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FUZZY TOPOLOGICAL PROPERTIES OF SPACES AND FUNCTIONS WITH RESPECT TO THE $frwg$-CLOSURE OPERATOR

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Abstract. We study the $frwg$-closure operator in fuzzy topological space, investigating the corresponding notions of regular, normal, compact, T_2-space and various classes of functions-closed, open, continuous, irresolute, strongly continuous, weakly continuous. We establish connections between the above mentioned properties of functions and the properties of fuzzy topological spaces.

1. Introduction

This paper deals with fuzzy regular weakly generalized closed set ($frwg$-closed set, for short) defined in [9]. In this paper we have shown some important properties of this set. Also the mutual relationship of this set with the sets defined in [2, 3, 5, 6, 7, 9, 11, 12] are established. Using this set as a basic tool, here we introduce $frwg$-closure operator which is seen to be an idempotent operator. It is also shown that $frwg$-closure operator of a fuzzy set is not an $frwg$-closed set.

Keywords and phrases: Fuzzy regular open set, fg-closed set, $frwg$-closed set, fuzzy R-open function, $frwg$-regular space, $frwg$-normal space, $frwg$-continuous function.
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Next we introduce two strictly larger collection of fuzzy functions, viz., $frwg$-open function and $frwg$-closed function than fuzzy open function [29] and fuzzy closed function [29] respectively. Next we introduce $frwg$-regular space, $frwg$-normal space and $frwg$-compact space which are strictly weaker than fuzzy regular space, fuzzy normal space and fuzzy compact space respectively. Afterwards, we introduce four different types of continuous-like functions and then we find the interrelations between these four functions. Lastly, we introduce a new type of separation axiom, viz., $frwg$-T_2-space which is strictly larger than fuzzy T_2-space [24]. Also the applications of the functions defined in this paper on the spaces defined in this paper and in [18, 19, 20, 22, 23, 24, 25, 26] are shown.

2. Preliminaries

Throughout this paper (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [18]. In [30], L.A. Zadeh introduced fuzzy set as follows: A fuzzy set A is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [30] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$ [30]. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [30] while AqB means A is quasi-coincident (q-coincident, for short) with B, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [28]. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy point x_t and a fuzzy set A, $x_t \in A$ means $A(x) \geq t$, i.e., $x_t \leq A$. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [18] and fuzzy interior [18] respectively. A fuzzy set A is called a fuzzy neighbourhood (fuzzy nbd, for short) of a fuzzy point x_α if there exists a fuzzy open set U in X such that $x_\alpha \in U \leq A$ [28]. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_α [28]. A fuzzy set A is called a fuzzy quasi neighbourhood (fuzzy q-nbd, for short) [28] of a fuzzy point x_α in an fts X if there is a fuzzy open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open q-nbd [28] of x_α. A fuzzy set A in X is called fuzzy regular open [1] (resp., fuzzy semiopen [1], fuzzy preopen [27], fuzzy α-open [17], fuzzy β-open [21], fuzzy γ-open [4]) if $A = int(clA)$ (resp.,

$A \leq cl(intA)$, $A \leq int(clA)$, $A \leq int(cl(intA))$, $A \leq cl(int(clA))$, $A \leq cl(intA) \bigvee int(clA))$. A fuzzy set A is called fuzzy π-open if A is the union of finite number of fuzzy regular open sets [8]. The complement of a fuzzy regular open (resp., fuzzy semiopen, fuzzy preopen, fuzzy α-open, fuzzy β-open, fuzzy γ-open) set is called fuzzy regular closed [1] (resp., fuzzy semiclosed [1], fuzzy preclosed [27], fuzzy α-closed [17], fuzzy β-closed [21], fuzzy γ-closed [4]). The intersection of all fuzzy semiclosed (resp., fuzzy preclosed, fuzzy α-closed, fuzzy β-closed, fuzzy γ-closed) sets containing a fuzzy set A is called fuzzy semiclosure [1] (resp., fuzzy preclosure [27], fuzzy α-closure [17], fuzzy β-closure [21], fuzzy γ-closure [4]) of A, to be denoted by $sclA$ (resp., $pclA$, αclA, βclA, γclA). The collection of all fuzzy open (resp., fuzzy regular open, fuzzy semiopen, fuzzy preopen, fuzzy α-open, fuzzy β-open, fuzzy γ-open, fuzzy π-open) sets in an fts (X,τ) is denoted by τ (resp., $FRO(X,\tau)$, $FSO(X,\tau)$, $FPO(X,\tau)$, $F\alpha O(X,\tau)$, $F\beta O(X,\tau)$, $F\gamma O(X,\tau)$, $F\pi O(X,\tau)$). The collection of all fuzzy closed (resp., fuzzy regular closed, fuzzy semiclosed, fuzzy preclosed, fuzzy α-closed, fuzzy β-closed, fuzzy γ-closed, fuzzy π-closed) sets in an fts X is denoted by τ^c (resp., $FRC(X,\tau)$, $FSC(X,\tau)$, $FPC(X,\tau)$, $F\alpha C(X,\tau)$, $F\beta C(X,\tau)$, $F\gamma C(X,\tau)$, $F\pi C(X,\tau)$).

3. $frwg$-Closed Set: Some Properties

In this section some important properties of $frwg$-closed sets are established. Afterwards, we establish the mutual relationship of this set with the sets defined in [2, 3, 5, 6, 7, 9, 11, 12].

First we recall the following definition from [9] for ready references.

Definition 3.1 [9]. Let (X,τ) be an fts and $A \in I^X$. Then A is called $frwg$-closed set in X if $cl(intA) \leq U$ whenever $A \leq U \in FRO(X,\tau)$.

The complement of $frwg$-closed set is called $frwg$-open set in X.

Remark 3.2. Union and intersection of two $frwg$-closed sets may not be so, as it seen from the following examples.

Example 3.3. Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.2, B(b) = 0.3$. Then (X,τ) is an fts. Here $FRO(X,\tau) = \{0_X, 1_X, A\}$. Consider two fuzzy sets C and D defined by $C(a) = 0.5, C(b) = 0, D(a) = 0, D(b) = 0.3$. As

$cl(intC) = cl(intD) = 0_X$, clearly C and D are $frwg$-closed sets in (X, τ). Let $E = C \bigvee D$. Then $E(a) = 0.5, E(b) = 0.3$. Then $E < A \in FRO(X, \tau)$. But $cl(intE) = 1_X \setminus A \not\leq A$ implies that E is not an $frwg$-closed set in (X, τ). Again consider two fuzzy sets U and V, defined by $U(a) = 0.4, U(b) = 0.5, V(a) = 0.6, V(b) = 0.4$. As $1_X \in FRO(X, \tau)$ only containing both U and V, clearly U and V are $frwg$-closed sets in (X, τ). Let $W = U \bigwedge V$. Then $W(a) = W(b) = 0.4$. Now $W < A \in FRO(X, \tau)$. But $cl(intW) = 1_X \setminus A \not\leq A$ implies that W is not an $frwg$-closed set in (X, τ).

So we can conclude that the family of all $frwg$-open sets in an fts (X, τ) does not form a fuzzy topology.

Note 3.4. It is clear from definitions that fuzzy closed set, fuzzy regular closed set, fuzzy preclosed set, fuzzy α-closed set in an fts (X, τ) are $frwg$-closed set in X. But the converses are not necessarily true, in general, follow from the following example.

Example 3.5. There exists an $frwg$-closed set that has none of the following properties : fuzzy closed set, fuzzy regular closed set, fuzzy α-closed set
Consider Example 3.3 and the fuzzy set C. Here C is $frwg$-closed set in X. But clearly C is not a fuzzy closed set, fuzzy regular closed set, fuzzy α-closed set.

Example 3.6. There exists an $frwg$-closed set which is not fuzzy preclosed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Consider the fuzzy set B defined by $B(A) = B(b) = 0.5$. As $1_X \in FRO(X, \tau)$ only containing B, clearly B is an $frwg$-closed set in X. But as $cl(intB) = 1_X \setminus A \not\leq B$ implies that $B \notin FPC(X, \tau)$.

Theorem 3.7. Let (X, τ) be an fts and $A, B \in I^X$. If $A \leq B \leq cl(intA)$ and A is $frwg$-closed set in X, then B is also $frwg$-closed set in X.

Proof. Let $U \in FRO(X, \tau)$ be such that $B \leq U$. Then by hypothesis, $A \leq B \leq U$. As A is $frwg$-closed set in X, $cl(intA) \leq U$. As $B \leq cl(intA)$, so $cl9intB) \leq cl(int(cl(intA))) \leq cl(intA) \leq U$

implies that B is $frwg$-closed set in X.

Theorem 3.8. Let (X, τ) be an fts and $A, B \in I^X$. If $int(clA) \leq B \leq A$ and A is $frwg$-open set in X, then B is also $frwg$-open set in X.

Proof. $int(clA) \leq B \leq A$ implies that $1_X \setminus A \leq 1_X \setminus B \leq 1_X \setminus int(clA) = cl(int(1_X \setminus A))$ where $1_X \setminus A$ is $frwg$-closed set in X. By Theorem 3.7, $1_X \setminus B$ is $frwg$-closed set in X and so B is $frwg$-open set in X.

Theorem 3.9. Let (X, τ) be an fts and $A \in I^X$. Then A is $frwg$-open set in X if and only if $K \leq int(clA)$ whenever $K \leq A$ and $K \in FRC(X, \tau)$.

Proof. Let $A(\in I^X)$ be $frwg$-open set in X and $K \leq A$ where $K \in FRC(X, \tau)$. Then $1_X \setminus A \leq 1_X \setminus K$ where $1_X \setminus K \in FRO(X, \tau)$ and $1_X \setminus A$ is $frwg$-closed set in (X, τ). By hypothesis, $cl(int(1_X \setminus A)) \leq 1_X \setminus K$ implies that $1_X \setminus int(clA) \leq 1_X \setminus K$ implies that $K \leq int(clA)$.

Conversely, let $K \leq int(clA)$ whenever $K \leq A$, $K \in FRC(X, \tau)$. Then $1_X \setminus A \leq 1_X \setminus K$ where $1_X \setminus K \in FRO(X, \tau)$. Now $1_X \setminus int(clA) \leq 1_X \setminus K$ implies that $cl(int(1_X \setminus A)) \leq 1_X \setminus K$ (by hypothesis) and so $1_X \setminus A$ is $frwg$-closed set in X. Consequently, A is $frwg$-open set in X.

Theorem 3.10. Let (X, τ) be an fts and $A, B \in I^X$. If A is $frwg$-closed set in X and $B \in FRC(X, \tau)$ with $A \ /qB$. Then $cl(intA) \not{q} B$.

Proof. By hypothesis, $A \not{q} B$ implies that $A \leq 1_X \setminus B \in FRO(X, \tau)$ and so $cl(intA) \leq 1_X \setminus B$. Hence $cl(intA) \not{q} B$.

Remark 3.11. The converse of Theorem 3.10 may not be true, in general, as it seen from the following example.

Example 3.12. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B, C\}$ where $A(a) = 0.3, A(b) = 0.4, B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5$. Then (X, τ) is an fts. Consider the fuzzy set D defined by $D(a) = 0.35, D(b) = 0.4$. Then $D < B \in FRO(X, \tau)$. But

$cl(intD) = 1_X \setminus C \not\leq B$ implies that D is not $frwg$-closed set in X. Again $D \not q (1_X \setminus C) \in FRC(X, \tau)$ and $cl(intD) = 1_X \setminus C \not q (1_X \setminus C)$ also.

Let us now recall the following definitions from [2, 3, 5, 6, 7, 9, 11, 12] for ready references.

Definition 3.13. Let (X, τ) be an fts and $A \in I^X$. Then A is called
(i) fg-closed set [2, 3] if $clA \leq U$ whenever $A \leq U \in \tau$,
the complement of an fg-closed set in X is called an fg-open set in X,
(ii) fgp-closed set [3] if $pclA \leq U$ whenever $A \leq U \in \tau$,
(iii) fpg-closed set [3] if $pclA \leq U$ whenever $A \leq U \in FPO(X, \tau)$,
(iv) $fg\alpha$-closed set [3] if $\alpha clA \leq U$ whenever $A \leq U \in \tau$,
(v) $f\alpha g$-closed set [3] if $\alpha clA \leq U$ whenever $A \leq U \in F\alpha O(X, \tau)$,
(vi) $fg\beta$-closed set [7] if $\beta clA \leq U$ whenever $A \leq U \in \tau$,
(vii) $f\beta g$-closed set [7] if $\beta clA \leq U$ whenever $A \leq U \in F\beta O(X, \tau)$,
(viii) fgs-closed set [3] if $sclA \leq U$ whenever $A \leq U \in \tau$,
(ix) fsg-closed set [3] if $sclA \leq U$ whenever $A \leq U \in FSO(X, \tau)$,
(x) fgs^*-closed set [5] if $clA \leq U$ whenever $A \leq U \in FSO(X, \tau)$,
(xi) fs^*g-closed set [6] if $clA \leq U$ whenever $A \leq U$ where U is fg-open set in X,
(xii) fmg-closed set [9] if $cl(intA) \leq U$ whenever $A \leq U$ and U is fg-open set in X,
(xiii) $fswg$-closed set [9] if $cl(intA) \leq U$ whenever $A \leq U \in FSO(X, \tau)$,
(xiv) $f\pi g$-closed set [9] if $clA \leq U$ whenever $A \leq U$ where $U \in F\pi O(X)$,
(xv) fwg-closed set [9] if $cl(intA) \leq U$ whenever $A \leq U \in \tau$,
(xvi) $fg\gamma$-closed set [11] if $\gamma clA \leq U$ whenever $A \leq U \in \tau$,
(xvii) $fg\gamma^*$-closed set [12] if $\gamma clA \leq U$ whenever $A \leq U \in FSO(X, \tau)$.

