"Vasile Alecsandri" University of Bacău
Faculty of Sciences
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ON p-OPEN SETS AND p^*-URYSOHN SPACES

KALLOL BHANDHU BAGCHI

Abstract. In this article we introduce the notions of p^*-T_1 space, p^*-T_2, p^*-regular space and p^*-Urysohn space, an analogue of the classical notion of T_1 space, T_2 space, regular space and Urysohn space respectively, where the role of open sets (resp., of the corresponding closure operator Cl) is played by p-open sets (resp., by the corresponding p-closure operator Cl^p). It is seen that the notion of p^*-Urysohn space is stronger than each of the notion Urysohn space, pre-Urysohn space , p-Urysohn space and weakly Hausdorff space. The notion of ordered pair of pre-open sets in a topological space is introduced along with and some important and interesting results have been obtained. Using p-open sets, one can introduce and study various notions in topological spaces.

1. Introduction

Unless otherwise mentioned, X stands for a topological space $(X, \mathscr{T})$. $Int(A)$ (resp. $Int_p(A)$) and $Cl(A)$ (resp., $Cl_p(A)$) denotes respectively the interior (resp., pre-interior) and closure (resp., pre-closure) of a subset A of a topological space X. Throughout this paper $o(A)$ will stand for $Int(Cl(A))$ for a subset A of a topological space X.

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A subset K of a topological space X is said to be pre-open (Corson et al. [5] and Mashhour et al. [9]) if there is an open set G such that $K \subseteq G \subseteq Cl(K)$. The complement of a pre-open set is called a pre-closed set ([9]). Equivalently, a set in a topological space is pre-open if and only if it is included in the interior of its closure. The notion of pre-open set is more general than that of open set. A significant number of notions and results in topological spaces have been generalized using the notion of pre-open set. Recently using the notion of pre-open set, Bagchi ([1]) introduced the notion of p-open sets: a subset G of a topological space X is said to be a p-open set if for each $x \in G$ there is a pre-open set K such that $x \in K \subseteq o(K) \subseteq G$. If G is p-open in a topological space X, then G is δ-open in X and conversely (Joseph [7] and Bagchi [1]). Again if G is a p-open set in a topological space X, then G is open in X, i. e., the notion of p-open sets is stronger than the notion of open sets (Bagchi [1]). Bagchi ([1]) also studied nearly-compact spaces through p-open sets.

In this article we introduce the notions of p^*-T_1 space, p^*-T_2, p^*-regular space and p^*-Urysohn space, an analogue of the classical notion of T_1 space, T_2 space, regular space and Urysohn space respectively, where the role of open sets (resp., of the corresponding closure operator Cl) is played by p-open sets (resp., by the corresponding p-closure operator Cl^p). It is seen that the notion of p^*-Urysohn space is stronger than each of the notion Urysohn space (Theorem 3.8), pre-Urysohn space (Theorem 3.7), p-Urysohn space (Theorem 3.9) and weakly Hausdorff space (Theorem 3.5). The notion of ordered pair of pre-open sets in a topological space is introduced along with and some important and interesting results have been obtained. Using p-open sets, one can introduce and study various notions in topological spaces.

2. Preliminaries

Firstly, we recall some useful definitions and results:

Definition 2.1. (Pal and Bhattacharyya [11]) A topological space X is called pre-Urysohn for every pair of points $x, y \in X, x \neq y$ there exist pre-open sets U, V such that $x \in U, y \in V$ and $Cl_p(U) \cap Cl_p(V) = \emptyset$. '

Definition 2.2. (Navalagi [10]) A topological space X is said to be p-Urysohn if for any two points x and y with $x \neq y$,there exist pre-open sets U and V such that $x \in U$ and $y \in V$ and $Cl(U) \cap Cl(V) = \emptyset$.

Definition 2.3. (Bagchi et al. [2]) A filterbase $\mathscr{F}$ on X is said to p-converge to a point $x \in X$, if for each pre-open set G of X, with $x \in G$, there exists $F \in \mathscr{F}$ such that $F \subseteq o(G)$.

