THERMOECONOMIC OPTIMIZATION OF THE ENERGETIC PLANT WITH INTERNAL COMBUSTION ENGINES BY USING LINEAR PROGRAMMING METHODS

DUMITRU GHEORGHE, CARAGHIULEA MARIANA

"Dunărea de Jos" University of Galați

Abstract: In the paper there are presented a linear mathematical model of the thermoeconomic optimization of the energetic plants with internal combustion engines and solving coresponding linear programming methods.

Keywords: Thermoeconomical optimization, Internal Combustion Engine, Linear programing.

1. INTRODUCTION

Otimization of the energetic plants with internal combustion engines may be formulated by two separate situation , at that objective functions are determined by different independent variable and restrictive conditions as following:

-designing of the onergetic plants with internal combustion engines;
-operating of the energetic plants with internal combustion engines;
Statement problem of the thermoeconomical optimization must be made on the combined analysis , thermodynamic and economic analysis of the energetic plants with the internal combustion engines.

2. WORDING OF THE LINEAR OPTIMIZATION PROBLEM FOR ENERGETIC PLANTS DESIGNING WITH INTERNAL COMBUSTION ENGINES

Considering energetic plants with internal combustion engines composed by following subsystems :

- internal combustion engine;
- power transmission;
- consumer of energetic plant;

result different statement of the optimization problem of the different energetic plants with internal combustion engines and distinct destinations.
In the general case there is formulated optimization problem by adoption of an optimization criterion and restrictive conditions with inegualities and/or egualities types and without restrictive conditions.
For energetic plants with internal combustion engines used on means of transportation there are adopting levelized operating and maintenance specific costs in the their long life time (durability , workin life).
Independent variables of the specific costs of the energetic plants designing with internal combustion engines can be considered follows :

- type of the internal combustion engines (spark ignition engine , compresion ignition engine, supercharged engine , etc.) $x_1 \in N$;

- constructive characteristics and materials used for internal combustion engines construction), $x_2 \in N$;

- designing technology of internal combustion engines, $x_3 \in N$;
- manufacturing technology of internal combustion engines, $x_4 \in N$;
- total capital investiment cost rate, $x_5 \in R$;
- operating and maintenance cost rate , $x_6 \in R$;
- fuel cost rate or exergetic efficiency of the internal combustion engines, $x_7 \in R$ or $x'_7 \in R$;
- type of rower transmission (mechanical, hydraulic, pneumatic, electric, combined, etc.), $x_8 \in N$;
- constructive characteristics and marerials used for power transmission, $x_9 \in N$;
- designing technology of the power transmission, $x_{10} \in N$;
- manufacturing technology of the power transmission, $x_{11} \in N$;
- total capital investiment cost rate, $x_{12} \in R$;
- operating and maintenance cost rate , $x_{13} \in R$;
- fuel cost rate or exergetic efficiency of the internal combustion engines, $x_{14} \in R$ or $x'_{14} \in R$;
- type of energetic plant consumer (driving wheels, driving axele, propeller, electric generator, compressor, etc.), $x_{15} \in N$;
- constructive characteristics and marerials used for consumer, $x_{16} \in N$;
- designing technology of the consumer , $x_{17} \in N$;
- manufacturing technology of the consumer , $x_{18} \in N$;
- total capital investiment cost rate, $x_{19} \in R$;
- operating and maintenance cost rate , $x_{20} \in R$;
- exergetic efficiency of the consumer or corresponding fuel cost rate, $x'_{21} \in R$; $x_{21} \in R$.

Levelized specific cost of the energetic plants with internal combustion engines can be done in €/ h, € / year , € / (t.km) , € / (kW.h),etc., for a fixed operating regime utilizing formula :

$$c(\vec{x}) = \sum_{i=1}^{21} c_i x_i = \vec{c}^{\,T} \vec{x} = f(x_1, x_2, \ldots x_{21}) \tag{1}$$

$$\vec{x} = (x_1, \ldots x_{21});$$

$$\vec{c} = (c_1, \ldots c_{21});$$

where : $\vec{x}$ -vector of the independent variables;

$\vec{c}$ - vector of the levelized specific costs;

Conditions type restrictions can be written in following forms :

- energetic plant mass with the internal combustion engines can’t exceed a admissible mass, b_1

$$a_{11}x_1 + a_{12}x_2 + a_{14}x_4 + a_{17}x_7 + a_{18}x_8 + a_{110}x_{10} + a_{113}x_{13} + a_{114}x_{14} \le b_1 \tag{2}$$

