NEW DEFINITIONS AND FLUID-ENERGETICS CORELATIONS

BENCHE VICTOR, HUMINIC ANGEL, HUMINIC GABRIELA

TRANSILVANIA University from Brasov

Abstract: Having as reference the image of the Euler-D'Alembert stream-tube, the paper deals with the motion of real fluids from the energetics point of view, being proposed the energetic stream-tube concept as a spatial representation of the power resource for a flowing fluid. In this way, there can be performed optimisations of the hydraulic circuits.

Keywords: flow in conduits, energetics stream-tube, energetics aureola

1. THEORETICAL CONSIDERATION

The energetic stream-tube is defined as that one what has the same geometrical axis and length of the real steam fluid and the cross section area equal with the stream power, N_h, in the same (geometrical) section (point in flow):

$$N_h = \dot{G} \cdot e = g \cdot \dot{m} \cdot e = \rho \cdot g \cdot Q \cdot e \; [W], \tag{1}$$

where:

N_h	$[W]$	- stream power in section;
$\dot{G}$	$[kgf/s]$	- gravimetric flow rate;
g	$[m/s^2]$	- acceleration due to gravity;
$\dot{m}$	$[kg/s]$	- mass flow rate;
ρ	$[kg/m^3]$	- density of fluid;
Q	$[m^3/s]$	- volumetric flow rate.

The specific energy (total head), e, of the fluid in the computational point is:

$$e = e_P + e_C = \left(z + \frac{p}{\rho g} \right) + \frac{v^2}{2g} \; [m.col.\ fluid] \tag{2}$$

where:

e_P	- potential energy;
e_C	- kinetic energy;
z	- geometrical height;
$\frac{p}{\rho g}$	- piezometric height;
$\frac{v^2}{2g}$	- kinetic height.

The energetic aureole (halo) of the physical fluid section (stream-tube) could have a rectangular section (see Figure 1) with the following dimensions: $H = e$ as height and $B = \dot{G}$ as base.

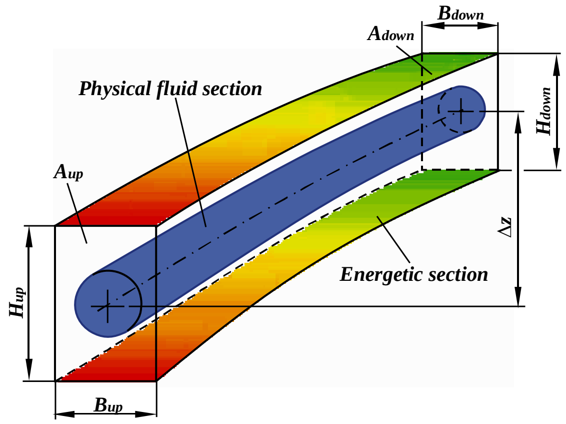

Fig. 1 - *The energetic aureole with rectangular section*

In this way the energetic section, A_e, can be write:

$$A_e = H \cdot B = N_h \; [W] \tag{3}$$

For circular sections (see Figure 2), the equivalent energetic diameter, D_e, results as:

$$D_e = \sqrt{\frac{4g}{\pi}}\sqrt{\dot{m} \cdot e} = 3.52\sqrt{\rho \cdot Q \cdot e} \; \left[W^{1/2}\right] \tag{4}$$

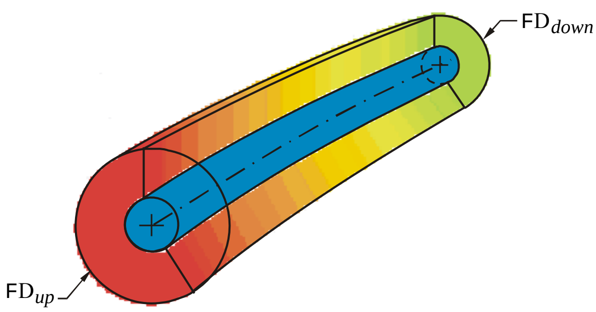

Fig. 2 - *The energetic aureole with circular section*

In the case of other sections an energetic hydraulic radius R_e can be defined as:

$$R_e = \frac{D_e}{4} = 0.88\sqrt{\rho Q e} \; \left[W^{1/2}\right] \tag{5}$$

The ideal energetic section is a straight one, by constant section, according with the image of the ideal flow (without energetics losses).