Definition 2.4. (Bagchi et al. [2]) A filter base $\mathscr{F}$ on X is said to p-accumulates to a point $x \in X$ if for each pre-open set A with $x \in A$, one has $F \cap o(A) \neq \emptyset$ for each $F \in \mathscr{F}$.

Definition 2.5. (Scarborough [12]) An open filter base $\mathscr{F}$ in a topological space X is a Urysohn filter base if and only if for each $p \in A(\mathscr{F})$ (where $A(\mathscr{F})$ denotes the set of accumulation points of $\mathscr{F}$) there is an open U containing p and some $V \in \mathscr{F}$ such that $Cl(U) \cap Cl(V) = \emptyset$.

Definition 2.6. (Berri et al. [3]) An open cover $\mathscr{U}$ of a topological space X is a Urysohn open cover if there exists an open cover $\mathscr{V}$ of X with the property that for each $V \in \mathscr{V}$, there is a $U \in \mathscr{U}$ such that $Cl(V) \subseteq U$.

A Urysohn space X is called Urysohn-closed provided X is a closed set in every Urysohn space in which it can be embedded.

Definition 2.7. (Bagchi [1]) Let X be a topological space and $K \subseteq X$. A point $x \in X$ is said to be a p-limit point of K if $K \cap [o(F) - \{x\}] \neq \emptyset$, for any pre-open set F containing x.

The set of all p-limit points of K is said to be p-derived set of K and is denoted by $D^p(K)$. And $Cl^p(K) = K \cup D^p(K)$ is said to be the p-closure of K.

Definition 2.8. (Bagchi [1]) A subset A of a topological space X is said to be p-closed if $D^p(A) \subseteq A$, i.e., A is said to be p-closed if A contains all of its p-limit points.

Definition 2.9. (Bagchi [1]) Let G be a subset of a topological space X and $x \in X$. x is said to be a p-interior point of G if there is a pre-open set H such that $x \in H \subseteq o(H) \subseteq G$.

The set of all p-interior points of G is said to be p-interior of G and is denoted by $Int^p(G)$.

It is clear that $Int^p(G) \subseteq G$ for every subset G of a topological space X.

Definition 2.10. (Bagchi [1]) Let G be a subset of a topological space X. G is said to be p-open if $G = Int^p(G)$.

Definition 2.11. (Herrington [6]) Let X be a topological space and let G, H be open sets containing a point $p \in X$. Then G and H will be called an ordered pair of open sets containing p (denoted by (G, H)) if $p \in G \subseteq Cl(G) \subseteq H$.

Let $\mathscr{F} = \{A_\alpha : \alpha \in \Delta\}$ be a filterbase in a topological space X. Then $\mathscr{F}$ u-converges to $x \in X$ if for each ordered pair of open sets (G, H) containing x there exists an $A_\alpha \in \mathscr{F}$ such that $A_\alpha \subseteq Cl(H)$. The filterbase $\mathscr{F}$ u-accumulates to x if for each ordered pair of open sets (G, H) containing x and for each $A_\alpha \in \mathscr{F}$, $A_\alpha \cap Cl(H) \neq \emptyset$.

Theorem 2.1. *(Bagchi et al. [2]) In a topological space X, the following statements are equivalent.*

(1) [(i)]
(2) X is nearly compact.
(3) Each maximal filter base p-converges in X.
(4) Each filter base p-accumulates to some $x_0 \in X$.
(5) For each family $\mathscr{F}$ of pre-closed sets with $\bigcap\{F : F \in \mathscr{F}\} = \emptyset$, there exists a finite subcollection $\mathscr{E}$ of $\mathscr{F}$ such that $\bigcup\{Cl(Int(E)) : E \in \mathscr{E}\} = \emptyset$.

