0,\ U,\ 1\}$be an intuitionistic fuzzy topology on X, where*

U=$\{<a, 0.7, 0.2>, <b, 0.6, 0.3>\}$.*Then the intuitionistic fuzzy set A*=$\{<a, 0.2, 0.7>, <b, 0.1, 0.8>\}$

is intuitionistic fuzzy α*gp-open in* (X, τ) *but it is not intuitionistic fuzzy open in* (X, τ).

Theorem 3.46. *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space* (X, τ) *is intuitionistic fuzzy* α*gp-open if* $F \subseteq \alpha cl(A)$ *whenever F is intuitionistic fuzzy pre-closed and* $F \subseteq A$.

Proof: Follows from definition 3.1and Lemma 2.1.

Theorem 3.47. *Let A be an intuitionistic fuzzy* α*gp-open set of an intuitionistic fuzzy topological space* (X, τ) *and* $\alpha int(A) \subseteq B \subseteq A$. *Then B is intuitionistic fuzzy* α*gp-open.*

Proof: Suppose A be an intuitionistic fuzzy α*gp-open in X and* $\alpha int(A) \subseteq B \subseteq A$. *Then* $A^c \subseteq B\ c \subseteq (\alpha int\ (A))^c$ *hence* $A^c \subseteq Bc \subseteq \alpha cl(A^c)$ *by Lemma 2.1(d) and* A^c *is intuitionistic fuzzy* α*gp-closed it follows from theorem 3.46 that* B^c *is intuitionistic fuzzy* α*gp-closed. Hence B is intuitionistic fuzzy* α*gp-open.*

4. INTUITIONISTIC FUZZY αGP-CONTINUOUS MAPPINGS

Definition 4.1. *A mapping* $f : (X, \tau) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy* α*gp- continuous if inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy* α*gp-closed set in X.*

Theorem 4.2. *A mapping* $f : (X, \tau) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy* α*gp- continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy* α*gp - open in X.*

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^C$ *for every intuitionistic fuzzy set U of Y.*

Remark 4.3. *Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy* α*gp-continuous, but converse is not true, as the next example shows*

Example 4.4. *Let* $X = \{a, b\}$, $Y = \{x, y\}$ *and intuitionistic fuzzy sets U and V be defined as follows :*

U= $\{<a, 0.7, 0.2>, <b, 0.6, 0.3>\}$ *V*= $\{<x, 0.7, 0.2>, <y, 0.8, 0.1>\}$

Let $\tau = \{0, U, 1\}$ *and* $\sigma = \{0, V, 1\}$ *be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f:* $(X, \tau) \rightarrow (Y, \sigma)$ *defined by* $f(a) = x$ *and* $f(b) = y$ *is intuitionistic fuzzy* α*gp- continuous but not intuitionistic fuzzy continuous.*

Remark 4.5. *Every intuitionistic fuzzy α-continuous mapping is intuitionistic fuzzy αgp-continuous, but the converse is not true , see the example below.*

Example 4.6. *Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and intuitionistic fuzzy sets U and V be defined as follows :*

$U = \{< a, 0.4, 0.5>, < b, 0.6, 0.3>, < c, 0.2, 0.7> \}$ $V = \{< x, 0.5, 0.2>, < y, 0.8, 0.1> < z, 0.4, 0.5> \}$

Let $\tau = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is intuitionistic fuzzy αgp-continuous but not intuitionistic fuzzy α-continuous.

Remark 4.7. *Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy αgp-continuous, , but converse is not true, as the next example shows*

Example 4.8. *Let $X = \{a, b\}$, $Y = \{x, y\}$ and intuitionistic fuzzy sets U and V be defined as follows :*

$U = \{< a, 0.5, 0.5>, < b, 0.4, 0.6> \}$, $V = \{< x, 0.5, 0.5>, < y, 0.3, 0.7> \}$ Let $\tau = \{0, U, 1\}$ and $\sigma = \{0, V, 1\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\tau) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy αgp-continuous but not intuitionistic fuzzy g-continuous.

Remark 4.9. *Every intuitionistic fuzzy αg-continuous mapping is intuitionistic fuzzy αgp-continuous, but the converse is not true, see the example below.*

Example 4.10. *Let $X = \{a, b, c, d\}$, $Y = \{p, q, r, s\}$ and intuitionistic fuzzy sets U , V and W be defined as follows :*

$U = \{< a, 0.5, 0.5>, < b, 0.4, 0.6 >, < c, 0.7, 0.2 >, < d, 0.2, 0.6 > \}$

$V = \{< a, 0.4, 0.4 >, < b, 0.4, 0.6 >, < c, 0.7, 0.2 >, < d, 0.8, 0.1 > \}$

$W = \{< p, 0.5, 0.4>, < q, 0.3, 0.7 >, < r, 0.5, 0.2 >, < s, 0.2, 0.6 > \}$

Let $\tau = \{0, U, V, 1\}$ and $\sigma = \{0, W, 1\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\tau) \to (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c) = r$ and $f(d) = s$ is intuitionistic fuzzy αgp- continuous but not intuitionistic fuzzy αg-continuous.