The real energetic section is a convergent one, because $A_{e\,down} < A_{e\,up}$, due to the head losses.

For fluids with the constant density, the following efficiency factors can be defined:

- hydraulic efficiency:

$$\eta_h = \frac{H_{down}}{H_{up}} = \frac{e_{down}}{e_{up}} \tag{6}$$

- volumetric efficiency:

$$\eta_V = \frac{B_{down}}{B_{up}} = \frac{Q_{down}}{Q_{up}} \tag{7}$$

According with Figure 1, there can be define the *indicated efficiency*, as energetic slope, with the ratio:

$$\eta = \frac{A_{e,down}}{A_{e,up}} = \eta_h \cdot \eta_V \tag{8}$$

In some studies concerning the forced pipe flow, it is convenient to use, sometimes, the specific energy diagram, which include the line of the pressure head and the line the total head (the line of energetic load). This diagram permits the visualisation of the flow from the energy point of view.
In order to have an image of the hydraulic power balance of the flow we are proposing a spatial image, 3D, through a third coordinate: $B = \dot{G}$ (this kind of representation can be extended also at other types of fluid motions e. g flows in open channels. There is starting from Equations of energy conservation written between two section, *up*- and *down*-stream:

$$\frac{v_{up}{}^2 - v_{down}{}^2}{2g} = \frac{\Delta p\big|_{up}^{down}}{\rho g} \; \left[m.col.\ fluid\right], \tag{9}$$

This Equation must be adjusted through two coefficients of losses: the coefficient of hydraulic losses η_h and the coefficient of volumetric losses η_V, also as well an coefficient of specific mass variation ε for expansible fluids. The specific potential energy variation could include also a variation of geometric height Δz :

$$\Delta e_P = \left|\Delta p \pm \rho g \Delta z\right|_{up}^{down} \tag{10}$$

In this way, the kinetic energy in the down-stream section $e_{C\ down}$ can be expressed as:

$$e_{C\ down} = k_e \Delta e_P\big|_{up}^{down} \; \left[m.col.fluid\right], \tag{11}$$

The dynamic pressure $p_{d\ down}$ in the same section is:

$$p_{d\ down} = \rho \frac{v_{down}^2}{2} = k_e \Delta p\big|_{up}^{down} \; \left[Pa\right] \; (\rho = ct.) \tag{12}$$

The correction factor ε, which is a function of the nature of fluid, state of it and the ratio p_{up} / p_{down}, is relating with the low-pressure changes Δp for gases and vapours having the speed up to $50\ m/s$. In the case of liquids ε is equal with the unit.

For the dimensionless factor of the energetic conversion, the following original expression is proposed:

$$k_e = \frac{\varepsilon^2}{\frac{1}{\eta_h^2} - \eta_V^2} \tag{13}$$

In the particular cases when $\rho = const.$ and $\varepsilon = 1$ the volumetric efficiency is $\eta_V = 1$. Consequently, the coefficient of energetic conversion becomes:

$$k_e = \frac{1}{\frac{1}{\eta_h^2} - 1} \tag{13'}$$

The hydraulic efficiency (coefficient of flow rate in some situations) has the expression:

$$\eta_h = \frac{1}{\sqrt{\alpha + \lambda \frac{l}{d} + \sum \xi}} = \frac{1}{\sqrt{\alpha + \lambda \frac{l_{tot}}{d}}} \tag{14}$$

where:
- l, d - the length and the diameter of the physical section (in the case others sections $d = 4R_h$;
- $\sum \xi$ - the sum of minor head losses coefficients;
- λ - the skin friction coefficient (Darcy-Weissbach);
- α - the Coriolis's coefficient.

For the particular cases $\sum \xi = 0$ and $l_{tot} = l$, the expression (14) becomes (14') and for effluent motions ($\lambda = 0$) (14"):

$$\eta_h = \frac{1}{\sqrt{\alpha + \lambda \frac{l}{d}}} \tag{14'}$$

$$\eta_h = \frac{1}{\sqrt{\alpha + \sum \xi}} \tag{14''}$$

The coefficient of proportionality k_e, propose, in equations (11) and (12) can be used as dimensionless indicator of interior energetics transformation of the section fluid, meaning the level of energetics conversion, taking also into consideration the head losses for real flows.
The pressure is the dynamic component of the hydraulic power, the kinematics component being the flow rate. We are considering the pressure as a specific hydraulic power, namely for a unitary flow rate. In this way, the relations (11) and (12) are representing the Equations of balances (or conversions) for specific hydraulic power: the equation (12) for a unitary volumetric flow rate and equation (11) for a unitary gravimeter flow rate.