Remark 3.14. It is clear from definitions that
(i) fg-closed set, fgp-closed set, fpg-closed set, $f\pi g$-closed set, fgs^*-closed set, fs^*g-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, fmg-closed set, fwg-closed set, $fswg$-closed set imply $frwg$-closed set. But the converses are not true, in general, follow from the following examples.
(ii) $frwg$-closed set is an independent concept of fgs-closed set, fsg-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set, $fg\beta$-closed set,

$f\beta g$-closed set follow from the following examples.

Example 3.15. There exists an $frwg$-closed set that none has the following properties : fg-closed set, $f\pi g$-closed set, fgs^*-closed set, fs^*g-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Consider the fuzzy set C defined by $C(a) = 0.5, C(b) = 0.3$. As $cl(intC) = 0_X$, clearly C is an $frwg$-closed set in X. Now $C < A \in \tau$ (resp., $A \in F\pi O(X, \tau)$, $A \in FSO(X, \tau)$, $A \in F\alpha O(X, \tau)$, A is an fg-open set in (X, τ)). But $clC = \alpha clC = 1_X \setminus A \not\leq A$ implies that C is not an fg-closed set, $f\pi g$-closed set, fgs^*-closed set, fs^*g-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set in X.

Example 3.16. There exists an $frwg$-closed set that none has the following properties : fsg-closed set, fgs-closed set, fpg-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Consider the fuzzy set B defined by $B(a) = B(b) = 0.5$. Then as $1_X \in FRO(X, \tau)$ only containing B, clearly B is an $frwg$-closed set in X. Now $FSO(X, \tau) = \{0_X, 1_X, U\}$ where $U \geq A$ and so $FSC(X, \tau) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \leq 1_X \setminus A$. Then $B < A \in \tau$ (resp., $A \in FSO(X, \tau)$). But as $sclB = 1_X \not\leq A$ implies that B is not an fgs-closed set as well as fsg-closed set in X. Again consider the fuzzy set C defined by $C(a) = C(b) = 0.6$. Then clearly C is $frwg$-closed set in X. Again $C \leq C \in FPO(X, \tau)$. But as $C \notin FPC(X, \tau)$, $pclC \not\leq C$ implies that C is not an fpg-closed set in X.

Example 3.17. There exists an $frwg$-closed set which is not an fgp-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.2, B(b) = 0.5$. Then (X, τ) is an fts. Consider the fuzzy set C defined by $C(a) = 0.4, C(b) = 0.5$. As $1_X \in FRO(X, \tau)$ only containing C, clearly C is $frwg$-closed set in X. Now $C < A \in \tau$. But $pclC = 1_X \setminus B \not\leq A$ and so C is not an fgp-closed set in X.

Example 3.18. There exists an $frwg$-closed set which is not an $fg\gamma^*$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.3$. Then (X, τ) is an fts. Consider the

fuzzy set C defined by $C(a) = 0.4, C(b) = 0.5$. As $1_X \in FRO(X,\tau)$ only containing C, clearly C is $frwg$-closed set in X. Again $C \leq C \in FSO(X,\tau)$. But $(cl(intC)) \bigwedge (int(clC)) = A \not\leq C$ implies that $C \notin F\gamma C(X,\tau)$ and so $\gamma clC \not\leq C$. Hence C is not an $fg\gamma^*$-closed set in X.

Example 3.19. There exists an $frwg$-closed set that none has the following properties :$fg\gamma$-closed set, fmg-closed set, fwg-closed set, $fswg$-closed set, $f\beta g$-closed set
Consider Example 3.17 and the fuzzy set C defined by $C(a) = 0.5, C(b) = 0.55$. As $1_X \in FRO(X,\tau)$ only containing C, clearly C is an $frwg$-closed set in X. Now $C < A \in \tau$ (resp., $A \in FSO(X,\tau)$ and A is an fg-open set in (X,τ))). But $\gamma clC = 1_X \not\leq A$ implies that C is not an $fg\gamma$-closed set in X. Also $cl(intC) = 1_X \setminus B \not\leq A$ implies that C is not an $fswg$-closed set as well as fmg-closed set, fwg-closed set in X. Again consider the fuzzy set D defined by $D(a) = D(b) = 0.6$. Then clearly D is an $frwg$-closed set in X. Now $D \leq D \in F\beta O(X,\tau)$. But as $D \notin F\beta C(X,\tau)$, $\beta clD \not\leq D$ implies that D is not an $f\beta g$-closed set in X.

Example 3.20. There exists an $frwg$-closed set which is not an $fg\beta$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.2, B(b) = 0.55$. Then (X,τ) is an fts. Consider the fuzzy set C defined by $C(a) = 0.5, C(b) = 0.55$. Aa $1_X \in FRO(X,\tau)$ only containing C, clearly C is an $frwg$-closed set in X. Now $C < A \in \tau$. But $\beta clC = 1_X \not\leq A$ implies that C is not an $fg\beta$-closed set in X.

Example 3.21. None of fgs-closed set, fsg-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set implies $frwg$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.3$. Then (X,τ) is an fts. Consider the fuzzy set C defined by $C(a) = C(b) = 0.4$. Then $C < A \in FRO(X,\tau)$. But $cl(intC) = 1_X \setminus A \not\leq A$ implies that C is not an $frwg$-closed set in X. Now $FSO(X,\tau) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus A$. So $FSC(X,\tau) = \{0_X, 1_X, 1_X \setminus U\}$ where $A \leq 1_X \setminus U \leq 1_X \setminus B$. Again $C < A \in \tau$ and $sclC = A \leq A$ implies that C is an fgs-closed set in X. Again consider the fuzzy set A. Then $A \leq A \in FRO(X,\tau)$ (resp.,

$A \in \tau$, $A \in FSO(X,\tau)$, $A \in F\gamma O(X,\tau)$). Now $cl(intA) = 1_X \setminus A \nleq A$ implies that A is not an $frwg$-closed set in X. But as $A \in FSC(X,\tau)$ as well as $A \in F\gamma C(X,\tau)$, A is an fsg-closed set, $fg\gamma$-closed set and $fg\gamma^*$-closed set.

Example 3.22. None of $fg\beta$-closed set, $f\beta g$-closed set implies $frwg$-closed set
Consider Example 3.12 and the fuzzy set B. Then $B \leq B \in FRO(X,\tau)$. But $cl(intB) = 1_X \setminus C \nleq B$ implies that B is not an $frwg$-closed set in X. But as $int(cl(intB)) = B$, $B \in F\beta C(X,\tau)$ implies that B is $fg\beta$-closed set as well as $f\beta g$-closed set in X.

Now we introduce the following concept.

Definition 3.23. An fts (X,τ) is called frT_g-space if every $frwg$-closed set in X is fuzzy closed set in X.

Now we recall the definitions of some spaces from [3, 5, 6, 8, 11, 12, 14, 15, 13, 16] in which the reverse implications in Remark 3.14 hold.

Definition 3.24. An fts (X,τ) is said to be
(i) $f\beta T_b$-space [8] if every $f\beta g$-closed set in X is fuzzy closed set in X,
(ii) fT_β-space [8] if every $fg\beta$-closed set in X is fuzzy closed set in X,
(iii) fT_α-space [3] if every $fg\alpha$-closed set in X is fuzzy closed set in X,
(iv) $f\alpha T_b$-space [3] if every $f\alpha g$-closed set in X is fuzzy closed set in X,
(v) fT_b-space [3] if every fgs-closed set in X is fuzzy closed set in X,
(vi) fT_{sg}-space [3] if every fsg-closed set in X is fuzzy closed set in X,
(vii) fT_γ-space [11] if every $fg\gamma$-closed set in X is fuzzy closed set in X,
(viii) fT_{γ^*}-space [12] if every $fg\gamma^*$-closed set in X is fuzzy closed set in X,
(ix) fsT_g-space [16] if every $fswg$-closed set in X is fuzzy closed set in X,
(x) fmT_g-space [13] if every fmg-closed set in X is fuzzy closed set in X,
(xi) fT_p-space [3] if every fgp-closed set in X is fuzzy closed set in X,
(xii) fpT_b-space [3] if every fpg-closed set in X is fuzzy closed set in

X,
(xiii) fT_w-space [15] if every fwg-closed set in X is fuzzy closed set in X,
(xiv) fT_π-space [14] if every $f\pi g$-closed set in X is fuzzy closed set in X,
(xv) fT_{s^*}-space [6] if every fs^*g-closed set in X is fuzzy closed set in X,
(xvii) fT_g-space [3] if every fg-closed set in X is fuzzy closed set in X.

Note 3.25. (i) In frT_g-space, $frwg$-closed set is fg-closed set, $f\pi g$-closed set, fs^*g-closed set, fgs^*-closed set, fgs-closed set, fsg-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set, $fg\beta$-closed set, $f\beta g$-closed set, fmg-closed set, fwg-closed set, $fswg$-closed set, fgp-closed set, fpg-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set.
(ii) In fT_b-space (resp., fT_{sg}-space, fT_γ-space, fT_{γ^*}-space, $f\beta T_b$-space, fT_β-space), fgs-closed set (resp., fsg-closed set, $fg\gamma$-closed set, $fg\gamma^*$-closed set, $f\beta g$-closed set, $fg\beta$-closed set) is $frwg$-closed set.

Let us now introduce a generalized version of neighbourhood structure of a fuzzy point in an fts.

Definition 3.26. Let (X, τ) be an fts and x_α, a fuzzy point in X. A fuzzy set A is called $frwg$-neighbourhood ($frwg$-nbd, for short) of x_α, if there exists an $frwg$-open set U in X such that $x_\alpha \leq U \leq A$. If, in addition, A is $frwg$-open set in X, then A is called an $frwg$-open nbd of x_α.

Definition 3.27. Let (X, τ) be an fts and x_α, a fuzzy point in X. A fuzzy set A is called $frwg$-quasi neighbourhood ($frwg$-q-nbd, for short) of x_α if there is an $frwg$-open set U in X such that $x_\alpha q U \leq A$. If, in addition, A is $frwg$-open set in X, then A is called an $frwg$-open q-nbd of x_α.

Note 3.28. (i) It is clear from definitions that every $frwg$-open set is an $frwg$-open nbd of each of its points. But every $frwg$-nbd of x_α may not be an $frwg$-open set containing x_α follows from the following example.
(ii) Also every fuzzy open nbd (resp., fuzzy open q-nbd) of a fuzzy point x_α is an $frwg$-open nbd (resp., $frwg$-open q-nbd) of x_α. But the converses are not necessarily true, in general, as it seen from the

following example.

Example 3.29. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.3, B(b) = 0.4$. Then (X, τ) is an fts. Consider the fuzzy set C defined by $C(a) = C(b) = 0.6$. Then C is not an $frwg$-open set in X (Indeed, $(1_X \setminus C)(a) = (1_X \setminus C)(b) = 0.4$. Then $1_X \setminus C < A \in FRO(X, \tau)$, but $cl(int(1_X \setminus C)) = 1_X \setminus A \not\leq A$ implies that $1_X \setminus C$ is not an $frwg$-closed set in X). Consider the fuzzy point $b_{0.5}$ and the fuzzy set D defined by $D(a) = D(b) = 0.5$. Then clearly D is an $frwg$-open set in X such that $b_{0.5} \in D < C$ implies that C is an $frwg$-nbd of $b_{0.5}$, but not an $frwg$-open nbd of $b_{0.5}$. Since there does not exist any fuzzy open set U in X with $b_{0.5} \in U \leq C$, C is not a fuzzy nbd as well as fuzzy open nbd of $b_{0.5}$. Again consider the fuzzy point $b_{0.6}$. Then $b_{0.6} q D \leq D$ implies that D is an $frwg$-open q-nbd of $b_{0.6}$. But as there does not exist any open set U in X with $b_{0.6} q U \leq D$ implies that D is not a fuzzy open q-nbd of $b_{0.6}$.

Theorem 3.30. Let (X, τ) be an fts and x_α, a fuzzy point in X. If $F (\in I^X)$ be $frwg$-closed set in X with $x_\alpha \in 1_X \setminus F$, then there exists an $frwg$-open nbd G of x_α in X such that $G \not{q} F$.

Proof. By hypothesis, $1_X \setminus F$ being an $frwg$-open set in X is an $frwg$-open nbd of x_α. So there exists an $frwg$-open set G in X such that $x_\alpha \in G \leq 1_X \setminus F$ implies that $G \not{q} F$.

4. $frwg$-Closure Operator, $frwg$-Open Function and $frwg$-Closed Function

Here we first introduce a new type of closure operator which is an idempotent operator. Afterwards, two new types of functions are introduced and characterized by this new closure operator.

Definition 4.1. Let (X, τ) be an fts and $A \in I^X$. Then $frwg$-closure and $frwg$-interior of A, denoted by $frwg\text{-}cl(A)$ and $frwg\text{-}int(A)$, are defined as follows:
$frwg\text{-}cl(A) = \bigwedge\{F : A \leq F, F \text{ is } frwg\text{-closed set in } X\}$,
$frwg\text{-}int(A) = \bigvee\{G : G \leq A, G \text{ is } frwg\text{-open set in } X\}$.

