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FUZZY p^*-PRECOMPACT TOPOLOGICAL SPACES

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Abstract. In this paper a new type of compactness in fuzzy topological spaces is introduced and studied by using p^*-preopen set [1] as a basic tool. We characterize this newly defined compactness by fuzzy net and prefilterbase. It is shown that this compactness implies fuzzy almost compactness [3] and the converse is true only on fuzzy p^*-preregular space [1]. Afterwards, it is shown that this compactness remains invariant under fuzzy p^*-preirresolute function [1].

1. Introduction

Fuzzy compactness is introduced by Chang [2]. Afterwards, many mathematicians have engaged themselves to introduce different types of fuzzy compactness. In [3], fuzzy almost compactness is introduced. In this paper we introduce fuzzy p^*-precompactness which is weaker than fuzzy almost compactness. Here we use fuzzy net [10] and prefilterbase [7] to characterize fuzzy p^*-precompactness. In this context we have to mention [6].

Keywords and phrases: Fuzzy p^*-preopen set, fuzzy p^*-preregular space, fuzzy regularly p^*-preclosed set, fuzzy p^*-precompact set (space), p^*p-adhere point of a prefilterbase, p^*p-cluster point of a fuzzy net.
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2. Preliminaries

Throughout this paper, (X, τ) or simply by X we shall mean an fts. In 1965, L.A. Zadeh introduced fuzzy set [11] A which is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [11] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [11] of a fuzzy set A in an fts X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [11] while AqB means A is quasi-coincident (q-coincident, for short) [10] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [2] and fuzzy interior [2] of A respectively. A fuzzy set A in X is called a fuzzy neighbourhood (fuzzy nbd, for short) [10] of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \in G \leq A$, i.e., $G(x) \geq t$ and $A \geq G$. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_t. A fuzzy set A is said to be a fuzzy q-nbd of a fuzzy point x_t in an fts X if there is a fuzzy open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy open, then A is called a fuzzy open q-nbd [10] of x_t.

A fuzzy set A in an fts (X, τ) is called fuzzy preopen [9] if $A \leq intclA$. The complement of a fuzzy preopen set is called fuzzy preclosed [9]. The union (intersection) of all fuzzy preopen (resp., fuzzy preclosed) sets contained in (resp., containing) a fuzzy set A is called fuzzy preinterior [9] (resp., fuzzy preclosure [9]) of A, denoted by $pintA$ (resp., $pclA$).

Let $(D, \geq)$ be a directed set and X be an ordinary set. Let J denote the collection of all fuzzy points in X. A function $S : D \to J$ is called a fuzzy net in X [10]. It is denoted by $\{S_n : n \in (D, \geq)\}$. A non empty family $\mathcal{F}$ of fuzzy sets in X is called a prefilterbase on X if (i) $0_X \notin \mathcal{F}$ and (ii) for any $U, V \in \mathcal{F}$, there exists $W \in \mathcal{F}$ such that $W \leq U \bigcap V$ [7].

3. Fuzzy p^*-Preopen Sets : Some Results

In this section we recall some definitions and results from [1, 2, 3, 5, 8] for ready references.

Definition 3.1 [1]. A fuzzy set A in an fts (X, τ) is called fuzzy

p^*-preopen if $A \leq pint(clA)$. The complement of this set is called fuzzy p^*-preclosed set.
The union (resp., intersection) of all fuzzy p^*-preopen (resp., fuzzy p^*-preclosed) sets contained in (containing) a fuzzy set A is called fuzzy p^*-preinterior (resp., fuzzy p^*-preclosure) of A, denoted by p^*pintA (resp., p^*pclA).

Definition 3.2 [1]. A fuzzy set A in an fts (X, τ) is called fuzzy p^*-pre nbd of a fuzzy point x_α in X if there exists a fuzzy p^*-preopen set U in X such that $x_\alpha \in U \leq A$. If, in addition, A is fuzzy p^*-preopen, then A is called fuzzy p^*-preopen nbd of x_α.

Definition 3.3 [1]. A fuzzy set A in an fts (X, τ) is called fuzzy p^*-pre q-nbd of a fuzzy point x_α in X if there exists a fuzzy p^*-preopen set U in X such that $x_\alpha q U \leq A$. If, in addition, A is fuzzy p^*-preopen, then A is called fuzzy p^*-preopen q-nbd of x_α.

Result 3.4 [1]. Union (resp., intersection) of any two fuzzy p^*-preopen (resp., fuzzy p^*-preclosed) sets is also so.

Result 3.5 [1]. $x_\alpha \in p^*pclA$ if and only if every fuzzy p^*-preopen q-nbd U of x_α, UqA.

Result 3.6 [1]. $p^*pcl(p^*pclA) = p^*pclA$ for any fuzzy set A in an fts (X, τ).

Result 3.7. $p^*pcl(A \bigvee B) = p^*pclA \bigvee p^*pclB$, for any two fuzzy sets A, B in X.

Proof. It is clear that

$$p^*pclA \bigvee p^*pclB \subseteq p^*pcl(A \bigvee B) ...(1)$$

Conversely, let $x_\alpha \in p^*pcl(A \bigvee B)$. Then for any fuzzy p^*-preopen q-nbd U of x_α, $Uq(A \bigvee B)$ which implies that there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1$. So either $U(y) + A(y) > 1$. Then UqA or $U(y) + B(y) > 1$. Then UqB. Accordingly, either $x_\alpha \in p^*pclA$ or $x_\alpha \in p^*pclB$. Hence $x_\alpha \in p^*pclA \bigvee p^*pclB$.

Result 3.8. For any fuzzy set A in an fts (X, τ),
(i) $p^*pcl(1_X \setminus A) = 1_X \setminus p^*pintA$,
(ii) $p^*pint(1_X \setminus A) = 1_X \setminus p^*pclA$.