2. APPLICATIONS

In the case of the effluent flows the hydraulic efficiency η_h became the flow rate factor μ which is dependent by the geometrical characteristics of the orifice, thickness of the wall and the nature and of the fluid state. The flow is determined of the difference of level $\Delta z = h$ and/or of a gauge pressure p_g, therefore:

$$e_p = h + \frac{p_g}{\rho g} \quad [m.col.fluid], \tag{15}$$

and

$$\Delta p = \rho g h + p_g \quad [Pa] \tag{16}$$

For supply pipes of hydraulic generators, also as for pipes in syphon (with the ascendancy aspiration branch) there are using as driving factor the relative pressure at aspiration:

$$\Delta p = |p_d| \quad [Pa], \tag{17}$$

In cases of exhaust pipes the additional pressure gauge is:

$$\Delta p = p_g \quad [Pa] \tag{18}$$

In the case of an axial wind turbine with area of rotor S placed in a stream with velocity v the maximum power is, according with Betz theory:

$$P_{max} = \frac{8}{9}\left(\frac{1}{2}\rho \cdot S \cdot v^3\right) = \frac{8}{9}\left(Q\frac{\rho \cdot v_{up}{}^2}{2}\right) = \frac{8}{9}\left(Q \cdot p_{d\,up}\right) \quad [W] \tag{19}$$

2.1. Numerical examples

It is considered an effluent motion through an annular orifice into a tank with small thickness wall, with the coefficient of flow $\mu = 0.64$, therefore $k_e = 0.41$, which indicate a small value of energy conversion. If a Borda annular nozzle with $\mu = 0.82$ is added to this orifice, the coefficient of energetic conversion becomes $k_e = 0.41$, which represent an increase with 64%. If a convergent nozzle formed after two circular arcs is used a, having $\mu = 0.95$ the coefficient of energetic conversion becomes $k_e = 0.9$.

For water jet having $Q = 1\ l/s$ and $v_s = 10\ m/s$, the energetic section, with rectangular sections, present:

in upstream: $H_{up} = 5.27\ m.col.H_2O$, $B_{up} = 1\ kg/s$, $A_{up} = 52.7\ W$;

in downstream: $H_{down} = 5\ m.col.H_2O$, $B_{down} = 1\ kg/s$, $A_{down} = 50\ W$ with rectangular sections.

The energetic aureole with annular sections is a stream-tube with small slope, having $D_{up} = 8.2\ \left[W^{1/2}\right]$ and $D_{down} = 8\ \left[W^{1/2}\right]$.

For a ducted axial wind-turbine (see Figure 3) having:

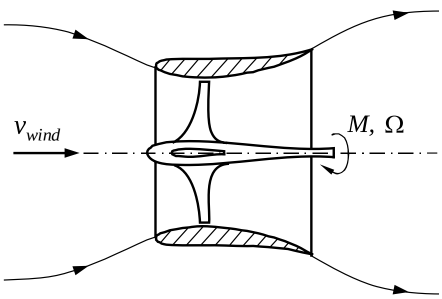

Fig. 3. *Ducted wind turbine*

$\mu = 0.5$	flow rate factor of inlet section,
$\zeta = 5\%$	volumetric losses due to the gap between end of the blades and annular minimum section of the tube, $\eta_V = 0.95$,
$v = 10\ m/s$	wind speed,
$S = 1\ m^2$	area of the rotor disc,

there are obtained the following parameters of the flow through the rotor of wind turbine: $Q = 10\ m^3/s$, $p_d = 60 Pa$, $N_1 = 540 W/m^2$, $k_e = 0.9$ and $P_{max} = \frac{8}{9} N_1 = 480 W/m^2$. Result an energetic section with $A_{up} = 600 W/m^2$ and $A_{down} = 480 W/m^2$ with an indicated efficiency $\eta = 0.8$.

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