Remark 4.2. It is clear from definition that for any $A \in I^X$, $A \leq frwg\text{-}cl(A) \leq clA$. If A is $frwg$-closed set in an fts X, then

$A = frwg\text{-}cl(A)$. Similarly, $intA \leq frwg\text{-}int(A) \leq A$. If A is $frwg$-open set in an fts X, then $A = frwg\text{-}int(A)$. It follows from Remark 3.2 that $frwg\text{-}cl(A)$ (resp., $frwg\text{-}int(A)$) may not be $frwg$-closed (resp., $frwg$-open) set in an fts X.

Theorem 4.3. Let (X, τ) be an fts and $A \in I^X$. Then for a fuzzy point x_t in X, $x_t \in frwg\text{-}cl(A)$ if and only if every $frwg$-open q-nbd U of x_t, UqA.

Proof. Let $x_t \in frwg\text{-}cl(A)$ for any fuzzy set A in an fts X and F be any $frwg$-open q-nbd of x_t. Then x_tqF implies that $x_t \notin 1_X \setminus F$ which is $frwg$-closed set in X. Then by Definition 4.1, $A \not\leq 1_X \setminus F$ and so there exists $y \in X$ such that $A(y) > 1 - F(y)$ implies that AqF.

Conversely, let for every $frwg$-open q-nbd F of x_t, FqA. If possible, let $x_t \notin frwg\text{-}cl(A)$. Then by Definition 4.1, there exists an $frwg$-closed set U in X with $A \leq U$, $x_t \notin U$. Then $x_tq(1_X \setminus U)$ which being $frwg$-open set in X is $frwg$-open q-nbd of x_t. By assumption, $(1_X \setminus U)qA$ implies that $(1_X \setminus A)qA$, a contradiction.

Theorem 4.4. Let (X, τ) be an fts and $A, B \in I^X$. Then the following statements are true:
(i) $frwg\text{-}cl(0_X) = 0_X$,
(ii) $frwg\text{-}cl(1_X) = 1_X$,
(iii) $A \leq B$ implies that $frwg\text{-}cl(A) \leq frwg\text{-}cl(B)$,
(iv) $frwg\text{-}cl(A \bigvee B) = frwg\text{-}cl(A) \bigvee frwg\text{-}cl(B)$,
(v) $frwg\text{-}cl(A \wedge B) \leq frwg\text{-}cl(A) \wedge frwg\text{-}cl(B)$, equality does not hold, in general, follows from Example 3.3,
(vi) $frwg\text{-}cl(frwg\text{-}cl(A)) = frwg\text{-}cl(A)$.

Proof. (i), (ii) and (iii) are obvious.
(iv) From (iii), $frwg\text{-}cl(A) \bigvee frwg\text{-}cl(B) \leq frwg\text{-}cl(A \bigvee B)$.
To prove the converse, let $x_\alpha \in frwg\text{-}cl(A \bigvee B)$. Then by Theorem 4.3, for any $frwg$-open set U in X with $x_\alpha qU$, $Uq(A \bigvee B)$ implies that there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1$ and so either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1$ implies that either UqA or UqB. So either $x_\alpha \in frwg\text{-}cl(A)$ or $x_\alpha \in frwg\text{-}cl(B)$. Consequently, $x_\alpha \in frwg\text{-}cl(A) \bigvee frwg\text{-}cl(B)$.
(v) Follows from (iii).

(vi) As $A \leq frwg\text{-}cl(A)$, for any $A \in I^X$, $frwg\text{-}cl(A) \leq frwg\text{-}cl(frwg\text{-}cl(A))$ (by (iii)).

Conversely, let $x_\alpha \in frwg\text{-}cl(frwg\text{-}cl(A)) = frwgcl(B)$ where $B = frwg\text{-}cl(A)$. Let U be any $frwg$-open set in X with $x_\alpha qU$. Then UqB implies that there exists $y \in X$ such that $U(y) + B(y) > 1$. Let $B(y) = t$. Then $y_t qU$ and $y_t \in B = frwg\text{-}cl(A)$ implies that UqA and so $x_\alpha \in frwg\text{-}cl(A)$. So $frwg\text{-}cl(frwg\text{-}cl(A)) \leq frwg\text{-}cl(A)$. Consequently, $frwg\text{-}cl(frwg\text{-}cl(A)) = frwg\text{-}cl(A)$.

Theorem 4.5. Let (X, τ) be an fts and $A \in I^X$. Then the following statements hold:
(i) $frwg\text{-}cl(1_X \setminus A) = 1_X \setminus frwg\text{-}int(A)$
(ii) $frwg\text{-}int(1_X \setminus A) = 1_X \setminus frwg\text{-}cl(A)$.

Proof (i). Let $x_t \in frwg\text{-}cl(1_X \setminus A)$ for a fuzzy set A in an fts (X, τ). If possible, let $x_t \notin 1_X \setminus frwg\text{-}int(A)$. Then $1 - (frwg\text{-}int(A))(x) < t$ implies that $[frwg\text{-}int(A)](x) + t > 1$ and so $frwg\text{-}int(A)qx_t$. So there exists at least one $frwg$-open set $F \leq A$ with $x_t qF$ and so $x_t qA$. As $x_t \in frwg\text{-}cl(1_X \setminus A), Fq(1_X \setminus A)$ implies that $Aq(1_X \setminus A)$, a contradiction. Hence $frwg\text{-}cl(1_X \setminus A) \leq 1_X \setminus frwg\text{-}int(A)$...(1) Conversely, let $x_t \in 1_X \setminus frwg\text{-}int(A)$. Then $1 - [(frwg\text{-}int(A)](x) \geq t$ implies that $x_t \not q(frwg\text{-}int(A))$ and so $x_t \not qF$ for every $frwg$-open set F contained in A ... (2).
Let U be any $frwg$-closed set in X such that $1_X \setminus A \leq U$. Then $1_X \setminus U \leq A$. Now $1_X \setminus U$ is $frwg$-open set in X contained in A. By (2), $x_t \not q(1_X \setminus U)$ implies that $x_t \in U$ and so $x_t \in frwg\text{-}cl(1_X \setminus A)$ and so $1_X \setminus frwg\text{-}int(A) \leq frwg\text{-}cl(1_X \setminus A)$...(3). Combining (1) and (3), (i) follows.
(ii) Putting $1_X \setminus A$ for A in (i), we get $frwg\text{-}cl(A) = 1_X \setminus frwg\text{-}int(1_X \setminus A)$ implies that $frwg\text{-}int(1_X \setminus A) = 1_X \setminus frwg\text{-}cl(A)$.

Let us now recall the following definition from [29] for ready references.

Definition 4.6 [29]. A function $f : X \to Y$ is called fuzzy open (resp., fuzzy closed) if $f(U)$ is fuzzy open (resp., fuzzy closed) set in Y for every fuzzy open (resp., fuzzy closed) set U in X.

Let us now introduce the following concept.

Definition 4.7. A function $h : X \to Y$ is called $frwg$-open function if $h(U)$ is $frwg$-open set in Y for every fuzzy open set U in X.

Remark 4.8. Since fuzzy open set is $frwg$-open set, we say that fuzzy open function is $frwg$-open function. But the converse need not be true, as it seen from the following example.

Example 4.9. There is an $frwg$-open function which is not a fuzzy open function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_2) is $frwg$-open set in (X, τ_2), clearly i is $frwg$-open function. But $A \in \tau_1$, $i(A) = A \notin \tau_2$ implies that i is not a fuzzy open function.

Theorem 4.10. For a bijective function $h : X \to Y$, the following statements are equivalent:
(i) h is $frwg$-open,
(ii) $h(intA) \leq frwg\text{-}int(h(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy open set U in X containing x_α, there exists an $frwg$-open set V in Y containing $h(x_\alpha)$ such that $V \leq h(U)$.

Proof (i) $\Rightarrow$ (ii). Let $A \in I^X$. Then $intA$ is a fuzzy open set in X. By (i), $h(intA)$ is $frwg$-open set in Y. Since $h(intA) \leq h(A)$ and $frwg\text{-}int(h(A))$ is the union of all $frwg$-open sets contained in $h(A)$, we have $h(intA) \leq frwg\text{-}int(h(A))$.
(ii) $\Rightarrow$ (i). Let U be any fuzzy open set in X. Then $h(U) = h(intU) \leq frwg\text{-}int(h(U))$ (by (ii))implies that $h(U)$ is $frwg$-open set in Y and hence h is $frwg$-open function.
(ii) $\Rightarrow$ (iii). Let x_α be a fuzzy point in X, and U, a fuzzy open set in X such that $x_\alpha \in U$. Then $h(x_\alpha) \in h(U) = h(intU) \leq frwg\text{-}int(h(U))$ (by (ii)). Then $h(U)$ is $frwg$-open set in Y. Let $V = h(U)$. Then $h(x_\alpha) \in V$ and $V \leq h(U)$.
(iii) $\Rightarrow$ (i). Let U be any fuzzy open set in X and y_α, any fuzzy point in $h(U)$, i.e., $y_\alpha \in h(U)$. Then there exists unique $x \in X$ such that $h(x) = y$ (as h is bijective). Then $[h(U)](y) \geq \alpha$ implies that $U(h^{-1}(y)) \geq \alpha$. So $U(x) \geq \alpha$. Thus $x_\alpha \in U$. By (iii), there exists $frwg$-open set V in Y such that $h(x_\alpha) \in V$ and $V \leq h(U)$.

Then $h(x_\alpha) \in V = frwg\text{-}int(V) \leq frwg\text{-}int(h(U))$. Since y_α is taken arbitrarily and $h(U)$ is the union of all fuzzy points in $h(U)$, $h(U) \leq frwg\text{-}int(h(U))$ implies that $h(U)$ is $frwg$-open set in Y. Consequently, h is an $frwg$-open function.

Theorem 4.11. If $h : X \to Y$ is $frwg$-open, bijective function, then the following statements are true:
(i) for each fuzzy point x_α in X and each fuzzy open q-nbd U of x_α in X, there exists an $frwg$-open q-nbd V of $h(x_\alpha)$ in Y such that $V \leq h(U)$,
(ii) $h^{-1}(frwg\text{-}cl(B)) \leq cl(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy open q-nbd of x_α in X. Then $x_\alpha q U = intU$ implies that $h(x_\alpha) q h(intU) \leq frwg\text{-}int(h(U))$ (by Theorem 4.10 (i)$\Rightarrow$(ii)) implies that there exists at least one $frwg$-open q-nbd V of $h(x_\alpha)$ in Y with $V \leq h(U)$.
(ii) Let x_α be any fuzzy point in X such that $x_\alpha \notin cl(h^{-1}(B))$ for any $B \in I^Y$. Then there exists a fuzzy open q-nbd U of x_α in X such that $U \not{q} h^{-1}(B)$. Now

$$h(x_\alpha) q h(U) ...(1)$$

where $h(U)$ is $frwg$-open set in Y. Now $h^{-1}(B) \leq 1_X \setminus U$ which is a fuzzy closed set in X and so $B \leq h(1_X \setminus U)$ (as h is injective) $\leq 1_Y \setminus h(U) \Rightarrow B \not{q} h(U)$. Let $V = 1_Y \setminus h(U)$. Then $B \leq V$ which is $frwg$-closed set in Y. We claim that $h(x_\alpha) \notin V$. If possible, let $h(x_\alpha) \in V = 1_Y \setminus h(U)$. Then $1 - [h(U)](h(x)) \geq \alpha$ implies that $h(U) \not{q} h(x_\alpha)$, contradicting (1). So $h(x_\alpha) \notin V$ and so $h(x_\alpha) \notin frwg\text{-}cl(B)$. Then $x_\alpha \notin h^{-1}(frwg\text{-}cl(B))$. Hence $h^{-1}(frwg\text{-}cl(B)) \leq cl(h^{-1}(B))$.

Theorem 4.12. An injective function $h : X \to Y$ is $frwg$-open if and only if for each $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$, there exists an $frwg$-closed set V in Y such that $B \leq V$ and $h^{-1}(V) \leq F$.

Proof. Let $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$. Then $1_X \setminus h^{-1}(B) \geq 1_X \setminus F$ where $1_X \setminus F$ is a fuzzy open set in X and so $h(1_X \setminus F) \leq h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus B$ (as h is injective) where $h(1_X \setminus F)$ is an $frwg$-open set in Y. Let $V = 1_Y \setminus h(1_X \setminus F)$. Then V is $frwg$-closed set in Y such that $B \leq V$. Now $h^{-1}(V) = h^{-1}(1_Y \setminus h(1_X \setminus F)) = 1_X \setminus h^{-1}(h(1_X \setminus F)) \leq F$.

Conversely, let F be a fuzzy open set in X. Then $1_X \setminus F$ is a fuzzy closed set in X. We have to show that $h(F)$ is an $frwg$-open set in Y. Now $h^{-1}(1_Y \setminus h(F)) \leq 1_X \setminus F$ (as h is injective). By assumption, there exists an $frwg$-closed set V in Y such that

$$1_Y \setminus h(F) \leq V ...(1)$$

and $h^{-1}(V) \leq 1_X \setminus F$. Therefore, $F \leq 1_X \setminus h^{-1}(V)$ implies that

$$h(F) \leq h(1_X \setminus h^{-1}(V)) \leq 1_Y \setminus V ...(2)$$

(as h is injective). Combining (1) and (2), $h(F) = 1_Y \setminus V$ which is an $frwg$-open set in Y. Hence h is $frwg$-open function.