Proof (i). Let $x_t \in p^*pcl(1_X \setminus A)$ for any $A \in I^X$. If possible, let $x_t \notin 1_X \setminus p^*pintA$. Then $x_t q p^*pintA$. Then there exists a fuzzy p^*-preopen set B in X with $B \leq A$ such that $x_t qB$. Then B is a fuzzy p^*-preopen q-nbd of x_t. By assumption, $Bq(1_X \setminus A)$ and so $Aq(1_X \setminus A)$, which is absurd.
Conversely, let $x_t \in 1_X \setminus p^*pintA$ for any $A \in I^X$. Then $x_t \not{q} p^*pintA$ and so $x_t \not{q} U$ for any fuzzy p^*-preopen set U in X with $U \leq A$. Then

$x_t \in 1_X \setminus U$ which is fuzzy p^*-preclosed set in X with $1_X \setminus A \leq 1_X \setminus U$. So $x_t \in p^*pcl(1_X \setminus A)$.
(ii) Writing $1_X \setminus A$ for A in (i), we get the result.

Definition 3.9. Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$, for each $x \in suppA$ [5]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy p^*-preopen), we call $\mathcal{U}$ is fuzzy open [5] (resp., fuzzy p^*-preopen) cover of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of X [2].

Definition 3.10. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to have a finite (resp., finite proximate) subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigvee \mathcal{U}_0 \geq A$ [5] (resp., $\bigvee\{clU : U \in \mathcal{U}_0\} \geq A$ [8]). In particular, if $A = 1_X$, we get $\bigvee \mathcal{U}_0 = 1_X$ (resp., $\bigvee\{clU : U \in \mathcal{U}_0\} = 1_X$ [3]).

Definition 3.11 [3]. An fts (X, τ) is called fuzzy almost compact space if every fuzzy open cover has a finite proximate subcover.

4. Fuzzy p^*-Precompact Space : Some Characterizations

In this section fuzzy p^*-precompactness is introduced and studied by fuzzy p^*-preopen and fuzzy regularly p^*-preopen sets and characterize this space via fuzzy net and prefilterbase.

Definition 4.1. A fuzzy set A in an fts (X, τ) is said to be a fuzzy p^*-precompact set if every fuzzy p^*-preopen cover $\mathcal{U}$ of A has a finite p^*p-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{p^*pclU : U \in \mathcal{U}_0\} \geq A$. If, in addition, $A = 1_X$, we say that the fts X is fuzzy p^*-precompact space.

Definition 4.2. Let x_α be a fuzzy point in an fts (X, τ). A prefilterbase $\mathcal{F}$ on X is said to
(a) p^*p-adhere at x_α, written as $x_\alpha \in p^*p\text{-ad}\mathcal{F}$, if for each fuzzy p^*-preopen q-nbd U of x_α and each $F \in \mathcal{F}$, Fqp^*pclU, i.e., $x_\alpha \in p^*pclF$, for each $F \in \mathcal{F}$;
(b) p^*p-converge to x_α, written as $\mathcal{F}\overrightarrow{p^*p}x_\alpha$, if to each fuzzy p^*-preopen q-nbd U of x_α, there corresponds some $F \in \mathcal{F}$ such that $F \leq p^*pclU$.

Definition 4.3. Let x_α be a fuzzy point in an fts (X, τ). A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to
(a) p^*p-adhere at x_α, denoted by $x_\alpha \in p^*p\text{-ad}(S_n)$, if for each fuzzy p^*-preopen q-nbd U of x_α and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m qp^*pclU$;
(b) p^*p-converge to x_α, denoted by $S_n\overrightarrow{p^*p}x_\alpha$, if for each fuzzy p^*-preopen q-nbd U of x_α, there exists $m \in D$ such that $S_n qp^*pclU$,

for all $n \geq m (n \in D)$.

Theorem 4.4. For a fuzzy set A in an fts X, the following statements are equivalent:
(a) A is a fuzzy p^*-precompact set,
(b) for every prefilterbase $\mathcal{B}$ in X, $[\bigwedge\{p^*pclB : B \in \mathcal{B}\}] \bigwedge A = 0_X \Rightarrow$ there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{p^*pintB : B \in \mathcal{B}_0\} \not{q} A$,
(c) for any family $\mathcal{F}$ of fuzzy p^*-preclosed sets in X with $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{p^*pintF : F \in \mathcal{F}_0\} \not{q} A$,
(d) every prefilterbase on X, each member of which is q-coincident with A, p^*p-adheres at some fuzzy point in A.

Proof (a) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{p^*pclB : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then for any $x \in suppA$, $[\bigwedge\{p^*pclB : B \in \mathcal{B}\}](x) = 0 \Rightarrow 1 - [\bigwedge\{p^*pclB(x) : B \in \mathcal{B}\}] = 1 \Rightarrow \bigvee[(1_X \setminus p^*pclB)(x) : B \in \mathcal{B}] = 1 \Rightarrow sup\{p^*pint(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1 \Rightarrow \{p^*pint(1_X \setminus B) : B \in \mathcal{B}\}$ is a fuzzy p^*-preopen cover of A. By (a), there exists a finite p^*p-proximate subcover $\{p^*pint(1_X \setminus B_1), p^*pint(1_X \setminus B_2), ..., p^*pint(1_X \setminus B_n)\}$ (say) of it for A. Thus $A \leq \bigvee_{i=1}^{n} p^*pcl(p^*pint(1_X \setminus B_i)) = \bigvee_{i=1}^{n}[1_X \setminus p^*pint(p^*pclB_i)] = 1_X \setminus \bigwedge_{i=1}^{n} p^*pint(p^*pclB_i) \Rightarrow \bigwedge_{i=1}^{n} p^*pint(p^*pclB_i) \leq 1_X \setminus A \Rightarrow A \not{q} \bigwedge_{i=1}^{n} p^*pint(p^*pclB_i) \Rightarrow A \not{q} \bigwedge_{i=1}^{n} p^*pintB_i$.