- maximum level of the chemical pollution of the energetic plant with internal combustion engines can’t exceed admissible level , b_2 :

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 \le b_2 \tag{3}$$

- maximum level of the acoustical pollution of the energetic plant with internal combustion engines can’t exceed admissible level , b_3 :

$$a_{31}x_1 + a_{32}x_2 + a_{34}x_4 + a_{36}x_6 + a_{37}x_7 + a_{38}x_8 + a_{39}x_9 + a_{311}x_{11} + a_{313}x_{13} + a_{314}x_{14} + a_{315}x_{15} + a_{316}x_{16} + a_{318}x_{18} + a_{320}x_{20} + a_{421}x_{21} \le b_3 \tag{4}$$

-durability of energetic plant with internal combustion enginea can’t be less than admissible durability, b_4 :

$$a_{41}x_1 + a_{42}x_2 + a_{44}x_4 + a_{36}x_6 + a_{47}x_7 + a_{48}x_8 + a_{49}x_9 + a_{411}x_{11} + a_{413}x_{13} + a_{414}x_{14} + a_{415}x_{15} + a_{416}x_{16} + a_{418}x_{18} + a_{420}x_{20} + a_{421}x_{21} \geq b_4 \quad (5)$$

where:

a_{ij}, i = 1,… , 4 ; j = 1,…….,21 – experimental coefficients.

In the general case, working optimization problem of the energetic plant with internal combustion engines has the from :

$$c(\vec{x}) = \sum_{i=1}^{n} c_i x_i = \vec{c}^{\,T}\vec{x} = f(x_1, \ldots x_n) \quad (6)$$

$$\vec{x} = (x_1, \ldots x_n);$$

$$\vec{c} = (c_1, \ldots c_n)$$

and conditions type restriction can be written in following forms :

$$g_j(x_1, \ldots, x_m) \leq 0, \; (j = 1, \ldots, m); \quad (7)$$

$$\vec{x} \geq 0; \quad (8)$$

or

$$g_j(x_1, \ldots, x_m) \geq 0, \; (j = 1, \ldots, m); \quad (9)$$

$$\vec{x} \geq 0;$$

or

$$g_j(x_1, \ldots, x_1) \leq 0, \; (j = 1, \ldots, 1); \quad (10)$$

$$g_j(x_{1+1}, \ldots, x_m) \geq 0, \; (j = 1+1, \ldots, m); \quad (11)$$

$$\vec{x} \geq 0;$$

or

$$g_j(x_1, \ldots, x_1) \leq 0, \; (j = 1, \ldots, 1); \quad (12)$$

$$g_j(x_{1+1}, \ldots, x_r) = 0, \; (j = 1+1, \ldots, r); \quad (13)$$

$$g_j(x_{r+1}, \ldots, x_m) \geq 0, \; (j = r+1, \ldots, m); \quad (14)$$

$$\vec{x} \geq 0;$$

In the case of linear problem of optimization there are:
$f(x_1, \ldots, x_n)$ – linear function of optimization:

$$\min c(\vec{x}) = \sum_{i=1}^{n} c_i x_i = \vec{c}^{\,T}\vec{x} = \text{optimum } c(\vec{x}) = \sum_{i=1}^{n} c_i x_i = \vec{c}^{\,T}\vec{x}; \quad (15)$$

restriction conditions:

$$A\vec{x} - \vec{b} \leq 0 \;\text{ or }\; A\vec{x} - \vec{b} \geq 0 \;\text{ or }\; A\vec{x} - \vec{b} = 0; \quad (16)$$

nonnegative independent variables:

$$\vec{x} \geq 0 ;$$

where:

$A = ((a_{ij}))$, $i = 1,\ldots,m$; $j = 1,\ldots,n$, matrix $m \times n$;

$\vec{b} = (b_1,\ldots,b_m)^T \in R^m$ – m- dimensional column vector;

$\vec{x} = (x_1,\ldots,x^n)^T \in R^n$ –n - dimensional column vector of the unknowns;