Definition 4.13. A function $h : X \to Y$ is called $frwg$-closed function if $h(A)$ is $frwg$-closed set in Y for each fuzzy closed set A in X.

Remark 4.14. Since fuzzy closed set is $frwg$-closed set in an fts, we can conclude that every fuzzy closed function is $frwg$-closed function, but the converse may not be true as it follows from Example 4.9. Here $1_X \setminus A \in \tau_1^c$, but $i(1_X \setminus A) = 1_X \setminus A \notin \tau_2^c$ and so i is not a fuzzy closed function. But since every fuzzy set in (X, τ_2) is $frwg$-closed set in (X, τ_2), clearly i is $frwg$-closed function.

Theorem 4.15. A bijective function $h : X \to Y$ is $frwg$-closed function if and only if $frwg$-$cl(h(A)) \leq h(clA)$, for all $A \in I^X$.

Proof. Let us suppose that $h : X \to Y$ be an $frwg$-closed function and $A \in I^X$. Then $h(cl(A))$ is $frwg$-closed set in Y. Since $h(A) \leq h(clA)$ and $frwg$-$cl(h(A))$ is the intersection of all $frwg$-closed sets in Y containing $h(A)$, we have $frwg$-$cl(h(A)) \leq h(clA)$.

Conversely, let for any $A \in I^X$, $frwg$-$cl(h(A)) \leq h(clA)$. Let U be any fuzzy closed set in X. Then $h(U) = h(clU) \geq frwg$-$cl(h(U)) \Rightarrow h(U)$ is an $frwg$-closed set in Y. Hence h is an $frwg$-closed function.

Theorem 4.16. If $h : X \to Y$ is an $frwg$-closed bijective function, then the following statements hold:
(i) for each fuzzy point x_α in X and each fuzzy closed set U in X with $x_\alpha \not{q} U$, there exists an $frwg$-closed set V in Y with $h(x_\alpha) \not{q} V$ such that $V \geq h(U)$,

(ii) $h^{-1}(frwg\text{-}int(B)) \geq int(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy closed set in X with $x_\alpha \ /qU = clU$ implies that $h(x_\alpha) \ /qh(clU) \geq frwg\text{-}cl(h(U))$ (by Theorem 4.15) and so $h(x_\alpha) \ /qV$ for some $frwg$-closed set V in Y with $V \geq h(U)$.
(ii). Let $B \in I^Y$ and x_α be any fuzzy point in X such that $x_\alpha \in int(h^{-1}(B))$. Then there exists a fuzzy open set U in X with $U \leq h^{-1}(B)$ such that $x_\alpha \in U$. Then $1_X \setminus U \geq 1_X \setminus h^{-1}(B)$. So $h(1_X \setminus U) \geq h(1_X \setminus h^{-1}(B))$ where $h(1_X \setminus U)$ is an $frwg$-closed set in Y. Let $V = 1_Y \setminus h(1_X \setminus U)$. Then V is an $frwg$-open set in Y and $V = 1_Y \setminus h(1_X \setminus U) \leq 1_Y \setminus h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus (1_Y \setminus B) = B$ (as h is injective). Now $U(x) \geq \alpha$ and so $x_\alpha \not{q}(1_X \setminus U)$. Then $h(x_\alpha) \not{q}h(1_X \setminus U)$. Hence $h(x_\alpha) \leq 1_Y \setminus h(1_X \setminus U) = V$. Consequently, $h(x_\alpha) \in V = frwg\text{-}int(V) \leq frwg\text{-}int(B)$ implies that $x_\alpha \in h^{-1}(frwg\text{-}int(B))$. Since x_α is taken arbitrarily, $int(h^{-1}(B)) \leq h^{-1}(frwg\text{-}int(B))$, for all $B \in I^Y$.

Remark 4.17. Composition of two $frwg$-closed (resp., $frwg$-open) functions need not be so, as it seen from the following example.

Example 4.18. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4$, $B(a) = 0.3, B(b) = 0.4, C(a) = C(b) = 0.6$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $frwg$-closed functions. Let $i_3 = i_2 \circ i_1 : (X, \tau_1) \to (X, \tau_3)$. We claim that i_3 is not an $frwg$-closed function. Now $1_X \setminus C \in \tau_1^c$. $(i_2 \circ i_1)(1_X \setminus C) = 1_X \setminus C \leq A \in FRO(X, \tau_3)$. But $cl_{\tau_3}(int_{\tau_3}(1_X \setminus C)) = 1_X \setminus A \not\leq A$. So $1_X \setminus C$ is not an $frwg$-closed set in (X, τ_3). Hence $i_2 \circ i_1$ is not an $frwg$-closed function.

Similarly we can show that $i_2 \circ i_1$ is not an $frwg$-open function though i_1 and i_2 are so.

Theorem 4.19. If $h_1 : X \to Y$ is fuzzy closed (resp., fuzzy open) function and $h_2 : Y \to Z$ is $frwg$-closed (resp., $frwg$-open) function, then $h_2 \circ h_1 : X \to Z$ is $frwg$-closed (resp., $frwg$-open) function.

Proof. Obvious.

Now to establish the mutual relationships of $frwg$-closed function with the functions defined in [3, 5, 6, 7, 11, 12, 14, 15, 13, 16], we have to recall the following definitions first.

Definition 4.20. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called an
(i) fg-closed function [3] if $h(A)$ is fg-closed set in Y for every $A \in \tau_1^c$,
(ii) $fg\beta$-closed function [7] if $h(A)$ is $fg\beta$-closed set in Y for every $A \in \tau_1^c$,
(iii) $f\beta g$-closed function [7] if $h(A)$ is $f\beta g$-closed set in Y for every $A \in \tau_1^c$,
(iv) $fg\alpha$-closed function [3] if $h(A)$ is $fg\alpha$-closed set in Y for every $A \in \tau_1^c$,
(v) $f\alpha g$-closed function [3] if $h(A)$ is $f\alpha g$-closed set in Y for every $A \in \tau_1^c$,
(vi) fgp-closed function [3] if $h(A)$ is fgp-closed set in Y for every $A \in \tau_1^c$,
(vii) fpg-closed function [3] if $h(A)$ is fpg-closed set in Y for every $A \in \tau_1^c$,
(viii) fgs-closed function [3] if $h(A)$ is fgs-closed set in Y for every $A \in \tau_1^c$,
(ix) fsg-closed function [3] if $h(A)$ is fsg-closed set in Y for every $A \in \tau_1^c$,
(x) fgs^*-closed function [5] if $h(A)$ is fgs^*-closed set in Y for every $A \in \tau_1^c$,
(xi) fs^*g-closed function [6] if $h(A)$ is fs^*g-closed set in Y for every $A \in \tau_1^c$,
(xii) $fg\gamma$-closed function [11] if $h(A)$ is $fg\gamma$-closed set in Y for every $A \in \tau_1^c$,
(xiii) $fg\gamma^*$-closed function [12] if $h(A)$ is $fg\gamma^*$-closed set in Y for every $A \in \tau_1^c$,
(xiv) fmg-closed function [13] if $h(A)$ is fmg-closed set in Y for every $A \in \tau_1^c$,
(xv) $fswg$-closed function [16] if $h(A)$ is $fswg$-closed set in Y for every $A \in \tau_1^c$,
(xvi) $f\pi g$-closed function [14] if $h(A)$ is $f\pi g$-closed set in Y for every $A \in \tau_1^c$,
(xvii) fwg-closed function [15] if $h(A)$ is fwg-closed set in Y for every $A \in \tau_1^c$.

Remark 4.21. It is clear from definitions that
(i) fg-closed function, fgp-closed function, fpg-closed function, $f\pi g$-closed function, fgs^*-closed function, fs^*g-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, fmg-closed function, fwg-closed function, $fswg$-closed function imply $frwg$-closed function. But the converses are not true, in general, follow from the following examples.
(ii) $frwg$-closed function is an independent concept of fgs-closed function, fsg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function, $fg\beta$-closed function, $f\beta g$-closed function follow from the following examples.

Example 4.22. There exists an $frwg$-closed function that none has the following properties :fg-closed function, $f\pi g$-closed function, fgs^*-closed function, fs^*g-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.7$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $1_X \setminus B \in \tau_1^c$, $i(1_X \setminus B) = 1_X \setminus B$. As $cl_{\tau_2}(int_{\tau_2}(1_X \setminus B)) = 0_X$, clearly $1_X \setminus B$ is $frwg$-closed set in (X,τ_2) implies that i is an $frwg$-closed function. Now $1_X \setminus B < A \in \tau_2$ (resp., $A \in F\pi O(X,\tau_2)$, $A \in FSO(X,\tau_2)$, A is an fg-open set in (X,τ_2), $A \in F\alpha O(X,\tau_2)$). But $cl_{\tau_2}(1_X \setminus B) = scl_{\tau_2}(1_X \setminus B) = \alpha cl_{\tau_2}(1_X \setminus B) = 1_X \setminus A \not\leq A$ implies that $1_X \setminus B$ is not an fg-closed set, $f\pi g$-closed set, fgs^*-closed set, fs^*g-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set in $(X,\tau_2) \Rightarrow i$ is not an fg-closed function, $f\pi g$-closed function, fgs^*-closed function, fs^*g-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function.

Example 4.23. There exists an $frwg$-closed function that none has the following properties :fgs-closed function, fsg-closed function, fpg-closed function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, B, C\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = B(b) = 0.5, C(a) = C(b) = 0.4$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $FSO(X,\tau_2) = \{0_X, 1_X, U\}$ where $U \geq A$ and so $FSC(X,\tau_2) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \leq 1_X \setminus A$. Now $1_X \setminus B, 1_X \setminus C \in \tau_1^c$, $i(1_X \setminus B) = 1_X \setminus B$ and $i(1_X \setminus C) = 1_X \setminus C$. Then $1_X \in FRO(X,\tau_2)$ only containing $1_X \setminus B$ and $1_X \setminus C$, clearly $1_X \setminus B$ and $1_X \setminus C$ are $frwg$-closed sets in (X,τ_2) implies that i is an $frwg$-closed function. Now $1_X \setminus B < A \in \tau_2$ (resp.,

$A \in FSO(X, \tau_2)$). But $scl_{\tau_2}(1_X \setminus B) = 1_X \not\leq A$ implies that $1_X \setminus B$ is not an fgs-closed set as well as fsg-closed set in (X, τ_2) and so i is not an fgs-closed function as well as an fsg-closed function. Also $1_X \setminus C \in \tau_1^c$, $i(1_X \setminus C) = 1_X \setminus C \leq 1_X \setminus C \in FPO(X, \tau_2)$. But as $1_X \setminus C \notin FPC(X, \tau_2)$, $pcl_{\tau_2}(1_X \setminus C) \not\leq 1_X \setminus C$ which shows that $1_X \setminus C$ is not an fpg-closed set in (X, τ_2) and hence i is not an fpg-closed function.

Example 4.24. There is an $frwg$-closed function which is not an fgp-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6$, $B(a) = 0.2, B(b) = 0.5, C(a) = 0.6, C(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_1^c$, $i(1_X \setminus C) = 1_X \setminus C < 1_X \in FRO(X, \tau_2)$ only and so $1_X \setminus C$ is an $frwg$-closed set in (X, τ_2) and so i is an $frwg$-closed function. Now $1_X \setminus C < A \in \tau_2$, But $pcl_{\tau_2}(1_X \setminus C) = 1_X \setminus B \not\leq A$ implies that $1_X \setminus C$ is not an fgp-closed set in (X, τ_2) and hence i is not an fgp-closed function.

Example 4.25. There is an $frwg$-closed function which is not an $fg\gamma^*$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4$, $B(a) = 0.4, B(b) = 0.3, C(a) = 0.6, C(b) = 0.5$. Then $X, \tau_1)$ and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_1^c, i(1_X \setminus C) = 1_X \setminus C < 1_X \in FRO(X, \tau_2)$ only and so $1_X \setminus C$ is an $frwg$-closed set in (X, τ_2) implies that i is an $frwg$-closed function. Now $1_X \setminus C \leq 1_X \setminus C \in FSO(X, \tau_2)$. But as $1_X \setminus C \notin F\gamma C(X, \tau_2)$, $\gamma cl_{\tau_2}(1_X \setminus C) \not\leq 1_X \setminus C$ and so $1_X \setminus C$ is not an $fg\gamma^*$-closed set in (X, τ_2). Hence i is not an $fg\gamma^*$-closed function.

Example 4.26. There exists an $frwg$-closed function that none has the following properties : $fg\gamma$-closed function, fmg-closed function, fwg-closed function, $fswg$-closed function, $f\beta g$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, C, D\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.2, B(b) = 0.5, C(a) = 0.5, C(b) = 0.45, D(a) = D(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C, 1_X \setminus D \in \tau_1^c, i(1_X \setminus C) = 1_X \setminus C$, $i(1_X \setminus D) = 1_X \setminus D$.