(b) $\Rightarrow$ (a). Let the condition (b) hold, and suppose that there exists a fuzzy p^*-preopen cover $\mathcal{U}$ of A having no finite p^*p-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $sup\{p^*pclU(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - sup\{(p^*pclU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0 \Rightarrow inf\{(1_X \setminus p^*pclU)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0} (1_X \setminus p^*pclU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ (=$\mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^{n} p^*pint(1_X \setminus p^*pclU_i) \not{q} A$, then $A \leq 1_X \setminus \bigwedge_{i=1}^{n} p^*pint(1_X \setminus p^*pclU_i) = \bigvee_{i=1}^{n}[1_X \setminus p^*pint(1_X \setminus p^*pclU_i)] =$

$\bigvee\limits_{i=1}^{n} p^*pcl(p^*pclU_i) = \bigvee\limits_{i=1}^{n} p^*pclU_i$ (by Result 3.6). Thus $\mathcal{U}$ has a finite p^*p-proximate subcover for A, contradicts our hypothesis. Hence for every finite subcollection $\{\bigwedge\limits_{U\in\mathcal{U}_1}(1_X\setminus p^*pclU), ..., \bigwedge\limits_{U\in\mathcal{U}_k}(1_X\setminus p^*pclU)\}$ of $\mathcal{B}$, where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subset of $\mathcal{U}$, we have $[\bigwedge\limits_{U\in\mathcal{U}_1\bigvee ... \bigvee\mathcal{U}_k} p^*pint(1_X\setminus p^*pclU)]qA$. By(b), $[\bigwedge\limits_{U\in\mathcal{U}} p^*pcl(1_X\setminus p^*pclU)]\bigwedge A \neq 0_X$. Then there exists $x \in suppA$, such that $\inf\limits_{U\in\mathcal{U}}[p^*pcl(1_X\setminus p^*pclU)](x) > 0 \Rightarrow 1-\inf\limits_{U\in\mathcal{U}}[p^*pcl(1_X\setminus p^*pclU)](x) < 1 \Rightarrow \sup\limits_{U\in\mathcal{U}}[1_X\setminus p^*pcl(1_X\setminus p^*pclU)](x) < 1 \Rightarrow \sup\limits_{U\in\mathcal{U}}U(x) \leq \sup\limits_{U\in\mathcal{U}}p^*pint(p^*pclU)(x) < 1$ which contradicts that $\mathcal{U}$ is a fuzzy p^*-preopen cover of A.

(a) $\Rightarrow$ (c). Let $\mathcal{F}$ be a family of fuzzy p^*-preclosed sets in X such that $\bigwedge\{F : F\in\mathcal{F}\}\bigwedge A = 0_X$. Then for each $x\in suppA$ and for each positive integer n, there exists some $F_n\in\mathcal{F}$ such that $F_n(x) < 1/n \Rightarrow 1-F_n(x) > 1-1/n \Rightarrow \sup\limits_{F\in\mathcal{F}}[(1_X\setminus F)(x)] = 1$ and so $\{1_X\setminus F : F\in\mathcal{F}\}$ is a fuzzy p^*-preopen cover of A. By (a), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigvee\limits_{F\in\mathcal{F}_0} p^*pcl(1_X\setminus F) \Rightarrow 1_X\setminus A \geq 1_X\setminus \bigvee\limits_{F\in\mathcal{F}_0} p^*pcl(1_X\setminus F) = \bigwedge\limits_{F\in\mathcal{F}_0}(1_X\setminus p^*pcl(1_X\setminus F)) = \bigwedge\limits_{F\in\mathcal{F}_0} p^*pintF$. Hence $A\not{q}(\bigwedge\limits_{F\in\mathcal{F}_0} p^*pintF)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(c) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{p^*pclB : B\in\mathcal{B}\}]\bigwedge A = 0_X$. Then the family $\mathcal{F} = \{p^*pclB : B\in\mathcal{B}\}$ is a family of fuzzy p^*-preclosed sets in X with $(\bigwedge F)\bigwedge A = 0_X$. By (c), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigwedge\{p^*pint(p^*pclB) : B\in\mathcal{B}_0\}]\not{q}A \Rightarrow (\bigwedge\limits_{B\in\mathcal{B}_0} p^*pintB)\not{q}A$.

(a) $\Rightarrow$ (d). Let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. If possible, let $\mathcal{F}$ do not p^*p-adhere at any fuzzy point in A. Then for each $x\in suppA$, there exists $n_x\in\mathcal{N}$ such that $x_{1/n_x}\in A$. Then there are a fuzzy p^*-preopen set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x}qU^x_{n_x}$ and $p^*pclU^x_{n_x}\not{q}F^x_{n_x}$. Thus $U^x_{n_x}(x) > 1-1/n_x$ so that $sup\{U^x_n(x) : n\in\mathcal{N}, n\geq n_x\} = 1$. Thus $\{U^x_n : n\in\mathcal{N}, n\geq n_x, x\in suppA\}$ forms a fuzzy p^*-preopen cover of

A. By (a), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in \mathcal{N}$ such that $A \leq \bigvee_{i=1}^{k} p^*pclU^{x_i}_{n_{x_i}}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigwedge_{i=1}^{k} F^{x_i}_{n_i}$. Then $F \not{q} [\bigvee_{i=1}^{k} p^*pclU^{x_i}_{n_{x_i}}]$, i.e., $F \not{q} A$, a contradiction.