Theorem 2.2. *(Bagchi [1]) Let K be a subset of a topological space X. Then K is p-open if and only if $X - K$ is p-closed.*

Theorem 2.3. *(Bagchi [1]) Let A be a subset of a topological space X. Then $Cl(A) \subseteq Cl^p(A)$.*

3. p^*-Urysohn Space

In this section, we introduce and study some separation axioms concerning p-open sets.

Definition 3.1. Let X be a topological space.

(1) [(i)]
(2) X is said to be p^*-T_1 if for each $x, y \in X$ with $x \neq y$, there exist p-open sets G and H such that $x \in G, y \in H, x \notin H$ and $y \notin G$.
(3) X is said to be p^*-T_2 if for each $x, y \in X$ with $x \neq y$, there exist p-open sets G and H such that $x \in G, y \in H$ and $G \cap H = \emptyset$.
(4) X is said to be p^*-regular if whenever A is p-closed in X and $x \in X$ with $x \notin A$, there exist p-open sets G and H such that $x \in G$, $A \subseteq H$ and $G \cap H = \emptyset$.
    A p^*-T_1, p^*-regular topological space is said to be p^*-T_3.
(5) X is said to be p^*-Urysohn space if for each $x, y \in X$ with $x \neq y$, there exist p-open sets G, H such that $x \in G, y \in H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$.

Theorem 3.1. *A topological space X is p^*-T_1 if and only if for each $x, y \in X$ with $x \neq y$, there exist regular open sets G and H such that $x \in G, y \in H, x \notin H$ and $y \notin G$.*

Proof. Let X be p^*-T_1 and $x \neq y$ in X. Then there exist p-open sets G, H such that $x \in G, y \in H, x \notin H$ and $y \notin G$. Since G and H are p-open sets containing x and y respectively, there exist pre-open sets U, V such that $x \in U \subseteq o(U) \subseteq G$ and $y \in V \subseteq o(V) \subseteq H$. Here $o(U)$ and $o(V)$ are the required regular open sets.

The converse part follows from the fact that every regular open set G in a topological space X is p-open. ∎

Theorem 3.2. *A topological space X is p^*-T_1 if and only if $\{x\}$ is p-closed for each $x \in X$.*

Proof. First suppose X be p^*-T_1 and $x \in X$. If possible, let $y \in Cl^p(\{x\})$ be such that $x \neq y$. Then there exist p-open sets G, H such that $x \in G, y \in H, x \notin H$ and $y \notin G$. Now $y \in H$ implies that there is a pre-open set U such that $y \in U \subseteq o(U) \subseteq H$. Since $x \notin H$, it follows that $x \notin o(U)$, i.e., $\{x\} \cap o(U) = \emptyset$. But this contradicts the fact that $y \in Cl^p(\{x\})$. Therefore $\{x\} = Cl^p(\{x\})$.

Next suppose for each x in a topological space X, $\{x\}$ is p-closed. Let $p, q \in X$ and $p \neq q$. Then $X - \{q\}$ and $X - \{p\}$ are p-open sets containing p, q respectively and $p \notin X - \{p\}$, $q \notin X - \{q\}$. Hence X is p^*-T_1. ∎

Theorem 3.3. *A topological space X is p^*-T_2 if and only if for each $x, y \in X$ with $x \neq y$, there exist regular open sets G and H such that $x \in G, y \in H$ and $G \cap H = \emptyset$.*

Proof. Let X be p^*-T_2 and $x \neq y$ in X. Then there exist p-open sets G, H such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$. Since G and H are p-open sets containing x and y respectively, there exist pre-open sets U, V such that $x \in U \subseteq o(U) \subseteq G$ and $y \in V \subseteq o(V) \subseteq H$. Clearly $o(U) \cap o(V) = \emptyset$. Here $o(U)$ and $o(V)$ are the required regular open sets.

The converse part follows from the fact that every regular open set G in a topological space X is p-open. ∎

Lemma 3.1. Let A be a subset of a topological space X. Then $o(A) \subseteq Cl^p(A)$.