Remark 4.11. *Every intuitionistic fuzzy rg-continuous mapping is intuitionistic fuzzy αgp-continuous, but the converse is not true, see the example below.*

Example 4.12. *Let $X = \{a, b, c, d, e\}$ and $Y = \{p, q, r, s, t\}$ and intuitionistic fuzzy sets O, U and W defined as follows*

O = { < a,0.9,0.1> ,< b,0.8,0.1> ,< c,0,1> ,<d,0,1> < e,0,1> }

U = { < a,0,1,> ,< b,0,1> ,< c,0.8,0.1> ,< d,0.7,0.2> < e,0,1> }

W= {< p,0.9,0.1 > ,< q,0, 1 > ,< r,0,1 > ,< s,0,1 > ,< t, 0,1> }

Let $\tau = \{0, O, U, 1\}$ and $\sigma = \{0, W, 1\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\tau) \to (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$ and $f(e) = t$ is intuitionistic fuzzy αgp -continuous but not intuitionistic fuzzy rg- continuous.

Remark 4.13. *Every intuitionistic fuzzy αgp-continuous mapping is intuitionistic fuzzy gp-continuous, but converse is not true, as the next example shows.*

Example 4.14. *Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$ and intuitionistic fuzzy sets O, U and V be defined as follows ;*

O = {< a, 0.9, 0.1 > ,< b, 0,1 > < c,0,1> }

U= {< a, 0, 1 > ,< b, 0.8, 0.1 > ,< c,0,1> }

V = {< x, 0.9, 0.1 > , < y,0.8, 0.1 > ,< z,0,1 > }

Let $\tau = \{0, O, U, 1\}$ and intuitionistic fuzzy sets U and V are defined as follows $\sigma = \{\{0, V, 1\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f: (X,\tau) \to (Y, \sigma)$ defined by $f(a) = x$ $f(b) = y$ and $f(c) = z$ is intuitionistic fuzzy gp-continuous but it is not intuitionistic fuzzy αgp- continuous.

Remark 4.15. *Every intuitionistic fuzzy αgp-continuous mapping is intuitionistic fuzzy gpr-continuous, but converse is not true, as the next example shows.*

Example 4.16. *Let $X = \{a, b, c\}$ $Y = \{p, q, r\}$ and intuitionistic fuzzy sets O, U, V, W, T be defined as follows:*

O = { < a,0.9,0.1> ,< b,0, 1 > ,< c, 0, 1 > }

U = { < a,0,1 > ,< b, 0.8, 0.1 > , < c, 0,1 > }

V = { < a, 0.9,0.1> ,< b,0.8,0.1> ,< c, 0 ,1 > }

W = { < a,0.9,0.1> ,< b,0.8,0.1 > ,< c,0.7,0.2 > }

T = {< p, 0, 1 > ,< q, 0, 1 > ,< r , 0.7, 0.2 > } Let $\tau = \{ 0, O, U, V, W, 1 \}$ and $\sigma = \{ 0, T, 1 \}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\tau) \to (Y, \sigma)$ defined by

f (a) = p , f (b) = q , f (c) = r is intuitionistic fuzzy gpr- continuous but not intuitionistic fuzzy αgp- continuous.

Remark 4.17. *Every intuitionistic fuzzy αgp-continuous mapping is intuitionistic fuzzy rgα-continuous, but converse is not true, as the next example shows*

Example 4.18. *Let X = {a, b, c, d, e} and Y= {w, x, y, z, t} and intuitionistic fuzzy sets P, Q, R, S, T and U be defined as follows:*