Now $1_X \in FRO(X, \tau_2)$ only containing $1_X \setminus C, 1_X \setminus D$ and so $1_X \setminus C, 1_X \setminus D$ are $frwg$-closed sets in (X, τ_2) and so i is an $frwg$-closed function. Now $1_X \setminus C < A \in \tau_2$ (resp., $A \in FSO(X, \tau_2)$ and A is an fg-open set in (X, τ_2)). But $\gamma cl_{\tau_2}(1_X \setminus C) = 1_X \not\leq A$ implies that $1_X \setminus C$ is not an $fg\gamma$-closed set in (X, τ_2). Hence i is not an $fg\gamma$-closed function. Again $cl_{\tau_2} int_{\tau_2}(1_X \setminus C) = 1_X \setminus B \not\leq A$ implies that $1_X \setminus C$ is not an fmg-closed set, fwg-closed set, $fswg$-closed set in (X, τ_2) and so i is not an fmg-closed function, fwg-closed function, $fswg$-closed function. Now $1_X \setminus D \leq 1_X \setminus D \in F\beta O(X, \tau_2)$. But as $1_X \setminus D \notin F\beta C(X, \tau_2)$, $\beta cl_{\tau_2}(1_X \setminus D) \not\leq 1_X \setminus D$. Consequently, $1_X \setminus D$ is not an $f\beta g$-closed set in (X, τ_2) and so i is not an $f\beta g$-closed function.

Example 4.27. There is an $frwg$-closed function which is not an $fg\beta$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6$, $B(a) = 0.2, B(b) = 0.55, C(a) = 0.5, C(b) = 0.45$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_1^c$, $i(1_X \setminus C) = 1_X \setminus C < 1_X \in FRO(X, \tau_2)$ only and so $1_X \setminus C$ is an $frwg$-closed set in (X, τ_2) implies that i is an $frwg$-closed function. Now $1_X \setminus C < A \in \tau_2$. But $\beta cl_{\tau_2}(1_X \setminus C) = 1_X \not\leq A$ and so $1_X \setminus C$ is not an $fg\beta$-closed set in (X, τ_2). Hence i is not an $fg\beta$-closed function.

Example 4.28. There is an fgs-closed function which is not an $frwg$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4$, $B(a) = 0.4, B(b) = 0.3, C(a) = C(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_1^c$, $i(1_X \setminus C) = 1_X \setminus C < A \in FRO(X, \tau_2)$. But $cl_{\tau_2} int_{\tau_2}(1_X \setminus C) = 1_X \setminus A \not\leq A$ and so $1_X \setminus C$ is not an $frwg$-closed set in (X, τ_2). Hence i is not an $frwg$-closed function. Now $FSO(X, \tau_2) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus A$ and so $FSC(X, \tau_2) = \{0_X, 1_X, 1_X \setminus U\}$ where $A \leq 1_X \setminus U \leq 1_X \setminus B$. Now $1_X \setminus C < A \in \tau_2$ and so $scl_{\tau_2}(1_X \setminus C) = A \leq A$ implies that $1_X \setminus C$ is an fgs-closed set in (X, τ_2). So i is an fgs-closed function.

Example 4.29. None of fsg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function implies $frwg$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, D\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.3, D(a) =$

$0.5, D(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus D \in \tau_1$, $i(1_X \setminus D) = 1_X \setminus D \leq A \in FRO(X, \tau_2)$. But $cl_{\tau_2} int_{\tau_2}(1_X \setminus D) = 1_X \setminus A \not\leq A$ implies that $1_X \setminus D$ is not an $frwg$-closed set in (X, τ_2) and so i is not an $frwg$-closed function. Now $1_X \setminus D \in FSC(X, \tau_2)$ and also $1_X \setminus D \in F\gamma C(X, \tau_2)$. So $1_X \setminus D$ is fsg-closed set as well as $fg\gamma$-closed set, $fg\gamma^*$-closed set in (X, τ_2). Hence i is an fsg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function.

Example 4.30. None of $fg\beta$-closed function, $f\beta g$-closed function implies $frwg$-closed function

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, D\}$, $\tau_2 = \{0_X, 1_X, A, B, C\}$ where $A(a) = 0.3, A(b) = 0.4, B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5, D(a) = D(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus D \in \tau_1^c$, $i(1_X \setminus D) = 1_X \setminus D \leq B \in FRO(X, \tau_2)$. But as $cl_{\tau_2} int_{\tau_2}(1_X \setminus D) = 1_X \setminus C \not\leq B$, $1_X \setminus D$ is not an $frwg$-closed set in $(X, \tau_2) \Rightarrow i$ is not an $frwg$-closed function. But as $int_{\tau_2} cl_{\tau_2} int_{\tau_2}(1_X \setminus D) = 1_X \setminus D$, $1_X \setminus D \in F\beta C(X, \tau_2)$ implies that $1_X \setminus D$ is an $fg\beta$-closed set as well as $f\beta g$-closed set in (X, τ_2) and so i is an $fg\beta$-closed function as well as $f\beta g$-closed function.

Remark 4.31. (i) Let $h : X \to Y$ be an $frwg$-closed function where Y is an frT_g-space. Then h is a fuzzy closed function, fg-closed function, $f\pi g$-closed function, fgs^*-closed function, fs^*g-closed function, fgs-closed function, fsg-closed function, $fg\alpha$-closed function, $f\alpha g$-closed function, $fg\beta$-closed function, $f\beta g$-closed function, fmg-closed function, fwg-closed function, $fswg$-closed function, fgp-closed function, fpg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function.
(ii) Let $h : X \to Y$ be a function where Y is an fT_b-space (resp., fT_{sg}-space, fT_γ-space, fT_{γ^*}-space, $f\beta T_b$-space, fT_β-space). Then if h is an fgs-closed function (resp., fsg-closed function, $fg\gamma$-closed function, $fg\gamma^*$-closed function, $f\beta g$-closed function, $fg\beta$-closed function), h is an $frwg$-closed function.

5. $frwg$-Regular, $frwg$-Normal and $frwg$-Compact Spaces

Here we introduce and study two new types of separation axioms and a new type of compactness. These three concepts are weak concepts of fuzzy regularity [24], fuzzy normality [23] and fuzzy compactness [18, 22].

Definition 5.1. An fts (X, τ) is said to be $frwg$-regular space if for any fuzzy point x_t in X and each $frwg$-closed set F in X with $x_t \notin F$, there exist $U, V \in FRO(X, \tau)$ such that $x_t \in U, F \leq V$ and $U \not{q} V$.

Theorem 5.2. In an fts (X, τ), the following statements are equivalent:
(i) X is $frwg$-regular,
(ii) for each fuzzy point x_t in X and any $frwg$-open q-nbd U of x_t, there exists $V \in FRO(X, \tau)$ such that $x_t \in V$ and $cl(intV) \leq U$,
(iii) for each fuzzy point x_t in X and each $frwg$-closed set A of X with $x_t \notin A$, there exists $U \in FRO(X, \tau)$ with $x_t \in U$ such that $cl(intU) \not{q} A$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any $frwg$-open q-nbd of x_t. Then $x_t q U$ implies that $U(x) + t > 1$, i.e., $x_t \notin 1_X \setminus U$ which is an $frwg$-closed set in X. By (i), there exist $V, W \in FRO(X, \tau)$ such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not{q} W$. Then $V \leq 1_X \setminus W$ and so $cl(intV) \leq cl(int(1_X \setminus W)) = 1_X \setminus W \leq U$.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A, an $frwg$-closed set in X with $x_t \notin A$. Then $A(x) < t$, i.e., $x_t q(1_X \setminus A)$ which being $frwg$-open set in X is $frwg$-open q-nbd of x_t. So by (ii), there exists $V \in FRO(X, \tau)$ such that $x_t \in V$ and $cl(intV) \leq 1_X \setminus A$. Then $cl(intV) \not{q} A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any $frwg$-closed set in X with $x_t \notin F$. Then by (iii), there exists $U \in FRO(X, \tau)$ such that $x_t \in U$ and $cl(intU) \not{q} F$. Then $F \leq 1_X \setminus cl(intU)$ (=V, say). Now $int(clV) = int(cl(1_X \setminus cl(intV))) = 1_X \setminus cl(int(cl(intU)))$. Now $cl(int(cl(intU))) \leq cl(cl(intU)) = cl(intU)$. Again $cl(int(cl(intU))) = cl(int(cl(intU))) \geq cl(int(intU)) = cl(intU)$. So $cl(int(cl(intU))) = cl(intU)$. Then $int(clV) = 1_X \setminus cl(intU) = V$. So $V \in FRO(X, \tau)$ and $V \not{q} U$ as $U \not{q}(1_X \setminus cl(intU))$. Consequently, X is $frwg$-regular space.

Definition 5.3. An fts (X, τ) is called $frwg$-normal space if for each pair of $frwg$-closed sets A, B in X with $A \ /qB$, there exist $U, V \in FRO(X, \tau)$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Theorem 5.4. An fts (X, τ) is $frwg$-normal space if and only if for every $frwg$-closed set F and $frwg$-open set G in X with $F \leq G$, there exists $H \in FRO(X, \tau)$ such that $F \leq H \leq cl(intH) \leq G$.

Proof. Let X be $frwg$-normal space and let F be $frwg$-closed set and G be $frwg$-open set in X with $F \leq G$. Then $F \not q(1_X \setminus G)$ where $1_X \setminus G$ is $frwg$-closed set in X. By hypothesis, there exist $H, T \in FRO(X, \tau)$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not q T$. Then $H \leq 1_X \setminus T \leq G$. Therefore, $F \leq H \leq cl(intH) \leq cl(int(1_X \setminus T)) = 1_X \setminus T \leq G$.

Conversely, let A, B be two $frwg$-closed sets in X with $A \not q B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists $H \in FRO(X, \tau)$ such that $A \leq H \leq cl(intH) \leq 1_X \setminus B$ implies that $A \leq H, B \leq 1_X \setminus cl(intH)$ (=V, say). Then $V \in FRO(X, \tau)$ and so $B \leq V$. Also as $H \not q(1_X \setminus cl(intH))$, $H \not q V$. Consequently, X is $frwg$-normal space.

Let us now recall the following definitions from [18, 22] for ready references.

Definition 5.5. Let (X, τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\bigcup \mathcal{U} \geq A$ [22]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy regular open, $frwg$-open) in X, then $\mathcal{U}$ is called a fuzzy open [22] (resp., fuzzy regular open [1], $frwg$-open) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [18].

Definition 5.6. Let (X, τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A (resp., of X) is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [22]. If, in particular $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [18].

Definition 5.7. Let (X, τ) be an fts and $A \in I^X$. Then A is called fuzzy compact [18] (resp., fuzzy almost compact [19], fuzzy nearly compact [25]) set if every fuzzy open (resp., fuzzy open, fuzzy regular open) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup\limits_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [18] (resp., fuzzy almost compact [19], fuzzy nearly compact [20]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup\limits_{U \in \mathcal{U}_0} clU = 1_X$,

$\bigcup \mathcal{U}_0 = 1_X$).

Let us now introduce the following concept.

Definition 5.8. Let (X, τ) be an fts and $A \in I^X$. Then A is called $frwg$-compact if every fuzzy cover $\mathcal{U}$ of A by $frwg$-open sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of $frwg$-compact space X.

Theorem 5.9. Every $frwg$-closed set in an $frwg$-compact space X is $frwg$-compact.

Proof. Let $A(\in I^X)$ be an $frwg$-closed set in an $frwg$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $frwg$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $frwg$-open sets of X. As X is $frwg$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Hence A is $frwg$-compact set.

Next we recall the following two definitions from [24, 23] for ready references.

Definition 5.10 [24]. An fts (X, τ) is called fuzzy regular space if for each fuzzy point x_t in X and each fuzzy closed set F in X with $x_t \notin F$, there exist $U, V \in \tau$ such that $x_t \in U$, $F \leq V$ and $U \not{q} V$.

Definition 5.11 [23]. An fts (X, τ) is called fuzzy normal space if for each pair of fuzzy closed sets A, B of X with $A \not{q} B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Remark 5.12. It is clear from above discussion that (i) $frwg$-regular (resp., $frwg$-normal) space is fuzzy regular (resp., fuzzy normal) space.
(ii) $frwg$-compact space is fuzzy compact, fuzzy almost compact and fuzzy nearly compact space.
But the converses are not true, in general, follow from the following example .

Example 5.13. Let $X = \{a\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Clearly (X, τ) is fuzzy regular space, fuzzy normal, fuzzy compact,

fuzzy almost compact and fuzzy nearly compact space. Here every fuzzy set is $frwg$-open as well as $frwg$-closed set in X. Consider the fuzzy set F defined by $F(a) = 0.1$ and the fuzzy point $a_{0.2}$. Then $a_{0.2} \notin F$ which is $frwg$-closed set in X. But there does not exist fuzzy regular open sets U, V in X such that $a_{0.2} \leq U, F \leq V$ and $U \not{q} V$. So (X, τ) is not an $frwg$-regular space.
Again consider two $frwg$-closed sets A, B defined by $A(a) = 0.4, B(a) = 0.5$. Then A, B are $frwg$-closed sets in X with $A \not{q} B$. But there do not exist $U, V \in FRO(X, \tau)$ such that $A \leq U, B \leq V$ and $U \not{q} V$. Hence (X, τ) is not an $frwg$-normal space. Next let $\mathcal{U} = \{U_n : n \in N\}$ where $U_n(a) = \frac{n}{n+1}$, for all $n \in N$. Then $\mathcal{U}$ is an $frwg$-open cover of X. But $\mathcal{U}$ has no finite subcover and so (X, τ) is not an $frwg$-compact space.