Proof. By Theorem 2.3, $Cl(A) \subseteq Cl^p(A)$. Since $o(A) \subseteq Cl(A)$, it follows that $o(A) \subseteq Cl^p(A)$. ∎

Theorem 3.4. *A topological space X is p^*-Urysohn if and only if whenever $x \neq y$ in X there exist regular open sets G, H such that $x \in G$ and $y \in H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$.*

Proof. Let $x \neq y$ in a p^*-Urysohn topological space X. Then there exist p-open sets G, H such that $x \in G, y \in H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$. Since G and H are p-open sets containing x and y respectively, there exist pre-open sets U, V such that $x \in U \subseteq o(U) \subseteq G$ and $y \in V \subseteq o(V) \subseteq H$. Here $o(U)$ and $o(V)$ are the required regular open sets.

The converse part follows from the fact that every regular open set G in a topological space X is p-open. ∎

Theorem 3.5. *Let X be a p^*-Urysohn topological space. Then X is weakly Hausdorff.*

Proof. Let $x \neq y$ in a p^*-Urysohn topological space X. Then there exist p-open sets G, H such that $x \in G$ and $y \in H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$. Now $x \in G$ implies that there is a pre-open set U such that $x \in U \subseteq o(U) \subseteq G$ and $y \in H$ means that there is a pre-open set V such that $y \in V \subseteq o(V) \subseteq H$. Thus $o(U) \cap o(V) = \emptyset$. Clearly U is a pre-open set such that $x \in o(U)$ and $y \in X - o(U)$. ∎

Lemma 3.2. Let A be a subset of a topological space X. Then $Cl^p(A)$ is p-closed.

Theorem 3.6. *Let X be a p^*-Urysohn topological space. Then X is p-T_1.*

Proof. Let $x \neq y$ in X. Then there exist p-open sets G, H such that $x \in G, y \in H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$. Now by the Lemma 3.2, $X - Cl^p(G)$ and $X - Cl^p(H)$ are p-open sets satisfying $x \notin X - Cl^p(G)$, $x \in X - Cl^p(H)$, $y \notin X - Cl^p(H)$ and $y \in X - Cl^p(G)$. Thus X is p-T_1. ∎

Theorem 3.7. *Let X be a p^*-Urysohn topological space. Then X is pre-Urysohn.*

Proof. The proof follows from the fact that $Cl_p(A) \subseteq Cl^p(A)$, for a subset A of a topological space X. ∎

Theorem 3.8. *Let X be a p^*-Urysohn topological space. Then X is Urysohn.*

Proof. Let $x \neq y$ in X. Then by the Theorem 3.4, there exist regular open sets G, H such that $x \in G$ and $y \in H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$.

Remaining part follows from the fact that $Cl(A) \subseteq Cl^p(A)$ for a subset A of X. ∎

Theorem 3.9. *Let X be a p^*-Urysohn topological space. Then X is p-Urysohn.*

Proof. The proof follows from the fact that $Cl(A) \subseteq Cl^p(A)$, for a subset A of a topological space X. ∎

Theorem 3.10. *Let X be a p^*-regular, p^*-T_1 topological space. Then X is p^*-Urysohn.*

Proof. Let $x \neq y$ in X. Then $\{y\}$ is a p-closed in X such that $x \notin \{y\}$. So there exist p-open sets G, H satisfying $x \in G, \{y\} \subseteq H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$. Clearly $x \in G, y \subseteq H$ and $Cl^p(G) \cap Cl^p(H) = \emptyset$. So X is p^*-Urysohn. ∎

Definition 3.2. A filter base $\mathscr{F}$ in a topological space X is said to p^*-converges to $x \in X$ if for any p-open set G containing x there is a $F \in \mathscr{F}$ such that $F \subseteq Cl^p(G)$.

$\mathscr{F}$ is said to p^*-accumulates to $x \in X$ if for each p-open set G containing x and for each $F \in \mathscr{F}$, $F \cap Cl^p(G) \neq \emptyset$.

It is easy to verify that if a filter base $\mathscr{F}$ in a topological space X, p^*-converges to $x \in X$, then $\mathscr{F}$ p^*-accumulates to x.