$P = \{ < a,0.9, 0.1 > ,< b,0,1 > ,< c,0,1 > ,< d, 0,1 > , < e, 0,1 > \}$

$Q = \{ < a ,0,1 > ,< b,0,1 > ,< c,0,1> ,< d,0.9,0.1 > , < e, 0,1 > \}$

$R = \{ < a, 0, 1 > ,< b, 0,1 > ,< c ,0,1> , < d,0,1> ,< e,0.8,0.2> \}$

$S = \{ < a,0.9,0.1> ,< b,0,1> ,< c,0,1> ,< d,0.9,0.1 > < e, 0,1 > \}$

$T = \{ < a,0.9,0.1> ,< b,0,1> ,< c,0,1> ,< d,0,1 > < e,0.8,0.2> \}$

$U = \{ < w,0, 1> ,< x,0, 1> ,< y,0,1> , < z,0.9,0.1 > , < t , 0.8,0.2 > \}$

Let τ = {0, P, Q, R, S, T, 1} and σ ={ 0, U , 1} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f: (X,τ) $\rightarrow$ (Y, σ) defined by f (a) = w , f (b) = x , f (c) = y f (d) = z and f (e) = t is intuitionistic fuzzy rgα-continuous but not intuitionistic fuzzy αgp-continuous.

Remark 4.19. *From the above discussion and known results we have the following diagram of implication*

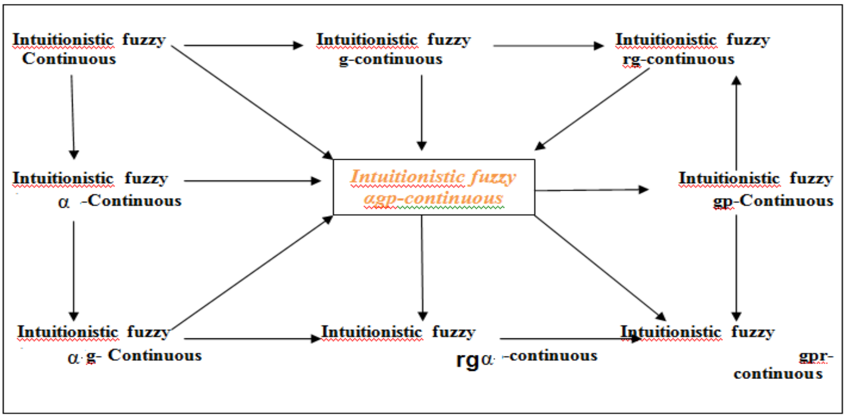

Fig.2 Relations between intuitionistic fuzzy α gp-continuous mappings and other existing intuitionistic fuzzy continuous mappings.

Theorem 4.20. *If f: $(X,\tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy αgp-continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ of X and each intuitionistic fuzzy preopen set V of Y such that $f(c(\alpha,\beta)) \subseteq V$ there exists an intuitionistic fuzzy αgp- open set U of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) \subseteq V$.*

Proof : Let $c(\alpha,\beta)$ be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy pre open set of Y such that $f(c(\alpha,\beta)) \subseteq V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy αgp- open set of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 4.21. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy αgp-continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$of X and each intuitionistic fuzzy open set V of Y such that $f(c(\alpha,\beta))qV$, there exists an intuitionistic fuzzy αgp- open set U of X such that $c(\alpha,\beta)qU$ and $f(U) \subseteq V$.*

Proof: Let $c(\alpha,\beta)$ be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta))q\ V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy αgp- open set of X such that $c(\alpha,\beta)q\ U$ and $f(U)= f(f^{-1}(V)) \subseteq V$.

Theorem 4.22. *If $f : (X,\tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy αgp-continuous and $g :(Y, \sigma) \rightarrow (Z, \mu)$ is intuitionistic fuzzy continuous. Then $gof : (X,\tau) \rightarrow (Z,\mu)$ is intuitionistic fuzzy αgp-continuous.*

Proof: Let A is an intuitionistic fuzzy closed set in Z. then $g^{-1}(A)$ is intuitionistic fuzzy closed in Y because g is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy αgp – closed in X. Hence gof is intuitionistic fuzzy αgp –continuous.

Theorem 4.23. *If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy αgp-continuous and $g : (Y,\sigma) \to (Z,\mu)$ is intuitionistic fuzzy g-continuous and (Y,σ) is intuitionistic fuzzy $T_{1/2}$ –space then $gof : (X,\tau) \to (Z,\mu)$ is intuitionistic fuzzy αgp-continuous.*

Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionistic fuzzy g-closed in Y. Since Y is intuitionistic fuzzy $T_{1/2}$-space, then g^{-1} (A) is intuitionistic fuzzy closed in Y. Hence $(gof)^{-1}(A) = f^{-1}$ $(g^{-1}(A))$ is intuitionistic fuzzy αgp – closed in X. Hence gof is intuitionistic fuzzy αgp – continuous.