(d) $\Rightarrow$ (a). If possible, let there exist a fuzzy p^*-preopen cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\bigvee\{p^*pclU : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigvee_{U \in \mathcal{U}_0} p^*pclU : \mathcal{U}_0$ is a finite subset of $\mathcal{U}\}$ is a prefilterbase on X such that FqA, for each $F \in \mathcal{F}$. By (d), $\mathcal{F}$ p^*p-adheres at some fuzzy point $x_\alpha \in A$. As $\mathcal{U}$ is a fuzzy cover of A, $\sup_{U \in \mathcal{U}} U(x) = 1 \Rightarrow$ there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1 - \alpha \Rightarrow x_\alpha q U_0$. As $x_\alpha \in p^*p$-ad$\mathcal{F}$ and $1_X \setminus p^*pclU_0 \in \mathcal{F}$, we have $p^*pclU_0 q (1_X \setminus p^*pclU_0)$, a contradiction.

Theorem 4.5. For a fuzzy set A in an fts X, the following implications hold :

(a) every fuzzy net in A p^*p-adheres at some fuzzy point in A,

$\Leftrightarrow$(b) every fuzzy net in A has a p^*p-convergent fuzzy subnet,

$\Leftrightarrow$ (c) every prefilterbase in A p^*p-adheres at some fuzzy point in A,

$\Rightarrow$ (d) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha \in \Lambda} p^*pclB_\alpha] \bigwedge A = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) \bigwedge A = 0_X$,

$\Rightarrow$ (e) A is fuzzy p^*-precompact set.

Proof (a) $\Rightarrow$ (b). Let a fuzzy net $\{S_n : n \in (D, \geq)\}$ in A where $(D, \geq)$ is a directed set, p^*p-adhere at a fuzzy point $x_\alpha \in A$. Let Q_{x_α} denote the set of the fuzzy p^*p-closures of all fuzzy p^*-preopen q-nbds of x_α. For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n q B$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n q B$. Then $(E, \gg)$ is a directed set where $(m, C) \gg (n, B)$ if and only if $m \geq n$ in D and $C \leq B$. Then $T : (E, \gg) \to (X, \tau)$ given by $T(n, B) = S_n$, is a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$. Let V be any fuzzy p^*-preopen q-nbd of x_α. Then there is $n \in D$ such that that $(n, p^*pclV) \in E$ and hence $S_n q p^*pclV$. Now, for any $(m, U) \gg (n, p^*pclV)$, $T(m, U) = S_m q U \leq p^*pclV$. Then $T(m, U) q p^*pclV$. Hence $T \xrightarrow{p^*p} x_\alpha$.

(b) $\Rightarrow$ (a). If a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not p^*p-adhere at a

fuzzy point x_α, then there is a fuzzy p^*-preopen q-nbd U of x_α and an $n \in D$ such that $S_m \not{q} p^*pclU$, for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can p^*p-converge to x_α.
(a) $\Rightarrow$ (c). Let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A. For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \in F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ if and only if $F_\alpha \leq F_\beta$. By (a), the fuzzy net S p^*p-adheres at some fuzzy point x_t $(0 < t \leq 1) \in A$. Then for any fuzzy p^*-preopen q-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} q p^*pclU$. Then $F_\beta q p^*pclU$ and hence $F_\alpha q p^*pclU$. Thus $\mathcal{F}$ p^*p-adheres at x_t.
(c) $\Rightarrow$ (a). Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. By (c), there exists a fuzzy point $a_\alpha \in A$ such that $\mathcal{F}$ p^*p-adhere at a_α. Then for each fuzzy p^*-preopen q-nbd U of a_α and each $F \in \mathcal{F}$, $F q p^*pclU$, i.e., $p^*pclU q T_n$, for all $n \in D$. Hence the given fuzzy net p^*p-adheres at a_α.
(c) $\Rightarrow$ (d). Let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ, $(\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha) \bigwedge A \neq 0_X$. Then $\mathcal{F} = \{(\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha) \bigwedge A : \Lambda_0 \text{ is a finite subset of } \Lambda\}$ is a prefilterbase in A. By (c), $\mathcal{F}$ p^*p-adheres at some fuzzy point $a_t \in A$ $(0 < t \leq 1)$. Then for each $\alpha \in \Lambda$ and each fuzzy p^*-preopen q-nbd U of a_t, $B_\alpha q p^*pclU$, i.e., $a_t \leq p^*pclB_\alpha$, for each $\alpha \in \Lambda$. Consequently, $(\bigwedge\limits_{\alpha \in \Lambda} p^*pclB_\alpha) \bigwedge A \neq 0_X$.
(d) $\Rightarrow$ (e). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy p^*-preopen cover of a fuzzy set A. Then by (d), $A \bigwedge [\bigwedge\limits_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = A \bigwedge [1_X \setminus \bigvee\limits_{\alpha \in \Lambda} U_\alpha] = 0_X$. If for some $\alpha \in \Lambda$, $1_X \setminus p^*pclU_\alpha = 0_X$, then we are done. If $1_X \setminus p^*pclU_\alpha$ $(=B_\alpha$, say$) \neq 0_X$, then for each $\alpha \in \Lambda$, $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigwedge\limits_{\alpha \in \Lambda} p^*pclB_\alpha \leq \bigwedge\limits_{\alpha \in \Lambda} (1_X \setminus U_\alpha)$. In fact, let x_t $(0 < t \leq 1)$ be a fuzzy point such that $x_t \in \beta pclB_\alpha = \beta pcl(1_X \setminus \beta pclU_\alpha)$. If $x_t q U_\alpha$, then $p^*pclU_\alpha q (1_X \setminus p^*pclU_\alpha)$, which is absurd. Hence $x_t \not{q} U_\alpha \Rightarrow x_t \in 1_X \setminus U_\alpha$. Then $[\bigwedge\limits_{\alpha \in \Lambda} p^*pclB_\alpha] \bigwedge A \leq A \bigwedge [\bigwedge\limits_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = 0_X$. By (d), there

exists a finite subset Λ_0 of Λ such that $[\bigwedge_{\alpha\in\Lambda_0} B_\alpha]\bigwedge A = 0_X$, i.e., $A \leq 1_X \setminus \bigwedge_{\alpha\in\Lambda_0} B_\alpha = \bigvee_{\alpha\in\Lambda_0}(1_X \setminus B_\alpha) = \bigvee_{\alpha\in\Lambda_0} p^*pclU_\alpha$ and (e) follows.

