NATURAL GAS DISTRIBUTION THROUGH POLYETHYLENE PIPELINES. REPARTITION PIPELINES CAPACITY OF TAKING OVER FLOW VARIATIONS

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Abstract: This paper proposes a genuine mathematical model for the study of the non-steady phenomena which characterize the slow motion through gas pipelines due to the change of gas inflow at the consumer. The software elaborated on the basis of this mathematical model allows the determination of the duration of the two stationary regimes between which the non-steady movement takes place

Keywords: gas, polyethylene, pipeline, flow, pressure, variation

1. MATHEMATICAL MODELING

Let us consider that all measures that characterize the gas in movement have constant average values in the pipeline's cross-section and variable values along pipeline's length and in time, except gas temperature and compressibility factor for which we will accept the average constant values T_m, Z_m.

As the polyethylene pipes are, from the hydraulic point of view, smooth, the friction coefficient λ values can be calculated as a function of *Reynolds* number either using the *Blasius* relationship (λ_1), for $3\cdot10^3 \le \mathrm{Re} \le 10^5$, or with *Nikuradse* formula (λ_2) for $\mathrm{Re} > 10^5$. The *Reynolds* number will be determined for the normal state according to the mass flow rate $\dot{m}$ expressed in kg/s and the pipe's diameter d in m. Consequently, we have the following relationships

$$\mathrm{Re}(d,\dot{m}) = 123{,}080\,\frac{\dot{m}}{d} \tag{1}$$

$$\lambda_1(d,\dot{m}) = 0.3164\,\mathrm{Re}(d,\dot{m})^{-0{,}25} \tag{2}$$

$$\lambda_2(d,\dot{m}) = 0.0032 + 0.221\,\mathrm{Re}(d,\dot{m})^{-0{,}237} \tag{3}$$

The mathematical model of the slow unsteady-state isothermal gas flow through pipes consists of the square pressure equation [3] associated with adequate initial and boundary conditions for pressure and mass flow rate

$$\frac{\partial P}{\partial t} = \alpha \sqrt{\frac{P}{\left|\frac{\partial P}{\partial x}\right|}} \frac{\partial^2 P}{\partial x^2} , \quad P(x,t) = p^2(x,t) , \quad \alpha = \sqrt{\frac{R\,T\,Y\,d}{\lambda}} \tag{4}$$

where

$$\alpha(d,\dot{m}) = \sqrt{R\,T_m\,Z_m}\; d^{0,5}\,\lambda(d,\dot{m})^{-0,5} . \tag{5}$$

The initial condition of the mathematical model expresses the square pressure repartition in the initial steady-state regime as

$$P(x,0) = P_1 - a_0\,\dot{m}_0^2\,x , \quad P_1 = p_1^2 , \quad a_0 = \frac{16\,R\,T_m\,Z_m}{\pi^2} d^{-5}\,\lambda(d,\dot{m}_0) . \tag{6}$$

The boundary conditions are written according to pipeline's work parameters: at the pipeline's initial end the pressure remains constant and at the final end the flow rate is variable:

$$t > 0 \begin{cases} P(0,t) = P_1 ; \\ \dfrac{\partial P}{\partial t}(l,t) = -a_2\,\dot{m}_2^2(t) , \quad a_2 = \dfrac{16RT_mZ_m}{\pi^2} d^{-5}\lambda(d,\dot{m}_2) . \end{cases} \tag{7}$$

where $\dot{m}_2(t)$ represents the gas mass flow delivered to the consumer, which is different from the initial one $\dot{m}_0$ corresponding to the original steady-state regime.

