A THEORY OF ROTATING CONDENSATION

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Abstract: An analysis is made for film condensation on a rotating disk situated in a large body of pure saturated vapor. The centrifugal field associated with the rotation sweeps the condensate outward along the disk surface, and gravity forces need not be involved. The problem is formulated as an exact solution of the complete Navier-Stokes and energy equations.

Keywords: heat-transfer

1. INTRODUCTION

Since the pioneering work of Nusselt in 1916 [1], studies of condensation heat transfer have been largely concerned with the situation of gravity-induced flow. In such natural condensation processes, it is the gravity force alone which brings about the removal of the condensed liquid from the cooled surface and, in this way, controls the condensation rate. Unfortunately, the magnitude of the gravity force is beyond our control, and so it appears that there are definite limits to any natural condensation process. This limitation becomes especially severe when applications at high altitudes are considered, e.g., space vehicle power plants. In these situations, the gravity force diminishes to a negligible value and natural condensation is impossible.

To overcome the limitations inherent in natural condensation, a conventional alternative such as pumping or blowing might be used. But, a more intriguing idea is to create an artificial "gravity" by use of a centrifugal field, and this is the problem on which attention will be focused here. The system to be studied, as shown schematically in the sketch, Fig. 1, is a cooled rotating disk at uniform temperature T_w situated in a large quiescent body of pure saturated vapor. It will be assumed that the condensed 0 liquid forms a continuous film on the disk. Fluid in this film will be moved radially outward along the disk under the action of the centrifugal force field (inqualitatively the same way that gravity causes condensate to flow downward in the natural condensation process).

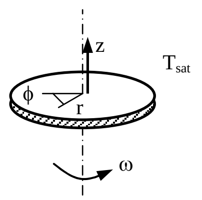

Fig. 1

The prime results of this investigation are the heat-transfer characteristics of the system. Heat-transfer coefficients are presented for fluids having Prandtl numbers in the range 0.003 to 100. Other results which are to be given include the film thickness, temperature profiles, and torque moment requirements. Readers who are primarily interested in results are invited to pass over the section on Analysis.

Aside from the practical aspects of the problem, there is a strong theoretical motivation, namely, that the problem can be formulated as an exact solution of the Navier-Stokes and energy equations. Very few cases are known which permit such rigorous determination of the temperature and velocity fields.

2. ANALYSIS

General Considerations. To achieve our ultimate goal of finding heat-transfer results, it is first necessary to analyze, in detail, the velocity and temperature distributions in the condensate layer. The problem is, of course, governed by the basic conservation laws: Mass, momentum, and energy; and it is these which constitute the starting point for our study. Since it will be supposed that the condensate is incompressible and that properties are constant, there will be a total of five unknowns to be considered: The three velocity components, the pressure, and the temperature. Correspondingly, there are five equations at our disposal: Three from the Navier-Stokes equations (momentum conservation), and one apiece from mass and energy conservation. Solving such a set of five nonlinear partial differential equations would appear, at first glance, a too formidable task. Fortunately, we can draw on the experience of von Karman [2], who successfully solved the simpler problem of a disk, rotating in a large quiescent body of single phase fluid.

The Conservation Equations and Their Transformation. The equations expressing conservation of mass, momentum, and energy for an incompressible, constant-property fluid may be written in cylindrical co-ordinates as

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{1}{r}\frac{\partial V_\phi}{\partial \phi}+\frac{\partial V_z}{\partial z}=0 \tag{1}$$

$$\rho\left(\frac{DV_r}{Dt}-\frac{V_\phi^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left(\nabla^2 V_r-\frac{2}{r^2}\frac{\partial V_\phi}{\partial \phi}-\frac{V_r}{r^2}\right) \tag{2}$$

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$$\rho\frac{DV_z}{Dt}=-\frac{\partial p}{\partial z}+\mu\nabla^2 V_z \tag{4}$$

$$\rho c_p\frac{DT}{Dt}=k\nabla^2 T \tag{5}$$

where
$$\frac{D}{Dt}=V_r\frac{\partial}{\partial r}+\frac{V_\phi}{r}\frac{\partial}{\partial \phi}+V_z\frac{\partial}{\partial z}$$

and
$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial z^2}$$

The viscous dissipation terms have been deleted from the energy equation on the assumption that their effect will be negligible. The retention of these terms would still permit the formulation of an exact solution. The gravity force has been omitted, but it could be included in equation (4) as part of the pressure without altering the analysis. Rather than deal with this formidable array of partial differential equations, we may transform them to a set of ordinary differential equations, which are easier to solve. The new variables which comprise the transformation are:
(a) new independent variable

$$\eta=\left(\frac{\omega}{\nu}\right)^{1/2} \tag{6a}$$

(b) new dependent variables

$$F(\eta)=\frac{V_r}{r\omega},\quad G(\eta)=\frac{V_\phi}{r\omega},\quad H(\eta)=\frac{V_z}{(\omega\nu)^{1/2}},\quad P(\eta)=\frac{p}{\mu\omega}, \tag{6b}$$

$$\theta(\eta) = \frac{T_{sat} - T}{T_{sat} - T_w} = \frac{T_{sat} - T}{\Delta T} \tag{6c}$$

Under the transformation, equations (1) through (5) become

$$H' = -2F \tag{1a}$$

$$F'' = HF' + F^2 - G^2 \tag{2a}$$

$$G'' = HG' + 2FG \tag{3a}$$

$$P' = H'' - HH' \tag{4a}$$

$$\theta'' = (\mathrm{Pr})H\theta' \tag{5a}$$

The primes denote differentiation with respect to η and Pr represents the Prandtl number. Inherent in the transformation, there are two important suppositions about the nature of the velocity and temperature fields which are worthy of discussion. First, there is the property of angular symmetry, i.e., $\partial/\partial\phi = 0$. Second (and more important) is that, except for a simple stretching of V_r and V_ϕ, the velocity and the temperature profiles do not change shape at different values of r. In particular, the assumption that the temperature distribution depends only on z (since $\eta \sim z$) implies the existence of a condensate layer whose thickness is uniform over the disk. The fact that equations (1a) through (5a) are ordinary differential equations shows that the transformation (6) is consistent with the basic conservation laws.