6. $frwg$-Continuous, $frwg$-Irresolute, Strongly $frwg$-Continuous and Weakly $frwg$-Continuous Functions

In this section four different types of functions are introduced and studied. The applications of these functions on $frwg$-regular space, $frwg$-normal space and $frwg$-compact spaces are shown.

Now we first introduce the following concept.

Definition 6.1. A function $h : X \to Y$ is said to be $frwg$-continuous function if $h^{-1}(V)$ is $frwg$-closed set in X for every fuzzy closed set V in Y.

Theorem 6.2. Let $h : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:
(i) h is $frwg$-continuous function,
(ii) for each fuzzy point x_α in X and each fuzzy open nbd V of $h(x_\alpha)$ in Y , there exists an $frwg$-open nbd U of x_α in X such that $h(U) \leq V$,
(iii) $h(frwg\text{-}cl(A)) \leq cl(h(A))$, for all $A \in I^X$,
(iv) $frwg\text{-}cl(h^{-1}(B)) \leq h^{-1}(clB)$, for all $B \in I^Y$.

Proof (i) $\Rightarrow$ (ii). Let x_α be a fuzzy point in X and V, any fuzzy open nbd of $h(x_\alpha)$ in Y. Then $x_\alpha \in h^{-1}(V)$ which is $frwg$-open set in X (by (i)). Let $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.

(ii) $\Rightarrow$ (i). Let A be any fuzzy open set in Y and x_α, a fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$ where A is a fuzzy open nbd of $h(x_\alpha)$ in Y. By (ii), there exists an $frwg$-open nbd U of x_α in X such that $h(U) \leq A$. Then $x_\alpha \in U \leq h^{-1}(A)$ implies that $x_\alpha \in U = frwg\text{-}int(U) \leq frwg\text{-}int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq frwg\text{-}int(h^{-1}(A))$, so that $h^{-1}(A)$ is an $frwg$-open set in X. Hence h is an $frwg$-continuous function.
(i) $\Rightarrow$ (iii). Let $A \in I^X$. Then $cl(h(A))$ is a fuzzy closed set in Y. By (i), $h^{-1}(cl(h(A)))$ is $frwg$-closed set in X. Now $A \leq h^{-1}(h(A)) \leq h^{-1}(cl(h(A)))$ and so $frwg\text{-}cl(A) \leq frwg\text{-}cl(h^{-1}(cl(h(A)))) = h^{-1}(cl(h(A)))$ implies that $h(frwg\text{-}cl(A)) \leq cl(h(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy closed set in Y. Put $U = h^{-1}(V)$. Then $U \in I^X$. By (iii), $h(frwg\text{-}cl(U)) \leq cl(h(U)) = cl(h(h^{-1}(V))) \leq clV = V$ implies that $frwg\text{-}cl(U) \leq h^{-1}(V) = U$. So U is $frwg$-closed set in X. Hence h is $frwg$-continuous function.
(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (iii), $h(frwg\text{-}cl(A)) \leq cl(h(A)) \Rightarrow h(frwg\text{-}cl(h^{-1}(B))) \leq cl(h(h^{-1}(B))) \leq clB$ implies that $frwg\text{-}cl(h^{-1}(B)) \leq h^{-1}(clB)$.
(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $h(A) \in I^Y$. By (iv), $frwg\text{-}cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A)))$. So $frwg\text{-}cl(A) \leq frwg\text{-}cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A)))$. Hence $h(frwg\text{-}cl(A)) \leq cl(h(A))$.

Remark 6.3. Composition of two $frwg$-continuous functions need not be so, as it seen from the following example.

Example 6.4. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.45, B(a) = 0.2, B(b) = 0.3, C(a) = 0.5, C(b) = 0.6$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Then clearly i_1 and i_2 are $frwg$-continuous functions. Now $1_X \setminus C \in \tau_3^c$. So $(i_2 \circ i_1)^{-1}(1_X \setminus C) = 1_X \setminus C \leq A \in FRO(X, \tau_1)$. But $cl_{\tau_1}(int_{\tau_1}(1_X \setminus C)) = 1_X \setminus A \not\leq A$ implies that $1_X \setminus C$ is not an $frwg$-closed set in (X, τ_1). Hence $i_2 \circ i_1$ is not an $frwg$-continuous function.

Let us now recall the following definition from [18] for ready references.

Definition 6.5 [18]. A function $h : X \to Y$ is called fuzzy continuous function if $h^{-1}(V)$ is fuzzy closed set in X for every fuzzy closed set V in Y.

Remark 6.6. Since every fuzzy closed set is $frwg$-closed set, it is clear that fuzzy continuous function is $frwg$-continuous function. But the converse is not necessarily true , follows from the following example.

Example 6.7. $frwg$-continuous function $\not\Rightarrow$ fuzzy continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $frwg$-closed set in (X, τ), clearly i is an $frwg$-continuous function. But $A \in \tau_2^c$, $i^{-1}(A) = A \notin \tau_1^c$ and hence i is not fuzzy continuous function.

Theorem 6.8. If $h_1 : X \to Y$ is $frwg$-continuous function and $h_2 : Y \to Z$ is fuzzy continuous function, then $h_2 \circ h_1 : X \to Z$ is $frwg$-continuous function.

Proof. Obvious.

Theorem 6.9. If a bijective function $h : X \to Y$ is $frwg$-continuous, fuzzy open function from an $frwg$-regular space X onto an fts Y, then Y is fuzzy regular space.

Proof. Let y_α be a fuzzy point in Y and F, a fuzzy closed set in Y with $y_\alpha \notin F$. As h is bijective, there exists unique $x \in X$ such that $h(x) = y$. So $h(x_\alpha) \notin F$ implies that $x_\alpha \notin h^{-1}(F)$ where $h^{-1}(F)$ is $frwg$-closed set in X (as h is an $frwg$-continuous function). As X is $frwg$-regular space, there exist $U, V \in FRO(X, \tau)$ such that $x_\alpha \in U, h^{-1}(F) \leq V$ and $U \not{q} V$. Since fuzzy regular open set is fuzzy open set, $U, V \in (X, \tau)$. Then $h(x_\alpha) \in h(U)$, $F = h(h^{-1}(F)) \leq h(V)$ (as h is bijective) and $h(U) \not{q} h(V)$ where $h(U)$ and $h(V)$ are fuzzy open sets in Y. (Indeed, $h(U) q h(V)$, then there exists $z \in Y$ such that $[h(U)](z) + [h(V)](z) > 1$ and so $U(h^{-1}(z)) + V(h^{-1}(z)) > 1$ as h is bijective which shows that UqV, a contradiction). Hence Y is a fuzzy regular space.

In a similar manner we can state the following theorem easily the proof of which are same as that of Theorem 6.9.

Theorem 6.10. If a bijective function $h : X \to Y$ is $frwg$-continuous, fuzzy open function from an $frwg$-normal space X onto an fts Y, then Y is fuzzy normal space.

Let us now recall the following definition from [10] for ready references.

Definition 6.11 [10]. A function $h : X \to Y$ is said to be fuzzy R-open function if $h(U) \in FRO(Y)$ for all $U \in FRO(X)$.

Theorem 6.12. If a bijective function $h : X \to Y$ is $frwg$-continuous, fuzzy R-open function from an $frwg$-regular (resp., $frwg$-normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.13. If a bijective function $h : X \to Y$ is $frwg$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal), frT_g-space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Definition 6.14. A function $h : X \to Y$ is called $frwg$-irresolute function if $h^{-1}(U)$ is an $frwg$-open set in X for every $frwg$-open set U in Y.

Theorem 6.15. A function $h : X \to Y$ is $frwg$-irresolute function if and only if for each fuzzy point x_α in X and each $frwg$-open nbd V in Y of $h(x_\alpha)$, there exists an $frwg$-open nbd U in X of x_α such that $h(U) \leq V$.

Proof. Let $h : X \to Y$ be an $frwg$-irresolute function. Let x_α be a fuzzy point in X and V be any $frwg$-open nbd of $h(x_\alpha)$ in Y. Then $h(x_\alpha) \in V$ implies that $x_\alpha \in h^{-1}(V)$ which being an $frwg$-open set in X is an $frwg$-open nbd of x_α in X. Put $U = h^{-1}(V)$. Then U is an $frwg$-open nbd of x_α in X and $h(U) = h(h^{-1}(V)) \leq V$.

Conversely, let A be an $frwg$-open set in Y and x_α be any fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$. By hypothesis, there exists an $frwg$-open nbd U of x_α in X such that $h(U) \leq A$ and so $x_\alpha \in U = frwg\text{-}int(U) \leq frwg\text{-}int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq frwg\text{-}int(h^{-1}(A))$. Then $h^{-1}(A) = frwg\text{-}int(h^{-1}(A))$. Then $h^{-1}(A)$ is $frwg$-open set in X. Hence h is an $frwg$-irresolute function.

Now we state the following two theorems easily the proofs of which are similar to that of Theorem 6.9.

Theorem 6.16. If a bijective function $h : X \to Y$ is $frwg$-irresolute, fuzzy R-open function from an $frwg$-regular (resp., $frwg$-normal) space X onto an fts Y, then Y is $frwg$-regular (resp., $frwg$-normal) space.

Theorem 6.17. If a bijective function $h : X \to Y$ is $frwg$-irresolute, fuzzy R-open function from an $frwg$-regular (resp., $frwg$-normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.18. If a bijective function $h : X \to Y$ is $frwg$-irresolute, fuzzy open function from a fuzzy regular (resp., fuzzy normal), frT_g-space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.19. Let $h : X \to Y$ be an $frwg$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $frwg$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \leq \bigcup\limits_{\alpha\in\Lambda} U_\alpha$ implies that $A \leq h^{-1}(\bigcup\limits_{\alpha\in\Lambda} U_\alpha) = \bigcup\limits_{\alpha\in\Lambda} h^{-1}(U_\alpha)$. Then $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $frwg$-open sets of X as h is an $frwg$-continuous function. As A is $frwg$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \leq \bigcup_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)$ and so $h(A) \leq h(\bigcup_{\alpha\in\Lambda_0} h^{-1}(U_\alpha) \leq \bigcup_{\alpha\in\Lambda_0} U_\alpha$. Hence $h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Since fuzzy open set is $frwg$-open, we can state the following theorems easily the proofs of which are same as that of Theorem 6.19.

Theorem 6.20. Let $h : X \to Y$ be an $frwg$-irresolute function from X onto an fts Y and $A(\in I^X)$ be an $frwg$-compact set in X. Then $h(A)$ is $frwg$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.21. Let $h : X \to Y$ be an $frwg$-continuous function from an $frwg$-compact space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.22. Let $h : X \to Y$ be an $frwg$-irresolute function from an $frwg$-compact space X onto an fts Y . Then Y is $frwg$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.23. Let $h : X \to Y$ be an $frwg$-continuous function from a fuzzy compact, frT_g-space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.24. Let $h : X \to Y$ be an $frwg$-irresolute function from a fuzzy compact, frT_g-space X onto an fts Y . Then Y is $frwg$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Remark 6.25. It is clear from definitions that (i) $frwg$-irresolute function is $frwg$-continuous function, but the converse may not be true, as it seen from the following example.
Also (ii) fuzzy continuity and $frwg$-irresoluteness are independent concepts follow from the following examples.

Example 6.26. None of Fuzzy continuous function, $frwg$-continuous function implies $frwg$-irresolute function

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.2, B(b) = 0.3$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is $frwg$-continuous as well as fuzzy continuous function. Now every fuzzy set in (X, τ_2) is $frwg$-closed set in $(X.\tau_2)$. Consider the fuzzy set C defined by $C(a) = 0.5, C(b) = 0.3$. Then C is $frwg$-closed set in (X, τ_2). Now $i^{-1}(C) = C < A \in FRO(X, \tau_1)$. But $cl_{\tau_1}(int_{\tau_1} C) = 1_X \setminus A \not\leq A$ and so C is not an $frwg$-closed set in (X, τ_1). Hence i is not an $frwg$-irresolute function.

Example 6.27. There exists an $frwg$-irresolute function which is not a fuzzy continuous function
Consider Example 6.7. Here i is not fuzzy continuous function. As every fuzzy set in (X, τ_1) is $frwg$-closed set in (X, τ_1), i is clearly an $frwg$-irresolute function.

Theorem 6.28. Let $h : X \to Y$ be an $frwg$-continuous function where Y is an frT_g-space. Then h is $frwg$-irresolute function.

Proof. Obvious.

It is clear from definition that composition of two $frwg$-irresolute functions is $frwg$-irresolute function.

Theorem 6.29. If $h_1 : X \to Y$ is $frwg$-irresolute function and $h_2 : Y \to Z$ is $frwg$-continuous function, then $h_2 \circ h_1 : X \to Z$ is an $frwg$-continuous function.

Proof. Obvious.

Now we introduce a strong version of $frwg$-continuous function as follows.

Definition 6.30. A function $h : X \to Y$ is called strongly $frwg$-continuous function if $h^{-1}(V)$ is fuzzy closed set in X for every $frwg$-closed set V in Y.

Theorem 6.31. A function $h : X \to Y$ is strongly $frwg$-continuous function if and only if for each fuzzy point x_α in X and each $frwg$-open nbd V in Y of $h(x_\alpha)$, there exists a fuzzy open nbd

U in X of x_α such that $h(U) \leq V$.