2. THE NUMERICAL MODEL

The equation (4) has obviously a non-linear character due to the irrational factor which makes impossible its analytical approach. This equation can be numerically solved using a finite-difference scheme associated with an adequate iterative procedure for eliminating the irrational factor. For this purpose we will transform the continuous domain of the square pressure values $C : [0 \le x \le l, 0 \le t \le T]$ into the discrete pattern $R_{i,j} : \lfloor x_i = (i-1)h,\ i = 1 \ldots N+1;\ t_j = j\,\tau,\ j = 0 \ldots M \rfloor$, where *i*, *j* are the spatial and temporal indexes, *h*, τ – the spatial and temporal steps, *N*, *M* – the total number of spatial and temporal steps respectively. Thus, the exact values of the square pressure *P*(*x*,*t*) will be replaced by the discrete approximate values $P_i^j = P(x_i, t_j)$.

We will use a finite difference computing scheme of the implicit type *Hyman–Kaplan*, known as being unconditionally stable and absolutely convergent [3]. The equation (4) transforms in the finite difference scheme

$$P_i^{j+1} - P_i^j = \alpha\,c\,A_i^{j+1}\left(P_{i-1}^{j+1} - 2\,P_i^{j+1} + P_{i+1}^{j+1}\right), \quad c = \frac{\tau\sqrt{2}}{h\sqrt{h}} , \tag{8}$$

where

$$A_i^{j+1} = \sqrt{\frac{P_i^{j+1}}{P_{i-1}^{j+1} - P_{i+1}^{j+1}}} \tag{9}$$

is the irrational factor introduced through the computing scheme.

The relationship (8) can also be written as

$$\Phi_i^{j+1} P_{i-1}^{j+1} - \Psi_i^{j+1} P_i^{j+1} + \Phi_i^{j+1} P_{i+1}^{j+1} = -P_i^j \ , \tag{10}$$

where, for $i = 2\ldots N$,

$$\Phi_i^{j+1} = c\,\alpha(d,\dot{m})A_i^{j+1} \ , \ \ \Psi_i^{j+1} = 2c\,\alpha(d,\dot{m})A_i^{j+1} + 1 \ . \tag{11}$$

The initial condition, for $i = 1\ldots N+1$, is

$$P_i^0 = P_1 - a_0\ \dot{m}_0^2\,(i-1)h \tag{12}$$

and the boundary conditions become

$$P_0^{j+1} = P_1 \ , \ \ P_{N-1}^{j+1} - 4P_N^{j+1} + 3P_{N+1}^{j+1} = -2\,h\,a(d,\dot{m}_2)\dot{m}_2^2 \ . \tag{13}$$

The relationships (12) and (13) complete the system generated by the scheme (8) thus allowing the calculation of the square pressure repartition P_i^{j+1} for $j = 1\ldots M$ and $i = 1\ldots N+1$ respectively. To solve the implicit equation system we will set the repartition of the square pressure corresponding to the known moment j as $P_0(i) = P_i^j$, and the repartition at the unknown moment $j+1$ as $P(i) = P_i^{j+1}$.

According to the solving procedure for the above mentioned implicit equation system we will get first successive approximate solutions which, by comparison, will give the right repartition. The approximate solutions successively obtained by solving the algebraic system generated by the adopted calculation scheme will be denoted as $P_1(i)$ for the first approximation, $P_2(i)$ and $P_3(i)$ for the following two approximations. Afterwards we will define the helping function $A(x)$ which allows the calculation of the A factor values generated by the scheme

$$A(x) = \sqrt{\frac{x(i)}{x(i-1) - x(i+1)}} \ , \tag{14}$$

for $i = 2\ldots N$, corresponding to the reduced pressure repartitions at the time steps imposed by calculus development. Thus, for launching the calculus we will put in equation (14) $x(i) = P_0(i)$ to get the first set of square pressure $P_1(i)$ values. For obtaining the second set of approximate values $P_2(i)$ we will put in equation (14) $x(i) = P_1(i)$ and finally the third set of approximate values $P_3(i)$ can be determined by putting $x(i) = P_2(i)$. These two last sets of values are compared as the iterative procedure imposes.