Definition 4.6. A fuzzy set A in an fts (X,τ) is said to be fuzzy regularly p^*-preopen if $A = p^*pint(p^*pclA)$. The complement of such a set is called fuzzy regularly p^*-preclosed.

Definition 4.7. A fuzzy point x_α in X is said to be a fuzzy p^*p-cluster point of a prefilterbase $\mathcal{B}$ if $x_\alpha \in p^*pclB$, for all $B \in \mathcal{B}$. If, in addition, $x_\alpha \in A$, for a fuzzy set A, then $\mathcal{B}$ is said to have a fuzzy p^*p-cluster point in A.

Theorem 4.8. A fuzzy set A in an fts (X,τ) is fuzzy p^*-precompact if and only if for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2, ..., F_n$ from $\mathcal{F}$ and for any fuzzy regularly p^*-preclosed set C containing A, one has $(F_1 \bigwedge ... \bigwedge F_n)qC$, $\mathcal{F}$ has a fuzzy p^*p-cluster point in A.

Proof. Let A be fuzzy p^*-precompact set and suppose $\mathcal{F}$ be a prefilterbase in X such that $[\bigwedge\{p^*pclF : F \in \mathcal{F}\}]\bigwedge A = 0_X$...(1). Let $x \in suppA$. Consider any $n \in \mathcal{N}$ such that $1/n < A(x)$, i.e., $x_{1/n} \in A$. By (1), $x_{1/n} \notin p^*pclF_x^n$, for some $F_x^n \in \mathcal{F}$. Then there exists a fuzzy p^*-preopen q-nbd U_x^n of $x_{1/n}$ such that $p^*pclU_x^n \not{q} F_x^n$. Now $U_x^n(x) > 1 - 1/n \Rightarrow sup\{U_x^n(x) : 1/n < A(x), n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{U} = \{U_x^n : x \in suppA, n \in \mathcal{N}\}$ forms a fuzzy p^*-preopen cover of A such that for U_x^n, there exists $F_x^n \in \mathcal{F}$ with $U_x^n \not{q} F_x^n$. Since A is fuzzy p^*-precompact, there exist finitely many members $U_{x_1}^{n_1},,,,, U_{x_k}^{n_k}$ of $\mathcal{U}$ such that $A \leq \bigvee_{i=1}^{k} p^*pclU_{x_i}^{n_i} = p^*pcl(\bigvee_{i=1}^{k} U_{x_i}^{n_i})$ (by Result 3.7) (=U, say). Now $F_{x_1}^{n_1}, ..., F_{x_k}^{n_k} \in \mathcal{F}$ such that $U_{x_i}^{n_i} \not{q} F_{x_i}^{n_i}$ for $i = 1, 2, ..., k$. Now U is a fuzzy regularly p^*-preclosed set containing A such that $p^*pclU \not{q} (F_{x_1}^{n_1} \bigwedge .. \bigwedge F_{x_k}^{n_k}) \Rightarrow U \not{q} (F_{x_1}^{n_1} \bigwedge .. \bigwedge F_{x_k}^{n_k})$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy p^*p-cluster point in A. Then by hypothesis, there is a fuzzy regularly p^*-preclosed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigwedge \mathcal{B}_0)\not{q}C$. Then $(\bigwedge \mathcal{B}_0)\not{q}A$. By Theorem 4.4 (b) $\Rightarrow$ (a), A is fuzzy p^*-precompact set.

From Theorem 4.4, Theorem 4.5 and Theorem 4.8, we have the characterizations of fuzzy p^*-precompact space as follows.

Theorem 4.9. For an fts X, the following statements are equivalent :

(a) X is fuzzy p^*-precompact,
(b) every fuzzy net in X p^*p-adheres at some fuzzy point in X,
(c) every fuzzy net in X has a p^*p-convergent fuzzy subnet,
(d) every prefilterbase in X p^*p-adheres at some fuzzy point in X,
(e) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha\in\Lambda} p^*pclB_\alpha] = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha\in\Lambda_0} B_\alpha) = 0_X$,
(f) for every prefilterbase $\mathcal{B}$ in X with $\bigwedge\{p^*pclB : B \in \mathcal{B}\} = 0_X$, there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{p^*pintB : B \in \mathcal{B}_0\} = 0_X$,
(g) for any family $\mathcal{F}$ of fuzzy p^*-preclosed sets in X with $\bigwedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{p^*pintF : F \in \mathcal{F}_0\} = 0_X$.

Theorem 4.10. An fts X is fuzzy p^*-precompact if and only if for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy p^*-preopen sets in X having finite intersection property $\bigwedge\{p^*pclF_\alpha : \alpha \in \Lambda\} \neq 0_X$.