$c=c_i$, $i = 1,n$, i-n- dimensional vector of specific costs for independent variables that are considered with deterministic character and in a developed from can be written standard optimization problems:

$$\min c = c^T \vec{x} \text{ ; } c{:}n \times 1$$

$$A\vec{x} - \vec{b} \leq 0,\ A{:}m \times n \text{ ; } b{:}\ m \times 1 \tag{17}$$

$$\vec{x} \geq 0 \text{ ; } \quad x : n \times 1;$$

A,c,b – deterministic matrix :

$$c = \begin{vmatrix} c_1 \\ . \\ . \\ . \\ c_n \end{vmatrix} ; \quad x = \begin{vmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{vmatrix} ; \quad A = \begin{vmatrix} a_{11}\ldots\ldots a_{1n} \\ \\ a_{m1}\ldots\ldots a_{mn} \end{vmatrix} ; \quad b = = \begin{vmatrix} b_1 \\ . \\ . \\ . \\ b_n \end{vmatrix} ; \tag{18}$$

or ,for m = 4 and n = 21 there are :

$$\min c = \sum_{i=1}^{21} c_i x_i \ (€ / h); \tag{15'}$$

$$\sum_{i=1}^{21} a_{1i} x_i \leq b_1 ;\ \sum_{i=1}^{21} a_{2i} x_i \leq b_2 ;\ \sum_{i=1}^{21} a_{3i} x_i \leq b_3 ;\ \sum_{i=1}^{21} a_{4i} x_i \leq b_4 ;\ x_i \leq b_4 ; \tag{16}$$

$$\begin{vmatrix} a_{11}\ldots\ldots a_{1,21} \\ . \\ . \\ a_{4,1}\ldots\ldots a_{4,21} \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ . \\ . \\ .x_{21} \end{vmatrix} \leq \begin{vmatrix} b_1 \\ . \\ . \\ b_4 \end{vmatrix} ; \tag{17'}$$

3. SOLVING METHOD FOR OPTIMIZATION PROBLEM BY LINEAR PROGRAMMING

It should be given solutions for problems:

-linear programming for canonic problems :

$$\inf_{\vec{x}\in X} \sum_{1-1}^{1=n} c_i \ x_i \ , \ X = \left\{ \vec{x} \in \mathbf{R}^n \middle| \ \vec{x} \geq 0, \sum a_{ij} \ x_i \leq b_i, \ j = 1,\ldots,m \ \right\}; \tag{18}$$

-linear programming for standard problems:

$$\inf_{\vec{x}\in X} \sum_{1-1}^{1=n} c_i \ x_i \ , \ X = \left\{ \vec{x} \in \mathbf{R}^n \middle| \ \vec{x} \geq 0, \sum a_{ij} \ x_i = b_i, \ j = 1,\ldots,m \ \right\}; \tag{19}$$

If there are noted by $\vec{P}_1,\ldots, \vec{P}_n$ n – dimensional column – vectors formed by column – elements of the matrix A, it can write:

$$\vec{P}_1 = \begin{vmatrix} a_{1i} \\ . \\ . \\ . \\ a_{mi} \end{vmatrix} ; \ i = 1,\ldots,n; \tag{20}$$

and restriction conditions can be written in form :

$$\sum_{i=1}^{1=n} x_i \ \vec{P}_i = \vec{b} \ ; \tag{21}$$

or

$$A\vec{x} - \vec{b} = 0 \ ; \tag{21'}$$

For r = rang A there are $\vec{P}_i$, i = r linear independent vectors and C_n^r admisible bases from column- vectors of the matrix A.

By admisible associated base of the solution $\vec{x}$ it is understood the independent vectors system $\vec{P}_i,\ldots,\vec{P}_{i\ r}$ that gives relation:

$$\sum_{k=1}^{m} x_{ik} \vec{P}_{ik} = \vec{b} \ . \tag{22}$$

There are known simplex method [1],[2],[3],[4],[5],[6], proposed by G. Dantzing, that builds successively basic solutions of the programing problem, as the every new solution obtained be better that previous and objective function have the minimum absolute value.

In the case of linear restriction A $\vec{x} - \vec{b} \geq 0$ there are used algorithm of Agmon – Motzkin – Schoenberg, [6] . Solving methods are in the tabular form and there are numerical solving methods, [5].

4. CONCLUSIONS

Thermoeconomic optimization of the energetic plants with internal combustion engines can be formulated as a optimization problem with using linear programming methods .That is the simplest working of the optimization problem and using of the algorithm proposed by Dantzig is the simplest solving of problem. It is possible used Mathlab program for solving a correct optimization formulated problem.

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