Proof. Let $h : X \to Y$ be a strongly $frwg$-continuous function. Let x_α be a fuzzy point in X and V be any $frwg$-open nbd of $h(x_\alpha)$ in Y. Then $h(x_\alpha) \in V$. So $x_\alpha \in h^{-1}(V)$ which being a fuzzy open set in X is a fuzzy open nbd of x_α in X. Put $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.

Conversely, let A be an $frwg$-open set in Y and x_α be any fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$. By hypothesis, there exists a fuzzy open nbd U of x_α in X such that $h(U) \leq A$ implies that $x_\alpha \in U = intU \leq int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq int(h^{-1}(A))$ implies that $h^{-1}(A) = int(h^{-1}(A))$. So $h^{-1}(A)$ is fuzzy open set in X. Hence h is a strongly $frwg$-continuous function.

Remark 6.32. It is clear from above discussion that
(i) composition of two strongly $frwg$-continuous functions is also so,
(ii) strongly $frwg$-continuous function implies fuzzy continuous, $frwg$-continuous and $frwg$-irresolute functions. But the converses are not true, in general, follow from the following examples.

Example 6.33. None of Fuzzy continuity, $frwg$-continuity implies strongly $frwg$-continuity
Consider Example 6.26. Here i is fuzzy continuous as well as $frwg$-continuous function. Now every fuzzy set in (X, τ_2) is $frwg$-closed set in (X, τ_2). Consider the fuzzy set C defined by $C(a) = 0.5, C(b) = 0.3$. Then C is $frwg$-closed set in (X, τ_2). Now $i^{-1}(C) = C \notin \tau_1^c$. Hence i is not strongly $frwg$-continuous function.

Example 6.34. There exists an $frwg$-irresolute function which is not a strongly $frwg$-continuous function
Consider Example 6.7. Since every fuzzy set in (X, τ_1) is $frwg$-closed set in (X, τ_1), i is clearly $frwg$-irresolute function. But A is $frwg$-closed set in (X, τ_2), $i^{-1}(A) = A \notin \tau_1^c$. Hence i is not strongly $frwg$-continuous function.

Theorem 6.35. If $h_1 : X \to Y$ is strongly $frwg$-continuous function and $h_2 : Y \to Z$ is $frwg$-continuous function, then $h_2 \circ h_1 : X \to Z$ is fuzzy continuous function.

Proof. Obvious.

Since fuzzy open set is $frwg$-open set, we have the following theorems.

Theorem 6.36. If a bijective function $h : X \to Y$ is strongly $frwg$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is $frwg$-regular (resp., $frwg$-normal) space.

Theorem 6.37. If a bijective function $h : X \to Y$ is strongly $frwg$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Since fuzzy regular open set is fuzzy open set, we state the following theorem easily.

Theorem 6.38. If a bijective function $h : X \to Y$ is strongly $frwg$-continuous, fuzzy R-open function from an $frwg$-regular (resp., $frwg$-normal) space X onto an fts Y, then Y is $frwg$-regular (resp., $frwg$-normal) space.

Theorem 6.39. Let $h : X \to Y$ be a strongly $frwg$-continuous function from an fts X onto an fts Y and $A(\in I^X)$ be a fuzzy compact set in X. Then $h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, $frwg$-compact) set in Y.

Theorem 6.40. Let $h : X \to Y$ be a strongly $frwg$-continuous function from a fuzzy compact space X onto an fts Y. Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, $frwg$-compact) space.

Now we introduce a weak form of $frwg$-continuous function.

Definition 6.41. A function $h : X \to Y$ is called weakly $frwg$-continuous function if $h^{-1}(U)$ is $frwg$-closed set in X for every $U \in FRC(Y)$.

Theorem 6.42. A function $h : X \to Y$ is weakly $frwg$-continuous if and only if for each fuzzy point x_α in X and each fuzzy regular open set V in Y with $h(x_\alpha) \in V$, there exists an $frwg$-open nbd U of x_α in X such that $h(U) \leq V$.

Proof. The proof is same as that of Theorem 6.31.

Note 6.43. Composition of two weakly $frwg$-continuous functions need not be so, as it seen from the following example.

Example 6.44. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, C, D\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.2, B(b) = 0.3, C(a) = 0.5, C(b) = 0.3, D(a) = 0.5, D(b) = 0.7$. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are weakly $frwg$-continuous functions. Now $1_X \setminus D \in FRC(X, \tau_3), (i_2 \circ i_1)^{-1}(1_X \setminus D) = 1_X \setminus D \leq A \in FRO(X, \tau_1)$. But $cl_{\tau_1}(int_{\tau_1}(1_X \setminus D)) = 1_X \setminus A \not\leq A$ implies that $1_X \setminus D$ is not an $frwg$-closed set in (X, τ_1). Hence $i_2 \circ i_1$ is not an weakly $frwg$-continuous function.

Remark 6.45. Fuzzy continuity, $frwg$-continuity, $frwg$-irresoluteness, strongly $frwg$-continuity imply weakly $frwg$-continuity, but the converses are not true, in general, follow from the following example.

Example 6.46. There exists a weakly $frwg$-continuous function that has none of the following properties : fuzzy continuous function, $frwg$-continuous function, $frwg$-irresolute function, strongly $frwg$-continuous function

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.3, B(b) = 0.4, C(a) = C(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FRO(X, \tau_1) = \{0_X, 1_X\}$. Clearly i is weakly $frwg$-continuous function. Now $1_X \setminus C \in \tau_2^c, i^{-1}(1_X \setminus C) = 1_X \setminus C < A \in FRO(X, \tau_1)$. But $cl_{\tau_1}(int_{\tau_1}(1_X \setminus C)) = 1_X \setminus A \not\leq A$. So $1_X \setminus C$ is an $frwg$-closed set in (X, τ_1). Hence i is not an $frwg$-continuous function. Again as $1_X \setminus C < 1_X \in FRO(X, \tau_2)$ only, $1_X \setminus C$ is an $frwg$-closed set in (X, τ_2). But as $1_X \setminus C$ is not an $frwg$-closed set in (X, τ_1), i is not an $frwg$-irresolute function. Also as $1_X \setminus C \notin \tau_1^c$, we conclude that i is not fuzzy continuous function as well as strongly $frwg$-continuous function.

Theorem 6.47. Let $h : X \to Y$ be a weakly $frwg$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $frwg$-compact set in X. Then $h(A)$ is a fuzzy nearly compact set in Y.

Theorem 6.48. Let $h : X \to Y$ be a weakly $frwg$-continuous function from an frT_g-space X onto an fts Y and $A(\in I^X)$ be a fuzzy compact set in X. Then $h(A)$ is a fuzzy nearly compact set in Y.

Theorem 6.49. Let $h : X \to Y$ be a weakly $frwg$-continuous function from an $frwg$-compact space X onto an fts Y. Then Y is a fuzzy nearly compact space.

Theorem 6.50. Let $h : X \to Y$ be a weakly $frwg$-continuous function from a fuzzy compact, frT_g-space X onto an fts Y. Then Y is a fuzzy nearly compact space.

Lastly to establish the mutual relationship of $frwg$-continuous function with the functions defined in [3, 5, 6, 7, 11, 12, 14, 15, 13, 16], we first recall the definitions of the functions defined in [3, 5, 6, 7, 11, 12, 14, 15, 13, 16].

Definition 6.51. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called
(i) fg-continuous function [3] if $h^{-1}(V)$ is fg-closed set in X for every $V \in \tau_2^c$,
(ii) $fg\beta$-continuous function [7] if $h^{-1}(V)$ is $fg\beta$-closed set in X for every $V \in \tau_2^c$,
(iii) $f\beta g$-continuous function [7] if $h^{-1}(V)$ is $f\beta g$-closed set in X for every $V \in \tau_2^c$,
(iv) fgp-continuous function [3] if $h^{-1}(V)$ is fgp-closed set in X for every $V \in \tau_2^c$,
(v) fpg-continuous function [3] if $h^{-1}(V)$ is fpg-closed set in X for every $V \in \tau_2^c$,
(vi) $fg\alpha$-continuous function [3] if $h^{-1}(V)$ is $fg\alpha$-closed set in X for every $V \in \tau_2^c$,
(vii) $f\alpha g$-continuous function [3] if $h^{-1}(V)$ is $f\alpha g$-closed set in X for every $V \in \tau_2^c$,
(viii) fgs-continuous function [3] if if $h^{-1}(V)$ is fgs-closed set in X

for every $V \in \tau_2^c$,
(ix) fsg-continuous function [3] if $h^{-1}(V)$ is fsg-closed set in X for every $V \in \tau_2^c$,
(x) fgs^*-continuous function [5] if $h^{-1}(V)$ is fgs^*-closed set in X for every $V \in \tau_2^c$,
(xi) fs^*g-continuous function [6] if $h^{-1}(V)$ is fs^*g-closed set in X for every $V \in \tau_2^c$,
(xii) $fg\gamma$-continuous function [11] if $h^{-1}(V)$ is $fg\gamma$-closed set in X for every $V \in \tau_2^c$,
(xiii) $fg\gamma^*$-continuous function [12] if $h^{-1}(V)$ is $fg\gamma^*$-closed set in X for every $V \in \tau_2^c$,
(xiv) $fswg$-continuous function [16] if $h^{-1}(V)$ is $fswg$-closed set in X for every $V \in \tau_2^c$,
(xv) fmg-continuous function [13] if $h^{-1}(V)$ is fmg-closed set in X for every $V \in \tau_2^c$,
(xvi) fwg-continuous function [15] if $h^{-1}(V)$ is fwg-closed set in X for every $V \in \tau_2^c$,
(xvii) $f\pi g$-continuous function [14] if $h^{-1}(V)$ is $f\pi g$-closed set in X for every $V \in \tau_2^c$.

Remark 6.52. It is clear from definitions that
(i) fg-continuous function, fgp-continuous function, fpg-continuous function, $f\pi g$-continuous function, fgs^*-continuous function, fs^*g-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function, fmg-continuous function, fwg-continuous function, $fswg$-continuous function imply $frwg$-continuous function. But the converses are not true, in general, follow from the following examples.
(ii) $frwg$-continuous function in as independent concept of fgs-continuous function, fsg-continuous function, $fg\gamma$-continuous function, $fg\gamma^*$-continuous function, $fg\beta$-continuous function, $f\beta g$-continuous function follow from the following examples.

Example 6.53. There exists an $frwg$-continuous function that has none of the following properties : fg-continuous function, $f\pi g$-continuous function, fgs^*-continuous function, fs^*g-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.7$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus B \in \tau_2^c$, $i^{-1}(1_X \setminus B) = 1_X \setminus B$. As $cl_{\tau_1}(int_{\tau_1}(1_X \setminus B)) = 0_X$,

clearly $1_X \setminus B$ is $frwg$-closed set in (X, τ_1). So i is an $frwg$-continuous function. Now $1_X \setminus B < A \in \tau_1$ (resp., $A \in F\pi O(X, \tau_1)$, $A \in FSO(X, \tau_1)$, A is an fg-open set in (X, τ_1), $A \in F\alpha O(X, \tau_1)$). But $cl_{\tau_1}(1_X \setminus B) = scl_{\tau_1}(1_X \setminus B) = \alpha cl_{\tau_1}(1_X \setminus B) = 1_X \setminus A \not\leq A$ and so $1_X \setminus B$ is not an fg-closed set, $f\pi g$-closed set, fgs^*-closed set, fs^*g-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set in (X, τ_1). Hence i is not an fg-continuous function, $f\pi g$-continuous function, fgs^*-continuous function, fs^*g-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function.

Example 6.54. There exists an $frwg$-continuous function that has none of the following properties : fgs-continuous function, fsg-continuous function, fpg-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B, C\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = B(b) = 0.5, C(a) = C(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Here $FRO(X, \tau_1) = \{0_X, 1_X\}$. So clearly i is an $frwg$-continuous function. Now $FSO(X, \tau_1) = \{0_X, 1_X, U\}$ where $U \geq A$ and so $FSC(X, \tau_1) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \leq 1_X \setminus A$. Now $1_X \setminus B \in \tau_2^c$, $i^{-1}(1_X \setminus B) = 1_X \setminus B < A \in \tau_1$ (resp., $A \in FSO(X, \tau_1)$). But $scl_{\tau_1}(1_X \setminus B) = 1_X \not\leq A$ and so $1_X \setminus B$ is not an fgs-closed set as well as fsg-closed set in (X, τ_1). Hence i is not an fgs-continuous function as well as an fsg-continuous function. Also $1_X \setminus C \in \tau_2^c$, $i^{-1}(1_X \setminus C) = 1_X \setminus C \leq 1_X \setminus C \in FPO(X, \tau_1)$. But as $1_X \setminus C \notin FPC(X, \tau_1)$, $pcl_{\tau_1}(1_X \setminus C) \not\leq 1_X \setminus C$ implies that $1_X \setminus C$ is not an fpg-closed set in (X, τ_1). Consequently, i is not an fpg-continuous function.