5. APPLICATION OF INTUITIONISTIC FUZZY αGP-CLOSED SETS

In this section we introduce intuitionistic fuzzy alpha pre $T_{1/2}$-space and intuitionistic fuzzy alpha pre $T{*}_{1/2}$ –space as an application of intuitionistic fuzzy αgp-closed set. We have derived some characterizations of intuitionistic fuzzy αgp-closed sets.

Definition 5.1. *An intuitionistic fuzzy topological space (X,τ) is called:*

(i) an intuitionistic fuzzy alpha pre $T_{1/2}$-space - if every intuitionistic fuzzy αgp-closet set is intuitionistic fuzzy closed.

(ii) an intuitionistic fuzzy alpha pre $T{*}_{1/2}$ -space - if every intuitionistic fuzzy αgp-closet set is intuitionistic fuzzy α-closed.

Theorem 5.2. *Every intuitionistic fuzzy alpha pre $T_{1/2}$- -space is intuitionistic fuzzy alpha pre $T{*}_{1/2}$.*

Proof: Let (X, τ) be an intuitionistic fuzzy alpha pre $T_{1/2}$ space and let A be intuitionistic fuzzy αgp-closed set in (X, τ).Then A is intuitionistic fuzzy closed, Since every intuitionistic fuzzy closed set is intuitionistic fuzzy α-closed , A is intuitionistic fuzzy α- closed in topological space (X, τ) . Hence (X, τ) is intuitionistic fuzzy alpha pre $T{*}_{1/2}$ space.

Remark 5.3. *The converse of the above theorem is not true, as the next example shows.*

Example 5.4. *Let $X = \{a, b, c\}$ and Let $\tau = \{0, A, B, 1\}$ be an intuitionistic fuzzy topology on X where A=$\{< a ,0.7, 0.5 >, < b ,0.3$*

,0.6 >, < c, 1, 0 >} And B={< a ,0.7, 0.3 >, < b ,0.0 ,0.1>, < c, 0, 1 >} . Then intuitionistic fuzzy topological space (X,τ) is alpha pre $T*_{1/2}$ *space but not intuitionistic fuzzy alpha pre* $T_{1/2}$*- space.*

Theorem 5.5. *Every intuitionistic fuzzy pre regular* $T_{1/2}$ *-space is intuitionistic fuzzy alpha pre* $T*_{1/2}$ *- space.*

Proof: Let (X, τ) be an intuitionistic fuzzy pre regular $T_{1/2}$ *and let A be intuitionistic fuzzy αgp-closed set in (X, τ). Since every intuitionistic fuzzy αgp closed is intuitionistic fuzzy gpr-closed Then A is intuitionistic Fuzzy gpr-closed in(X, τ) . Now (X, τ) be an intuitionistic fuzzy pre regular* $T_{1/2}$ *space, Then by definition of intuitionistic fuzzy pre regular* $T_{1/2}$ *space , A is intuitionistic fuzzy pre closed topological space (X, τ) , now every intuitionistic fuzzy pre closed is intuitionistic α-closed, therefore A is intuitionistic fuzzy α-closed in (X, τ). Hence (X, τ) is intuitionistic fuzzy alpha pre* $T*_{1/2}$ *-space.*

Remark 5.6. *The converse of the above theorem is not true, as the next example shows.*

Example 5.7. *Let* $X = \{a, b\}$ *and Let* $\tau = \{0, A, B, 1\}$ *be an intuitionistic fuzzy topology on X where A={< a ,0.7, 0.5 >,< b ,0.3 ,0.6 >} and B={< a ,0.7, 0.3 >,< b ,0.0 ,0.1 >} .Then intuitionistic fuzzy topological space (X,τ) is intuitionistic fuzzy alpha pre* $T*_{1/2}$ *-space but not intuitionistic fuzzy pre regular* $T_{1/2}$ *–space.*

Theorem 5.8. *Every intuitionistic fuzzy semi pre regular* $T_{1/2}$ *-space is intuitionistic fuzzy alpha pre* $T*_{1/2}$ *-space.*

Proof: Let (X, τ) be an intuitionistic fuzzy semi pre regular $T_{1/2}$ *and let A be intuitionistic fuzzy αgp-closed set in (X, τ). Since every intuitionistic fuzzy αgp closed is intuitionistic fuzzy gspr-closed we see that A is intuitionistic fuzzy gspr-closed in(X, τ) . Since (X, τ) be an intuitionistic fuzzy semi pre regular* $T_{1/2}$ *space, then by definition of intuitionistic fuzzy semi pre regular T1/2 space , A is intuitionistic fuzzy closed set in (X, τ and therefore , A is is intuitionistic fuzzy alpha -closed set in (X, τ) . Hence (X, τ) is intuitionistic fuzzy alpha pre* $T*_{1/2}$ *-space.*