Thus the values of $\Phi(x)$ and $\Psi(x)$ expressions corresponding to the three sets of values of the square pressure repartitions

$$\Phi(i) = \Phi[x(i)] = c\,\alpha(d,\dot{m})A[x(i)] \ , \ \ \Psi(i) = \Psi[x(i)] = 2\,c\,\alpha(d,\dot{m})A[x(i)] + 1 \ , \tag{15}$$

can be calculated. The conditions (12) and (13) are written as

$$P_i^j = P_0(i) \ , \ \ i = 1\ldots N+1 \ ; \tag{16}$$

$$P(1) = P_1 \ ; \ \ \ P(N-1) - 4P(N) + 3P(N+1) = -2\,h\,a(d,\dot{m}_2)\dot{m}_2^2 \ . \tag{17}$$

In this case, the calculation scheme (8) becomes

$$-\Phi(i)P(i-1) + \Psi(i)P(i) - \Phi(i)P(i+1) = P_0(i) \tag{18}$$

and, associated with the conditions (16) and (17), generates the following system of equations

$$P(1) = P_1 \tag{19.a}$$

$$+\Psi(2)P(2) - \Phi(2)P(3) = P_0(2) + \Phi(2)P_1 \tag{19.b}$$

$$-\Phi(i)P(i-1) + \Psi(i)P(i) - \Phi(i)P(i+1) = P_0(i)\,,\quad i = 3\ldots N\,, \tag{19.c}$$

$$\left[-4\,\Phi(N) + \Psi(N)\right]P(N) + 2\,\Phi(N)P(N+1) = P_0(N) - 2\,h\,a(d,\dot{m})\dot{m}_2^2\,\Phi(N)\,. \tag{19.d}$$

The solution of the algebraic system (19) is the launching repartition $P_1(i)$ for $i = 2\ldots N+1$. The algebraic system has a *Jacobi* type (tridiagonal) coefficient matrix and can be easily solved by the *Tomas* procedure [2].
The computer program TRANZPOL has been developed on the basis of the previous algorithm. The program calculates the pressure repartition along a polyethylene gas pipe during the transient flow regime generated by the flow rate variation at the pipe's final end. The program is developed in the Delphi 5 programming medium.

3. SIMULATION OF A TRANSIENT REGIME IN A POLYETHYLENE GAS PIPE

Let us consider a polyethylene repartition gas pipe which transports the flow rate $\dot{m}_0$, running in a steady-state regime. The repartition of the square pressure along the pipe is

$$P(x) = P_1 - a_0\,\dot{m}_0^2\,x\ . \tag{20}$$

At a certain time, a new consumer starts working, which results in a flow rate change at the final end of the pipe

$$\dot{m}_2 = \dot{m}_0\,f\ , \tag{21}$$

where f is a greater-than-unity flow rate variation coefficient. From that moment, the gas movement will reach an unsteady-state character and in the pipe a transient process begins which will last until the square pressure repartition along the pipe will correspond to the new flow rate value

$$P(x) = P_1 - a\,\dot{m}_2^2\,x\ . \tag{22}$$

The control value of gas square pressure at the pipe's final end will be

$$P_c = P_1 - a\,\dot{m}_2^2\,l\ , \tag{23}$$

controlling the process' duration, because the moment of reaching this value corresponds to the end of the transient movement.
In order to establish the transient regime's duration, on the basis of the TRANZPOL computer program the specialized software named **TRANZ-1** was developed. This software is conversational and can be run for the following set of input data requested to the operator: pipe's length and diameter in meters, gas pressure at pipe's beginning in bar absolute, initial flow rate in m^3/h, and flow rate variation coefficient.
The program **TRANZ-1** was run was run to simulate the transient regime in a Dn 200 (d = 0,164 m) repartition gas pipe made of polyethylene, having the length of 4,000 m and a pressure at the initial end of 5 bar absolute which was kept constant during the simulation. The initial flow rate was 5,443 m^3/h and increased by 5%.
The results are shown in figure 1, the diagrams indicating the two pressure variation curves along the pipe as well as the duration of the transient regime generated by the new flow rate value.