Theorem 3.11. *Let X be a p^*-Urysohn topological space and $\mathscr{F}$ be a filter base p^*-converges to $x \in X$. Then x is unique.*

Proof. Let $x \neq y$ in X such that $\mathscr{F}$ p^*-converges to y also. Then for the p^*-open sets G, H containing x and y respectively, there exist $E, F \in \mathscr{F}$ such that $E \subseteq Cl^p(G)$ and $F \subseteq Cl^p(H)$. Thus $Cl^p(G), Cl^p(H) \in \mathscr{F}$ and so $Cl^p(G) \cap Cl^p(H) \neq \emptyset$. This contradicts the fact that X is a p^*-Urysohn topological space. Therefore $x = y$. ∎

Theorem 3.12. *Let $\mathscr{F}$ be a filter base in a topological space X and $x \in X$. Then following statements are true.*

(1) [(i)]
(2) If $\mathscr{F}$ p-converges to x, then $\mathscr{F}$ p^*-converges to x.
(3) If $\mathscr{F}$ p-accumulates to x, then $\mathscr{F}$ p^*-accumulates to x.

Proof. Proofs follow from the fact that $G \subseteq Int(Cl(G)) \subseteq Cl(G) \subseteq Cl^p(G)$, for a pre-open set G in a topological space X. ∎

Theorem 3.13. *Let X be a nearly compact topological space. Then following statements are true.*

(1) [(i)]

(2) *Each maximal filter base p^*-converges in X.*
(3) *Each filter base p^*-accumulates to some $x_o \in X$.*

Proof. The claim follows from Theorem 2.1 along with Theorem 3.12. ∎

4. u^*-convergence

Let X be a topological space and let G and H be pre-open sets in X containing a point $p \in X$. Then G and H will be called an ordered pair of pre-open sets containing p (denoted by (G, H)) if $p \in G \subseteq o(G) \subseteq H$.

Definition 4.1. Let X be a topological space and let $\mathscr{F}$ be a filter base in X. $\mathscr{F}$ u^*-converges to $x \in X$ if for each ordered pair of pre-open sets (G, H) containing x there exists a $F \in \mathscr{F}$ such that $F \subseteq o(H)$.

The filter base $\mathscr{F}$ u^*-accumulates to $x \in X$ if for each ordered pair of pre-open sets (G, H) containing x and for each $F \in \mathscr{F}$, $F \cap o(H) \neq \emptyset$.

Theorem 4.1. *Let $\mathscr{F}$ be a filter base in a topological space X and $x \in X$. Then following are true.*

(1) *[(i)]*
(2) *If $\mathscr{F}$ u^*-converges to x, then $\mathscr{F}$ u-converges to x.*
(3) *If $\mathscr{F}$ u^*-accumulates to x, then $\mathscr{F}$ u-accumulates to x.*

Proof. The proof follows from the fact that every open set in a topological space is pre-open and $o(G) \subseteq Cl(G)$ for a subset G of a topological space. ∎

Lemma 4.1. Let X be a topological space and $x \in X$. If (G, H) is an ordered pair of open sets containing x, then (G, H) is an ordered pair of pre-open sets containing x

Proof. Clearly $x \in G \subseteq Cl(G) \subseteq H$ implies that $x \in G \subseteq o(G) \subseteq H$. The conclusion follows from the fact that each open set in a topological space is a pre-open set. ∎

Theorem 4.2. *If a filter base $\mathscr{F}$ in a topological space X u^*-converges to $x \in X$ then for each ordered pair (G, H) of regular open sets containing x there is a set $F \in \mathscr{F}$ satisfying $F \subseteq H$.*

Proof. Let $\mathscr{F}$ be a filter base in a topological space X u^*-converges to $x \in X$ and (G, H) be an order pair of regular open sets containing x.