Example 6.55. There exists an $frwg$-continuous function which is not an fgp-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.6$, $B(a) = 0.2, B(b) = 0.5, C(a) = 0.6, C(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c$, $i^{-1}(1_X \setminus C) = 1_X \setminus C < 1_X \in FRO(X, \tau_1)$ only and so $1_X \setminus C$ is an $frwg$-closed set in (X, τ_1) and so i is an $frwg$-continuous function. Now $1_X \setminus C < A \in \tau_1$, But $pcl_{\tau_1}(1_X \setminus C) = 1_X \setminus B \not\leq A$ implies that $1_X \setminus C$ is not an fgp-closed set in (X, τ_1). Hence i is not an fgp-continuous function.

Example 6.56. There exists an $frwg$-continuous function which is not an $fg\gamma^*$-continuous function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.4$, $B(a) = 0.4, B(b) = 0.3, C(a) = 0.6, C(b) = 0.5$. Then $X, \tau_1)$ and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c, i^{-1}(1_X \setminus C) = 1_X \setminus C < 1_X \in FRO(X, \tau_1)$ only and so $1_X \setminus C$ is an $frwg$-closed set in (X, τ_1) which shows that i is an $frwg$-continuous function. Now $1_X \setminus C \leq 1_X \setminus C \in FSO(X, \tau_1)$. But as $1_X \setminus C \notin F\gamma C(X, \tau_1)$, $\gamma cl_{\tau_1}(1_X \setminus C) \not\leq 1_X \setminus C$ implies that $1_X \setminus C$ is not an $fg\gamma^*$-closed set in (X, τ_1). Hence i is not an $fg\gamma^*$-continuous function.

Example 6.57. There exists an $frwg$-continuous function that has none of the following properties : $fg\gamma$-continuous function, fmg-continuous function, fwg-continuous function, $fswg$-continuous function, $f\beta g$-continuous function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C, D\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.2, B(b) = 0.5, C(a) = 0.5, C(b) = 0.45, D(a) = D(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C, 1_X \setminus D \in \tau_2^c, i^{-1}(1_X \setminus C) = 1_X \setminus C$, $i^{-1}(1_X \setminus D) = 1_X \setminus D$. Now $1_X \in FRO(X, \tau_1)$ only containing $1_X \setminus C, 1_X \setminus D$ and so $1_X \setminus C, 1_X \setminus D$ are $frwg$-closed sets in (X, τ_1). So i is an $frwg$-continuous function. Now $1_X \setminus C < A \in \tau_1$ (resp., $A \in FSO(X, \tau_1)$ and A is an fg-open set in (X, τ_1)). But $\gamma cl_{\tau_1}(1_X \setminus C) = 1_X \not\leq A$ implies that $1_X \setminus C$ is not an $fg\gamma$-closed set in (X, τ_1). Then i is not an $fg\gamma$-continuous function. Again $cl_{\tau_1}(int_{\tau_1}(1_X \setminus C)) = 1_X \setminus B \not\leq A$ implies that $1_X \setminus C$ is not an fmg-closed set, fwg-closed set, $fswg$-closed set in (X, τ_2). Hence i is not an fmg-continuous function, fwg-continuous function, $fswg$-continuous function. Now $1_X \setminus D \leq 1_X \setminus D \in F\beta O(X, \tau_1)$. But as $1_X \setminus D \notin F\beta C(X, \tau_1)$, $\beta cl_{\tau_1}(1_X \setminus D) \not\leq 1_X \setminus D$. So $1_X \setminus D$ is not an $f\beta g$-closed set in (X, τ_1). Consequently, i is not an $f\beta g$-continuous function.

Example 6.58. There exists an $frwg$-continuous function which is not an $fg\beta$-continuous function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.6$, $B(a) = 0.2, B(b) = 0.55, C(a) = 0.5, C(b) = 0.45$. Then (X, τ_1) and (X, τ_2) are fts's. Consider

the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c$, $i^{-1}(1_X \setminus C) = 1_X \setminus C < 1_X \in FRO(X, \tau_1)$ only and so $1_X \setminus C$ is an $frwg$-closed set in (X, τ_1). As a result, i is an $frwg$-continuous function. Now $1_X \setminus C < A \in \tau_1$. But $\beta cl_{\tau_1}(1_X \setminus C) = 1_X \not\leq A$. So $1_X \setminus C$ is not an $fg\beta$-closed set in (X, τ_1). Hence i is not an $fg\beta$-continuous function.

Example 6.59. There exists an fgs-continuous function which is not an $frwg$-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.4$, $B(a) = 0.4, B(b) = 0.3, C(a) = C(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c$, $i(1_X \setminus C) = 1_X \setminus C < A \in FRO(X, \tau_1)$. But $cl_{\tau_1}(int_{\tau_1}(1_X \setminus C)) = 1_X \setminus A \not\leq A$ and so $1_X \setminus C$ is not an $frwg$-closed set in (X, τ_1). Hence i is not an $frwg$-continuous function. Now $FSO(X, \tau_1) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus A$ and so $FSC(X, \tau_1) = \{0_X, 1_X, 1_X \setminus U\}$ where $A \leq 1_X \setminus U \leq 1_X \setminus B$. Now $1_X \setminus C < A \in \tau_1$ and so $scl_{\tau_1}(1_X \setminus C) = A \leq A$ ad so $1_X \setminus C$ is an fgs-closed set in (X, τ_1). Hence i is an fgs-continuous function.

Example 6.60. There exist fsg-continuous function, $fg\gamma$-continuous function, $fg\gamma^*$-continuous function none of which implies $frwg$-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, D\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.3, D(a) = 0.5, D(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus D \in \tau_2$, $i^{-1}(1_X \setminus D) = 1_X \setminus D \leq A \in FRO(X, \tau_1)$. But $cl_{\tau_1}(int_{\tau_1}(1_X \setminus D)) = 1_X \setminus A \not\leq A$. So $1_X \setminus D$ is not an $frwg$-closed set in (X, τ_1). Hence i is not an $frwg$-continuous function. Now $1_X \setminus D \in FSC(X, \tau_1)$ and also $1_X \setminus D \in F\gamma C(X, \tau_1)$. So $1_X \setminus D$ is fsg-closed set as well as $fg\gamma$-closed set, $fg\gamma^*$-closed set in (X, τ_1). So i is an fsg-continuous function, $fg\gamma$-continuous function, $fg\gamma^*$-continuous function.

Example 6.61. There exist $fg\beta$-continuous function, $f\beta g$-continuous function none of which is $frwg$-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B, C\}$, $\tau_2 = \{0_X, 1_X, D\}$ where $A(a) = 0.3, A(b) = 0.4, B(a) = B(b) = 0.4, C(a) = 0.6, C(b) = 0.5, D(a) = D(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's.

Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus D \in \tau_2^c$, $i^{-1}(1_X \setminus D) = 1_X \setminus D \leq B \in FRO(X, \tau_1)$. But as $cl_{\tau_1}(int_{\tau_1}(1_X \setminus D)) = 1_X \setminus C \not\leq B$, $1_X \setminus D$ is not an $frwg$-closed set in (X, τ_1) and so i is not an $frwg$-continuous function. But as $int_{\tau_1}(cl_{\tau_1}(int_{\tau_1}(1_X \setminus D))) = 1_X \setminus D$, $1_X \setminus D \in F\beta C(X, \tau_1)$ implies that $1_X \setminus D$ is an $fg\beta$-closed set as well as $f\beta g$-closed set in (X, τ_1). Hence i is an $fg\beta$-continuous function as well as $f\beta g$-continuous function.

Remark 6.62. (i) Let $h : X \to Y$ be an $frwg$-continuous function where X is an frT_g-space. Then h is a fuzzy continuous function, fg-continuous function, $f\pi g$-continuous function, fgs^*-continuous function, fs^*g-continuous function, fgs-continuous function, fsg-continuous function, $fg\alpha$-continuous function, $f\alpha g$-continuous function, $fg\beta$-continuous function, $f\beta g$-continuous function, fmg-continuous function, fwg-continuous function, $fswg$-continuous function, fgp-continuous function, fpg-continuous function, $fg\gamma$-continuous function, $fg\gamma^*$-continuous function.
(ii) Let $h : X \to Y$ be a function where X is an fT_b-space (resp., fT_{sg}-space, fT_γ-space, fT_{γ^*}-space, $f\beta T_b$-space, fT_β-space). Then if h is an fgs-continuous function (resp., fsg-continuous function, $fg\gamma$-continuous function, $fg\gamma^*$-continuous function, $f\beta g$-continuous function, $fg\beta$-continuous function), h is an $frwg$-continuous function.

7. $frwg$-T_2-Space

A new type of generalized version of fuzzy T_2-space is introduced here which is a strong version of fuzzy T_2-space. Afterwards, the applications of functions defined in earlier sections on this space are established here.

We first recall the following definition and theorem from [24, 25] for ready references.

Definition 7.1 [24]. An fts (X, τ) is called fuzzy T_2-space if for any two distinct fuzzy points x_α and y_β; when $x \neq y$, there exist fuzzy open sets U_1, U_2, V_1, V_2 such that $x_\alpha \in U_1, y_\beta q V_1, U_1 \not{q} V_1$ and $x_\alpha q U_2, y_\beta \in V_2, U_2 \not{q} V_2$; when $x = y$ and $\alpha < \beta$ (say), there exist fuzzy open sets U and V in X such that $x_\alpha \in U, y_\beta q V$ and $U \not{q} V$.

Theorem 7.2 [25]. An fts (X, τ) is fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha qU$, $y_\beta qV$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Now we introduce the following concept.

Definition 7.3. An fts (X, τ) is called $frwg$-T_2-space if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $frwg$-open sets U, V in X such that $x_\alpha qU$, $y_\beta qV$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has an $frwg$-open nbd U and y_β has an $frwg$-open q-nbd V such that $U \not{q} V$.

Theorem 7.4. If an injective function $h : X \to Y$ is $frwg$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is $frwg$-T_2-space.

Proof. Let x_α and y_β be two distinct fuzzy points in X. Then $h(x_\alpha)$ $(= z_\alpha$, say) and $h(y_\beta)$ $(= w_\beta$, say) are two distinct fuzzy points in Y.
Case I. Suppose $x \neq y$. Then $z \neq w$. Since Y is fuzzy T_2-space, there exist fuzzy open sets U, V in Y such that $z_\alpha qU, w_\beta qV$ and $U \not{q} V$. As h is $frwg$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $frwg$-open sets in X with $x_\alpha qh^{-1}(U), y_\beta qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_\alpha qU$ implies that $U(z) + \alpha > 1$ and so $U(h(x)) + \alpha > 1$. Then $[h^{-1}(U)](x) + \alpha > 1$. So $x_\alpha qh^{-1}(U)$. Again, $h^{-1}(U)qh^{-1}(V)$ implies that there exists $t \in X$ such that $[h^{-1}(U)](t) + [h^{-1}(V)](t) > 1$ and then $U(h(t)) + V(h(t)) > 1$. Then UqV, a contradiction].
Case II. Suppose $x = y$ and $\alpha < \beta$ (say). Then $z = w$ and $\alpha < \beta$. Since Y is fuzzy T_2-space, there exist a fuzzy open nbd U of z_α and a fuzzy open q-nbd V of w_β such that $U \not{q} V$. Then $U(z) \geq \alpha$ implies that $[h^{-1}(U)](x) \geq \alpha$. Then $x_\alpha \in h^{-1}(U), y_\beta qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ where $h^{-1}(U)$ and $h^{-1}(V)$ are $frwg$-open sets in X as h is $frwg$-continuous function. Consequently, X is $frwg$-T_2-space.

Similarly we can state the following theorems easily the proofs of which are similar to that of Theorem 7.4.

Theorem 7.5. If a bijective function $h : X \to Y$ is $frwg$-irresolute function from an fts X onto an $frwg$-T_2-space (resp., fuzzy T_2-space)

Y, then X is $frwg$-T_2-space.

Theorem 7.6. If a bijective function $h : X \to Y$ is $frwg$-continuous function from an frT_g-space X onto a fuzzy T_2-space Y, then X is fuzzy T_2-space.

Theorem 7.7. If a bijective function $h : X \to Y$ is $frwg$-irresolute function from an frT_g-space X onto an $frwg$-T_2-space (resp., fuzzy T_2-space) Y, then X is fuzzy T_2-space.

Theorem 7.8. If a bijective function $h : X \to Y$ is $frwg$-open function from a fuzzy T_2-space X onto an fts Y, then Y is $frwg$-T_2-space.

Theorem 7.9. If a bijective function $h : X \to Y$ is $frwg$-open function from a fuzzy T_2-space X onto an frT_g-space Y, then Y is fuzzy T_2-space.

Theorem 7.10. If a bijective function $h : X \to Y$ is strongly $frwg$-continuous function from an fts X onto an $frwg$-T_2-space (resp., fuzzy T_2-space) Y, then X is fuzzy T_2-space.

Remark 7.11. It is clear from the fact that fuzzy open set is $frwg$-open set, every fuzzy T_2-space is $frwg$-T_2-space, but the converse is not necessarily true, follows from the following example.

Example 7.12. Let $X = \{a\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Clearly (X, τ) is not a fuzzy T_2-space. Here every fuzzy set in (X, τ) is $frwg$-open set in (X, τ). Consider two fuzzy points $a_{0.1}$ and $a_{0.4}$. Then there exist two $frwg$-open sets U, V in X where $U(a) = 0.2, V(a) = 0.61$ such that $a_{0.1} \in U$, $a_{0.4} q V$ and $U \not{q} V$ and this is true for every pair of distinct fuzzy points in X. So (X, τ) is an $frwg$-T_2-space.

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