Proof. Let X be fuzzy p^*-precompact space and $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy p^*-preopen sets in X with finite intersection property. Suppose $\bigwedge\{p^*pclF_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\{1_X \setminus p^*pclF_\alpha : \alpha \in \Lambda\}$ is a fuzzy p^*-preopen cover of X. By hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \bigvee\{p^*pcl(1_X \setminus p^*pclF_\alpha) : \alpha \in \Lambda_0\} = \bigvee\{1_X \setminus p^*pint(p^*pclF_\alpha) : \alpha \in \Lambda_0\} \leq \bigvee\{1_X \setminus F_\alpha : \alpha \in \Lambda_0\} = 1_X \setminus \bigwedge_{\alpha\in\Lambda_0} F_\alpha \Rightarrow \bigwedge_{\alpha\in\Lambda_0} F_\alpha = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.
Conversely, suppose that X is not fuzzy p^*-precompact space. Then there is a fuzzy p^*-preopen cover $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of X such that for every finite subset Λ_0 of Λ, $\bigvee\{p^*pclF_\alpha : \alpha \in \Lambda_0\} \neq 1_X$. Then $1_X \setminus \bigvee\{p^*pclF_\alpha : \alpha \in \Lambda_0\} \neq 0_X \Rightarrow \bigwedge_{\alpha\in\Lambda_0} (1_X \setminus p^*pclF_\alpha) \neq 0_X$, for every finite subset Λ_0 of Λ. Thus $\{1_X \setminus p^*pclF_\alpha : \alpha \in \Lambda\}$ is a collection of fuzzy p^*-preopen sets with finite intersection property. By hypothesis, $\bigwedge_{\alpha\in\Lambda} p^*pcl(1_X \setminus p^*pclF_\alpha) \neq 0_X$, i.e., $1_X \setminus \bigvee_{\alpha\in\Lambda} p^*pint(p^*pclF_\alpha) \neq 0_X \Rightarrow \bigvee_{\alpha\in\Lambda} p^*pint(p^*pclF_\alpha) \neq 1_X$. Hence $\bigvee_{\alpha\in\Lambda} F_\alpha \neq 1_X$, a contradiction as $\mathcal{F}$ is a fuzzy p^*-preopen cover of X.

Definition 4.11. Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy p^*-preopen sets in X, i.e., for each member n of a directed set $(D, \geq)$,

S_n is a fuzzy p^*-preopen set in X. A fuzzy point x_α in X is said to be a fuzzy p^*p-cluster point of the fuzzy net if for every $n \in D$ and every fuzzy p^*-preopen q-nbd V of x_α, there exists $m \in D$ with $m \geq n$ such that $S_m q V$.

Theorem 4.12. An fts X is fuzzy p^*-precompact if and only if every fuzzy net of fuzzy p^*-preopen sets in X has a fuzzy p^*p-cluster point in X.

Proof. Let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy p^*-preopen sets in a fuzzy p^*-precompact space X. For each $n \in D$, let us put $F_n = p^*pcl[\bigvee\{S_m : m \in D \text{ and } m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$ is a family of fuzzy p^*-preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{p^*pintF : F \in \mathcal{F}_0\} \neq 0_X$. By Theorem 4.9 (a)$\Rightarrow$ (g), $\bigwedge_{n\in D} F_n \neq 0_X$. Let $x_\alpha \in \bigwedge_{n\in D} F_n$. Then $x_\alpha \in F_n$, for all $n \in D$. Thus for any fuzzy p^*-preopen q-nbd A of x_α and any $n \in D$, $Aq[\bigvee\{S_m : m \geq n\}]$ and so there exists some $m \in D$ with $m \geq n$ and $AqS_m \Rightarrow x_\alpha$ is a fuzzy p^*p-cluster point of $\mathcal{U}$.

Conversely, let $\mathcal{F}$ be a collection of fuzzy p^*-preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{p^*pintF : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '$\gg$' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ if and only if $F_1 \leq F_2$. Let $F^* = p^*pintF$, for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy p^*-preopen sets of X. By hypothesis, $\mathcal{U}$ has a fuzzy p^*p-cluster point, say x_α. We claim that $x_\alpha \in \bigwedge \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy p^*-preopen q-nbd of x_α. Since $F \in \mathcal{F}^*$ and x_α is a fuzzy p^*p-cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and $G^*qA \Rightarrow GqA \Rightarrow FqA \Rightarrow x_\alpha \in p^*pclF = F$, for each $F \in \mathcal{F} \Rightarrow x_\alpha \in \bigwedge \mathcal{F} \Rightarrow \bigwedge \mathcal{F} \neq 0_X$. By Theorem 4.9 (g) $\Rightarrow$ (a), X is fuzzy p^*-precompact space.

Definition 4.13. A fuzzy cover $\mathcal{U}$ by fuzzy p^*-preclosed sets of an fts (X, τ) will be called a fuzzy p^*p-cover of X if for each fuzzy point x_α $(0 < \alpha < 1)$ in X, there exists $U \in \mathcal{U}$ such that U is a fuzzy p^*-preopen nbd of x_α.

Theorem 4.14. An fts (X, τ) is fuzzy p^*-precompact if and only if every fuzzy p^*p-cover of X has a finite subcover.

Proof. Let X be fuzzy p^*-precompact space and $\mathcal{U}$ be any fuzzy p^*p-cover of X. Then for each $n \in \mathcal{N}$ with $n > 1$, there exist $U_x^n \in \mathcal{U}$ and a fuzzy β-preopen set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Then $V_x^n(x) \geq 1 - 1/n \Rightarrow sup\{V_x^n(x) : n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{V} = \{V_x^n :$

$x \in X, n \in \mathcal{N}, n > 1\}$ is a fuzzy p^*-preopen cover of X. As X is fuzzy p^*-precompact, there exist finitely many points $x_1, x_2, ..., x_m \in X$ and $n_1, n_2, ..., n_m \in N \setminus \{1\}$ such that $1_X = \bigvee_{k=1}^{m} p^*pclV_{x_k}^{n_k} \le \bigvee_{k=1}^{m} p^*pclU_{x_k}^{n_k} = \bigvee_{k=1}^{m} U_{x_k}^{n_k}$.