Remark 5.9. *The converse of the above theorem is not true, as the next example shows.*

Example 5.10. *Let* $X = \{a, b, c, d\}$ *and Let* $\tau = \{\{0, A, B, 1\}$ *be an intuitionistic fuzzy topology on X , where A={< a ,0.7, 0.5 >, < b ,0.3 ,0.6 >, < c, 1, 0 > < d, 0,1>} B={< a ,0.7, 0.3 >, < b ,0.0 ,0.1>, < c, 0, 1> < d, 0.5,0.5>} . Then intuitionistic fuzzy* *topological space (X, τ) is intuitionistic fuzzy alpha pre $T*_{1/2}$ space but not intuitionistic fuzzy semi pre regular $T_{1/2}$ space.*

Theorem 5.11. *Every intuitionistic fuzzy $pT_{1/2}$ -space is intuitionistic fuzzy alpha pre $T*_{1/2}$ - space.*

Proof: Let (X, τ) be an intuitionistic fuzzy $pT_{1/2}$-space and let A be intuitionistic fuzzy αgp-closed set in (X, τ). Since every intuitionistic fuzzy αgp closed is intuitionistic fuzzy gp-closed Then A is intuitionistic fuzzy gp-closed in(X, τ) . Now (X, τ) be an intuitionistic fuzzy $pT_{1/2}$ space, Then by definition of intuitionistic fuzzy $pT_{1/2}$ space , A is intuitionistic fuzzy closed set in (X, τ) and therefore A is intuitionistic fuzzy α-closed set in (X, τ) Hence (X, τ) is intuitionistic fuzzy alpha pre $T*_{1/2}$ space.

Remark 5.12. *The converse of the above theorem is not true, as the next example shows.*

Example 5.13. *Let X = $\{a, b\}$ and Let τ = $\{0, O, 1\}$ be an intuitionistic fuzzy topology on X, where O=$\{$< a ,0.5, 0.5 >,< b ,0.4 ,0.6>$\}$. Then intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy alpha pre $T*_{1/2}$ - space but not intuitionistic fuzzy $pT_{1/2}$ –space.*

Theorem 5.14. *Every intuitionistic fuzzy rgα $T_{1/2}$ -space is intuitionistic fuzzy alpha pre $T*_{1/2}$ - space.*

Proof: Let (X, τ) be an intuitionistic fuzzy rgα $T_{1/2}$-space and let A be intuitionistic fuzzy αgp-closed set in (X, τ). Since every intuitionistic fuzzy αgp closed is intuitionistic fuzzy rgα-closed It follows that A is intuitionistic Fuzzy rgα-closed in(X, τ) . Since (X, τ) be an intuitionistic fuzzy rgα $T_{1/2}$ space, A is intuitionistic fuzzy closed set in (X, τ) and therefore , A is is intuitionistic fuzzy α-closed set in (X, τ) . Hence (X, τ) is intuitionistic fuzzy alpha pre $T*_{1/2}$ space.

Remark 5.15. *The converse of the above theorem is not true, as the next example shows.*

Example 5.16. *Let X = $\{a, b, c\}$ and Let τ = $\{0, A, B, 1\}$ be an intuitionistic fuzzy topology on X where A=$\{$< a ,0.7, 0.5 >,< b ,0.3 ,0.6>, < c, 1, 0 >$\}$ and B=$\{$ < a ,0.7, 0.3 >,< b ,0.0 ,0.1 >, < c, 0, 1>$\}$ .Then intuitionistic fuzzy topological space (X,τ) is alpha pre $T*_{1/2}$ space but not intuitionistic fuzzy rgα $T_{1/2}$ -–space.*

6. CONCLUSION

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as fuzzy topology and intuitionistic fuzzy topology. The present paper investigated a new form of intuitionistic fuzzy closed sets called intuitionistic fuzzy αgp-closed sets which contain the classes of intuitionistic fuzzy closed sets, intuitionistic fuzzy pre-closed sets, intuitionistic fuzzy α-closed sets, intuitionistic fuzzy regular closed, intuitionistic fuzzy g*-closed sets, , and contained in the classes of intuitionistic fuzzy gp-closed sets, intuitionistic fuzzy gpr-closed sets , intuitionistic fuzzy gspr-closed sets , intuitionistic fuzzy sgp-and class of all intuitionistic fuzzy gsp-closed sets. Several properties and application of intuitionistic fuzzy αgp-closed sets and intuitionistic fuzzy αgp-continuous mappings are studied. Many examples are given to justify the result.

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