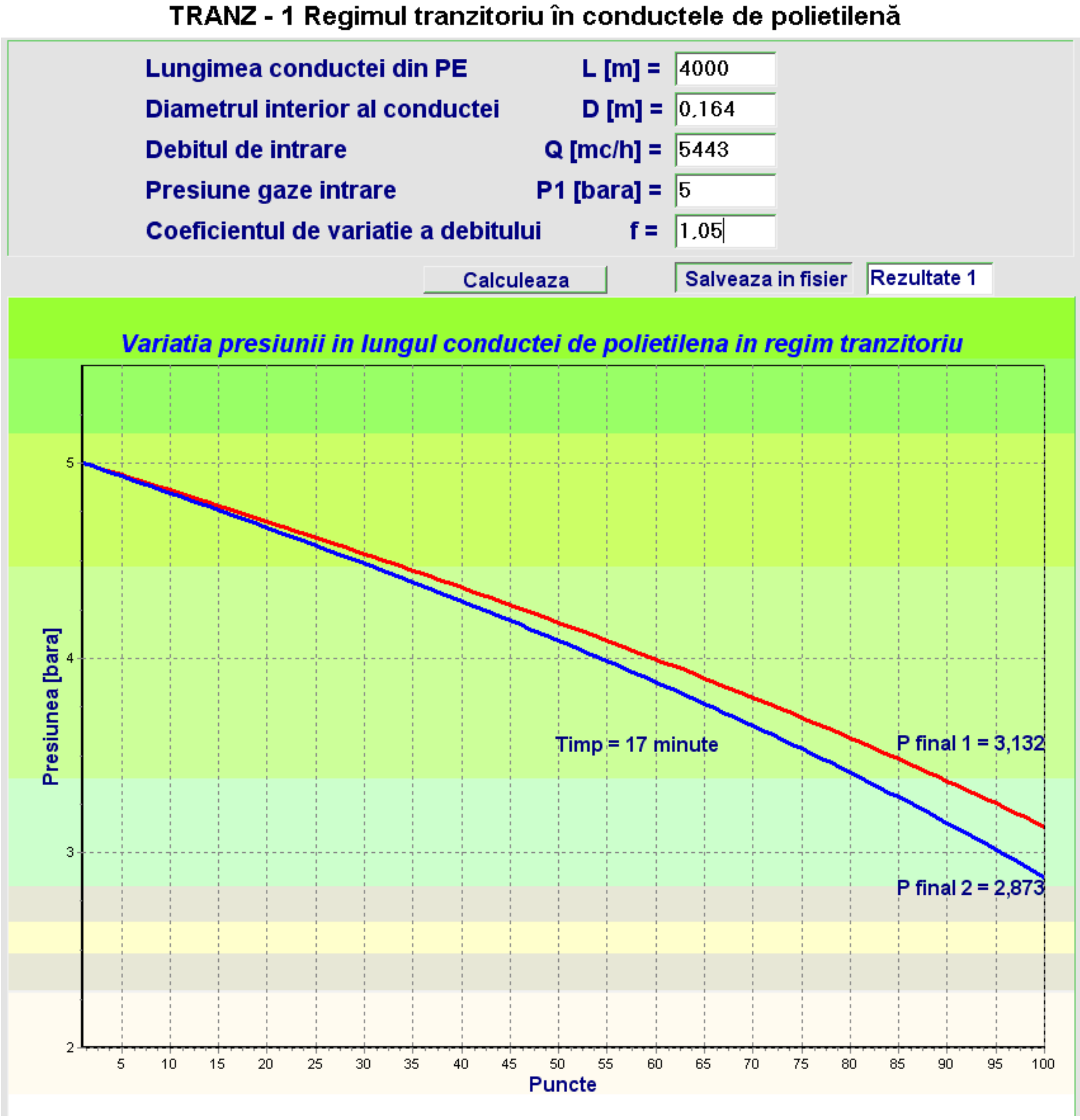

Figure 1. Pressure variation along a Dn 200 polyethylene pipe during the transient regime

4. CONCLUSIONS

Using TRANZ-1 specialized software transient regimes were studied in polyethylene pipes having various geometrical shapes and working in different conditions. The corresponding stabilizing times were determined. The results emphasized the following aspects:

1. Pipe's geometry essentially affects the transient regime duration. Thus, in identical work conditions (pressure at the input end and initial flow rate) as well as with the same flow variation coefficient, increasing pipe's length results in an increase of the re-establishment time. Moreover, to a greater diameter corresponds a shorter re-establishment duration.