Conversely, let $\mathcal{U}$ be fuzzy p^*-preopen cover of X. For any fuzzy point x_α $(0 < \alpha < 1)$ in X, as $\sup_{U \in \mathcal{U}} U(x) = 1$, there exists $U_{x_\alpha} \in \mathcal{U}$ such that $U_{x_\alpha}(x) \ge \alpha$ $(0 < \alpha < 1)$. Then $\mathcal{V} = \{p^*pclU : U \in \mathcal{U}\}$ is a fuzzy p^*p-cover of X and the rest is clear.

The following theorem gives a necessary condition for an fts to be fuzzy p^*-precompact.

Theorem 4.15. If an fts X is fuzzy p^*-precompact, then every prefilterbase on X with at most one p^*p-adherent point is p^*p-convergent.

Proof. Let $\mathcal{F}$ be a prefilterbase with at most one p^*p-adherent point in a fuzzy p^*-precompact fts X. Then by Theorem 4.9, $\mathcal{F}$ has at least one p^*p-adherent point in X. Let x_α be the unique p^*p-adherent point of $\mathcal{F}$ and if possible, let $\mathcal{F}$ do not p^*p-converge to x_α. Then for some fuzzy p^*-preopen q-nbd U of x_α and for each $F \in \mathcal{F}$, $F \not\le p^*pclU$, so that $F \bigwedge \{1_X \setminus p^*pclU\} \ne 0_X$. Then $\mathcal{G} = \{F \bigwedge (1_X \setminus p^*pclU) : F \in \mathcal{F}\}$ is a prefilterbase in X and hence has a p^*p-adherent point y_t (say) in X. Now $p^*pclU \not{q} G$, for all $G \in \mathcal{G}$ so that $x_\alpha \ne y_t$. Again, for each fuzzy p^*-preopen q-nbd V of y_t and each $F \in \mathcal{F}$, $p^*pclV q (F \bigwedge (1_X \setminus p^*pclU)) \Rightarrow p^*pclV qF \Rightarrow y_t$ is a fuzzy p^*p-adherent point of $\mathcal{F}$, where $x_\alpha \ne y_t$. This contradicts the fact that x_α is the only fuzzy p^*p-adherent point of $\mathcal{F}$.

Some results on fuzzy p^*-precompactness of an fts are given by the following theorem.

Theorem 4.16. Let (X, τ) be an fts and $A \in I^X$. Then the following statements are true :

(a) If A is fuzzy p^*-precompact, then so is p^*pclA,

(b) Union of two fuzzy p^*-precompact sets is also so,

(c) If X is fuzzy p^*-precompact, then every fuzzy regularly p^*-preclosed set A in X is fuzzy p^*-precompact.

Proof (a). Let $\mathcal{U}$ be a fuzzy p^*-preopen cover of p^*pclA. Then $\mathcal{U}$ is also a fuzzy p^*-preopen cover of A. As A is fuzzy p^*-precompact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \le \bigvee \{p^*pclU : U \in \mathcal{U}_0\} = p^*pcl\{\bigvee U : U \in \mathcal{U}_0\} \Rightarrow p^*pclA \le p^*pcl\{p^*pcl[\bigvee \{U : U \in \mathcal{U}_0\}]\} = p^*pcl\{\bigvee U : U \in \mathcal{U}_0\} = \bigvee \{p^*pclU : U \in \mathcal{U}_0\}$. Hence the proof.

(b). Obvious.

(c). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy p^*-preopen cover of a fuzzy regularly p^*-preclosed set A in X. Then for each $x \notin suppA$, $A(x) = 0 \Rightarrow (1_X \setminus A)(x) = 1 \Rightarrow \mathcal{U} \bigvee \{(1_X \setminus A)\}$ is a fuzzy p^*-preopen cover of X. Since X is fuzzy p^*-precompact, there are finitely many members $U_1, U_2, ..., U_n$ in $\mathcal{U}$ such that $1_X = (p^*pclU_1 \bigvee ... \bigvee p^*pclU_n) \bigvee p^*pcl(1_X \setminus A)$. We claim that $p^*pintA \leq p^*pclU_1 \bigvee ... \bigvee p^*pclU_n$. If not, there exists a fuzzy point $x_t \in p^*pintA$, but $x_t \not\leq (p^*pclU_1 \bigvee ... \bigvee p^*pclU_n)$, i.e., $t > max\{(p^*pclU_1)(x), ..., (p^*pclU_n)(x)\}$. As $1_X = (p^*pclU_1 \bigvee ... \bigvee p^*pclU_n) \bigvee p^*pcl(1_X \setminus A)$, $[p^*pcl(1_X \setminus A)](x) = 1 \Rightarrow 1 - p^*pintA(x) = 1 \Rightarrow p^*pintA(x) = 0 \Rightarrow x_t \notin p^*pintA$, a contradiction. Hence $A = p^*pcl(p^*pintA) \leq p^*pcl(p^*pclU_1 \bigvee .... \bigvee p^*pclU_n) = p^*pclU_1 \bigvee ... \bigvee p^*pclU_n$ (by Result 3.6 and Result 3.7) $\Rightarrow A$ is fuzzy p^*-precompact set.

5. **Mutual Relationship**

Here we establish the mutual relationship between fuzzy almost compactness [3] and fuzzy p^*-precompactness. Then it is shown that fuzzy p^*-precompactness implies fuzzy almost compactness, but converse is true in fuzzy p^*-preregular space [1]. It is also established that fuzzy p^*-precompactness remains invariant under fuzzy p^*-preirresolute function [1].