Our prime interest is in the temperature distribution and the heat transfer, and hence our objective is the solution of equation (5a). Unfortunately, the integration of equation (5a) cannot be carried out without a prior (or simultaneous) knowledge of the velocity function $H(\eta)$. But H is intimately connected with the other velocity functions F and G through equations (1a), (2a), and (3a). There is no complete escape from the formidable computational undertaking, but the task may be somewhat lightened by combining equations (1a), (2a), and (3a) to give

$$H''' = HH'' - (H')^2/2 + 2G^2 \tag{7}$$

$$G'' = HG' - H'G \tag{8}$$

Simultaneous solution furnishes G and H, but only the latter is needed for integration of equation (5a) for the temperature distribution.

Boundary Conditions. To complete the statement of the problem, it is necessary to specify the boundary conditions. At the surface of the disk (z = 0), the standard viscous flow conditions are imposed, namely, that there is no motion relative to the solid surface. Moreover, the liquid immediately adjacent to the disk is required to take on the surface temperature T_w.

It has already been observed that the condensate forms a layer of uniform thickness, denoted by δ, over the surface of the rotating disk. At the interface between the condensate and the vapor, the usual condition of negligible shear will be imposed. Further, the condensate and vapor will share the common temperature T_{sat} at the interface. The formal statement of these boundary conditions is

$$\left.\begin{aligned} V_r &= 0 \\ V_\phi &= r\omega \\ V_z &= 0 \\ T &= T_w \end{aligned}\right\} z = 0 \qquad \left.\begin{aligned} \tau_{zr} &= 0 \\ \tau_{z\phi} &= 0 \\ T &= T_{sat} \end{aligned}\right\} z = \delta \tag{9a}$$

The thickness δ of the condensate layer remains to be determined from the analysis. In terms of the new independent and dependent variables pertinent to equations (5a), (7), and (8), these conditions become

$$\left.\begin{array}{l} H=0 \\ H'=0 \\ G=1 \\ \theta=1 \end{array}\right\}\eta=0 \qquad \left.\begin{array}{l} H''=0 \\ G'=0 \\ \theta=0 \end{array}\right\}\eta=\eta_\delta \tag{9b}$$

where η_δ is the value of η corresponding to $z = \delta$.
With these boundary conditions, we then may proceed to solve equations (5a), (7), and (8), provided that numerical values are first chosen for two parameters: Prandtl number and η_δ. From the mathematical standpoint, this is a completely satisfactory situation. But, from the practical view, the problem is incomplete, since the dimensionless condensate thickness η_δ would not be known a priori. So, there still remains the task of relating η_δ to another parameter which contains quantities which are all known. Such a relationship is found later.

The Parameter $c_p\Delta T/h_{fg}$. To relate η_δ (and hence δ) to known physical quantities, we invoke an over-all energy balance as follows

$$h_{f_g}\int_0^\delta \rho 2\pi r V_r dz + \int_0^\delta \rho 2\pi r V_r c_p (T_{sat} - T)dz = k\left(\frac{\partial T}{\partial z}\right)_{n=0} \pi r^2 \tag{10}$$

The first term on the left is the energy released as latent heat, while the second term is the energy liberated by subcooling of the condensate. The right hand represents the heat transferred from the condensate to the disk over a span from r = 0 to r = r. In writing equation (10), the assumption of negligible heat conduction across the vapor-liquid interface, which is standard in condensation theory, has been used. In terms of the variables of the analysis as defined by equations (6), the over-all heat balance becomes

$$\frac{c_p\Delta T}{h_{fg}} = \Pr\frac{H(\eta_\delta)}{\theta'(\eta_\delta)} \tag{10a}$$

where $H(\eta_\delta)$ and $\theta'(\eta_\delta)$ represent the values of H and $d\theta/d\eta$ at $\eta = \eta_\delta$.
From a solution of equations (5a), (7), and (8) corresponding to specified values of Pr and η_δ, the values of $H(\eta_\delta)$ and $\theta'(\eta_\delta)$ are available, and the right side of equation (10a) may be computed. So, $c_p\Delta T/h_{fg}$ is determined. Now, if the Prandtl number is fixed and equations (5a), (7), and (8) are solved for several different values of η_δ, we are able to compute a corresponding set of values for $c_p\Delta T/h_{fg}$. In other words, for a fixed Prandtl number, there is a unique relationship between η_δ and $c_p\Delta T/h_{fg}$. So, if we wish, we can think of our solutions as depending upon Pr and $cp\Delta T/h_{fg}$, rather than on Pr and η_δ.
Solutions. Numerical solutions of equations (5a), (7), and (8) subject to the boundary conditions (9b) have been carried out on a computer using the numerical integration procedures outlined in reference [3]. The solutions were obtained for Prandtl numbers of 0.003, 0.008, 0.03, 1.0,10, and 100; the first three span the range of liquid metals, while the last three correspond to ordinary liquids. Values of the parameter $c_p\Delta T/h_{fg}$ ranged from 0.0001 to 0.1 for the liquid metals and from 0.001 to 1.0 for the ordinary liquids. Based on these solutions, results for the heat transfer, condensate layer thickness, torque moment, and temperature and velocity profiles will be given.

REFERENCES

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