2. The working conditions also influence the re-establishment duration for the same pipe geometry. Thus, the increase of the flow rate variation coefficient yields in an increase of the transient process duration.

In figure 2 the results of the transient regimes simulations on a PE Dn 250 pipe are presented. The pipe is 4 km long, the gas pressure at the initial end is 5 bar absolute and the flow rate has an initial value of 5,000 m^3/h

which is successively increased by 5% amounts. One can see the rise of the steady-state regime re-establishing duration concomitant with the decrease of the final end pressure.

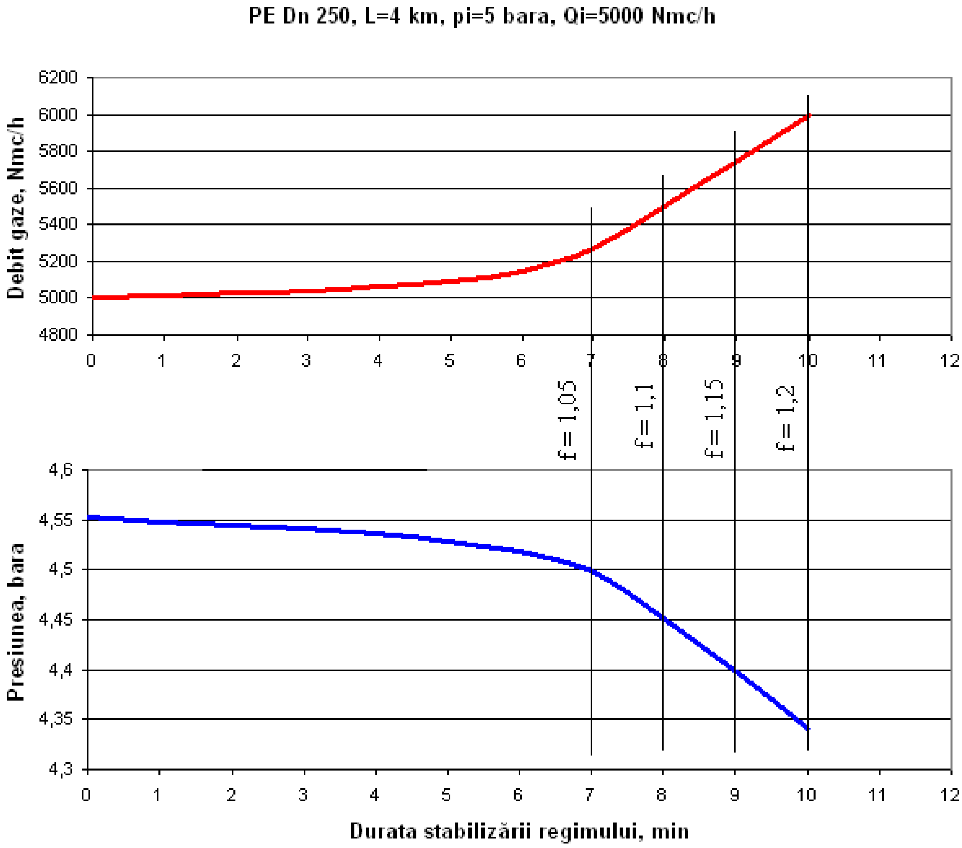

Figure 2. Final end pressure decrease and steady-state regime re-establishing duration decrease as a function of flow rate

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