By Lemma 4.1, (G, H) is an ordered pair of pre-open sets containing x and so there is a set $F \in \mathscr{F}$ such that $F \subseteq o(H) = H$. ∎

Theorem 4.3. *Let G be an open set in a topological space X and $x \in X$. Then (G, G) is an ordered pair containing x if and only if G is a clopen set containing x.*

Proof. Let (G, G) be an ordered pair containing X. Then $x \in G \subseteq Cl(G) \subseteq G$. So $Cl(G) = G$, i.e., G is a clopen set containing x.

Now let G be a clopen set containing x. Then $x \in G \subseteq Cl(G) \subseteq G$, i.e., (G, G) is an ordered pair containing x. ∎

Theorem 4.4. *Let G be a pre-open set in a topological space X and $x \in X$. If (G, G) is an ordered pair of pre-open sets containing x, then G is a regular open set containing x.*

Proof. (G, G) is an ordered pair of pre-open sets containing x implies that $x \in G \subseteq o(G) \subseteq G$. Therefore $G = o(G)$ and thus G is regular open set containing x. ∎

Theorem 4.5. *Let $\mathscr{F}$ be a filter base in a topological space X and $x \in X$. Then following statements are true.*

(1) [(i)]
(2) If $\mathscr{F}$ p-converges to x, then $\mathscr{F}$ u^*-converges to x.
(3) If $\mathscr{F}$ p-accumulates to x, then $\mathscr{F}$ u^*-accumulates to x.

Proof. (1) [(i)]
(2) Let (G, H) be an ordered pair of pre-open sets containing x. Then $x \in G \subseteq o(G) \subseteq H$. So H is a pre-open set containing x. Since $\mathscr{F}$ p-converges to x, there is a $F \in \mathscr{F}$ such that $F \subseteq o(H)$. Thus $\mathscr{F}$ u^*-converges to x.
(3) Let (G, H) be an ordered pair of pre-open sets containing x. Then $x \in G \subseteq o(G) \subseteq H$. So H is a pre-open set containing x. Since $\mathscr{F}$ p-accumulates to x, for each $F \in \mathscr{F}$ we have $F \cap o(H) \neq \emptyset$. Thus $\mathscr{F}$ u^*-accumulates to x.

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Theorem 4.6. *Let X be a nearly compact topological space. Then following statements are true.*

(1) [(i)]
(2) Each maximal filter base u^*-converges in X.
(3) Each filter base u^*-accumulates to some $x_o \in X$.

Proof. The claim follows from Theorem 2.1 and Theorem 4.5. ∎

Definition 4.2. Let $\mathscr{F}$ be a filter base of pre-open sets in a topological space X. $\mathscr{F}$ is said to be a pre-Urysohn filter base if for each $x \notin A_p(\mathscr{F})$ (where $A_p(\mathscr{F})$ denotes the set of p-accumulation points of $\mathscr{F}$) there is a pre-open set G containing x and some $F \in \mathscr{F}$ such that $Cl(G) \cap Cl(F) = \emptyset$.

Theorem 4.7. *Let $\mathscr{F}$ be a pre-Urysohn filter base in a topological space X and $x \in X$. Then $\mathscr{F}$ p-accumulates to x if and only if $\mathscr{F}$ u^*-accumulates to x.*

Proof. Assume that $\mathscr{F}$ p-accumulates to x. Then by the Theorem 4.5, $\mathscr{F}$ u^*-accumulates to x.

Now suppose that $\mathscr{F}$ does not p-accumulate to x. then there is a pre-open set G containing x and a $F \in \mathscr{F}$ such that $Cl(G) \cap Cl(F) = \emptyset$. Then $x \in G \subseteq o(G) \subseteq Cl(G) \subseteq H$, where $H = X - Cl(F)$. Then (G, H) is an ordered pair of pre-open sets containing x satisfying $F \cap o(H) = \emptyset$. So $\mathscr{F}$ does not u^*-accumulate to x. ∎

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Kallol Bhandhu Bagchi
Department of Mathematics
Kalipada Ghosh Tarai Mahavidyalaya
Siliguri, W. Bengal-734014 , India
e-mail: kbagchi.789@gmail.com