Since for any fuzzy set A in an fts X, $p^*pclA \leq clA$ (as every fuzzy closed set is fuzzy p^*-preclosed [1]), we can state the following theorem easily.

Theorem 5.1. Every fuzzy p^*-precompact space is fuzzy almost compact.

To get the converse we have to recall the following definition and theorem for ready references.

Definition 5.2 [1]. An fts (X, τ) is said to be fuzzy p^*-preregular if for each fuzzy p^*-preclosed set F in X and each fuzzy point x_α in X with $x_\alpha q(1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy p^*-preopen set V in X such that $x_\alpha qU$, $F \leq V$ and $U \not{q} V$.

Theorem 5.3 [1]. An fts (X, τ) is fuzzy p^*-preregular iff every fuzzy p^*-preclosed set is fuzzy closed.

Theorem 5.4. A fuzzy p^*-preregular, fuzzy almost compact space X is fuzzy p^*-precompact.

Proof. Let $\mathcal{U}$ be a fuzzy p^*-preopen cover of a fuzzy p^*-preregular, fuzzy almost compact space X. Then by Theorem 5.3, $\mathcal{U}$ is a fuzzy

open cover of X. As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{clU : U \in \mathcal{U}_0\} = \bigvee\{p^*pclU : U \in \mathcal{U}_0\}$ (by Theorem 5.3) $= 1_X \Rightarrow X$ is fuzzy p^*-precompact.
Next we recall the following definition and theorem for ready references.

Definition 5.5 [1]. A function $f : X \to Y$ is said to be fuzzy p^*-preirresolute if the inverse image of every fuzzy p^*-preopen set in Y is fuzzy p^*-preopen in X.

Theorem 5.6 [1]. For a function $f : X \to Y$, the following statements are equivalent :
(i) f is fuzzy p^*-preirresolute,
(ii) $f(p^*pclA) \leq p^*pcl(f(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy p^*-preopen q-nbd V of $f(x_\alpha)$ in Y, there exists a fuzzy p^*-preopen q-nbd U of x_α in X such that $f(U) \leq V$.

Theorem 5.7. Fuzzy p^*-preirresolute image of a fuzzy p^*-precompact space is fuzzy p^*-precompact.

Proof. Let $f : X \to Y$ be fuzzy p^*-preirresolute surjection from a fuzzy p^*-precompact space X to an fts Y, and let $\mathcal{V}$ be a fuzzy p^*-preopen cover of Y. Let $x \in X$ and $f(x) = y$. Since $sup\{V(y) : V \in \mathcal{V}\} = 1$, for each $n \in \mathcal{N}$, there exists some $V_x^n \in \mathcal{V}$ with $V_x^n(y) > 1 - 1/n$ and so $y_{1/n} q V_x^n$. By fuzzy p^*-preirresoluteness of f, by Theorem 5.6 (i)$\Rightarrow$ (iii), $f(U_x^n) \leq V_x^n$, for some fuzzy p^*-preopen set U_x^n in X q-coincident with $x_{1/n}$. Since $U_x^n(x) > 1 - 1/n$, $sup\{U_x^n(x) : n \in \mathcal{N}\} = 1$. Then $\mathcal{U} = \{U_x^n : n \in \mathcal{N}, x \in X\}$ is a fuzzy p^*-preopen cover of X. By fuzzy p^*-precompactness of X, $\bigvee\limits_{i=1}^{k} p^*pclU_{x_i}^{n_i} = 1_X$, for some finite subcollection $\{U_{x_1}^{n_1}, ..., U_{x_k}^{n_k}\}$ of $\mathcal{U}$. Then $1_Y = f(\bigvee\limits_{i=1}^{k} p^*pclU_{x_i}^{n_i}) = \bigvee\limits_{i=1}^{k} f(p^*pclU_{x_i}^{n_i}) \leq \bigvee\limits_{i=1}^{k} p^*pcl(f(U_{x_i}^{n_i}))$ (by Theorem 5.6 (i) $\Rightarrow$ (ii)) $\leq \bigvee\limits_{i=1}^{k} p^*pclV_{x_i}^{n_i} \Rightarrow Y$ is fuzzy p^*-precompact space.

References

[1] Bhattacharyya, Anjana; p^***-preregular space in fuzzy setting**, The 16th International Conference Proceeding, MSAST 2022, December 21-23, 79-87.

[2] Chang, C.L.; **Fuzzy topological spaces**, J. Math. Anal. Appl., 24 (1968), 182-190.

[3] DiConcillio, A. and Gerla, G.; **Almost compactness in fuzzy topological spaces**, Fuzzy Sets and Systems, 13 (1984), 187-192.

[4] Fath Alla, M.A.; **On fuzzy topological spaces**, Ph.D. Thesis, Assiut Univ., Sohag, Egypt (1984).

[5] Ganguly, S. and Saha, S.; **A note on compactness in fuzzy setting**, Fuzzy Sets and Systems, 34 (1990), 117-124.

[6] Jaber, Saad Mahdi; **Fuzzy precompact space**,Journal of Physics: Conference Series, 1591 (2020), 012073.

[7] Lowen, R.; **Convergence in fuzzy topological spaces**,General Topology and Its Appl., 10 (1979), 147-160.

[8] Mukherjee, M.N. and Sinha, S.P.; **Almost compact fuzzy sets in fuzzy topological spaces**, Fuzzy Sets and Systems, 38 (1990), 389-396.

[9] Nanda, S.; **Strongly compact fuzzy topological spaces**, Fuzzy Sets and Systems, 42 (1991), 259-262.

[10] Pu, Pao Ming and Liu, Ying Ming; **Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith Convergence**, J. Math Anal. Appl. 76 (1980), 571-599.

[11] Zadeh, L.A.; Fuzzy Sets, **Inform. Control**, 8 (1965), 338-353.

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