THEORETICAL STUDY ABOUT THE NEWTON'S BUCKET

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Abstract: The paper presents a theoretical study about the Newton's bucket and his applications today. It is know that Isaac Newton conducted more experiments with a bucket containing water, water and rocks, which he described in 1689. His observations and conclusions are presented.

Keywords: Bucket, rope, water surface

1. NEWTON'S EXPERIENCES

The experiment is quite simple and any reader of this article can try the experiment for themselves. All one needs to do is to half fill a bucket with water and suspend it from a fixed point with a rope. Rotate the bucket, twisting the rope more and more. When the rope has taken all the twisting that it can take, hold the bucket steady and let the water settle, then let go. What happens? The bucket starts to rotate because of the twisted rope. At first the water in the bucket does not rotate with the bucket but remains fairly stationary. Its surface remains flat. Slowly, however, the water begins to rotate with the bucket and as it does so the surface of the water becomes concave. Here is Newton's own description: *"... the surface of the water will at first be flat, as before the bucket began to move; but after that, the bucket by gradually communicating its motion to the water, will make it begin to revolve, and recede little by little from the centre, and ascend up the sides of the bucket, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same time with the vessel, it becomes relatively at rest in it".*

Soon the spin of the bucket slows as the rope begins to twist in the opposite direction. The water is now spinning faster than the bucket and its surface remains concave.

What is the problem? Is this not precisely what we would expect to happen? Newton asked the simple question: why does the surface of the water become concave? One is inclined to reply to Newton: that is an easy question - the surface becomes concave since the water is spinning. But after a moment's thought one has to ask what spinning means. It certainly doesn't mean spinning relative to the bucket as is easily seen. After the bucket is released and starts spinning then the water is spinning relative to the bucket yet its surface is flat. When friction between the water and the sides of the bucket has the two spinning together with no relative motion between them then the water is concave. After the bucket stops and the water goes on spinning relative to the bucket then the surface of the water is concave. Certainly the shape of the surface of the water is not determined by the spin of the water relative to the bucket.

Newton then went a step further with a thought experiment. Try the bucket experiment in empty space. He suggested a slightly different version for this thought experiment. Tie two rocks together with a rope, he suggested, and go into deep space far from the gravitation of the Earth or the sun. One certainly can't physically try this today any more than one could in 1689. Rotate the rope about its centre and it will become taut as the rocks pull outwards. The rocks will create an outward force pulling the rope tight. If one does this in an empty universe then what can it mean for the system to be rotating. There is nothing to measure rotation with respect to. Newton deduced from this thought experiment that there had to be something to measure rotation with respect to, and that something had to be space itself. It was his strongest argument for the idea of absolute space.

Now Newton returned to his bucket experiment. What one means by spin, he claimed, was spin with respect to absolute space. When the water is not rotating with respect to absolute space then its surface is flat but when it spins with respect to absolute space its surface is concave. However he wrote in the *Principia*: "*I do not define time, space, place, and motion, as they are well known to all. Absolute space by its own nature, without reference to anything external, always remains similar and unmovable*".

He was not too happy with this as perhaps one can see from other things he wrote:
"It is indeed a matter of great difficulty to discover and effectually to distinguish the true motions of particular bodies from the apparent, because the parts of that immovable space in which these motions are performed do by no means come under the observations of our senses".

2. OTHER EXPERIMENTS

Leibniz, on the other hand, did not believe in absolute space. He argued that space only provided a means of encoding the relation of one object to another. It made no sense to claim that the universe was rotating or moving through space. He supported his argument with philosophical reasoning, but faced with Newton's bucket, he had no answer. He was forced to admit: "*I grant there is a difference between absolute true motion of a body and a mere relative change of its situation with respect to another body*".

For around 200 years Newton's arguments in favors of absolute space were hardly challenged. One person to question Newton was George Berkeley. He claimed that the water became concave not because it was rotating with respect to absolute space but rather because it was rotating with respect to the fixed stars. This did not convince many people that Newton might have been wrong. In 1870 Carl Neumann suggested a similar situation to the bucket when he imagined that the whole universe consisted only of a single planet. He suggested: wouldn't it be shaped like an ellipsoid if it rotated and a sphere if at rest? The first serious challenge to Newton, however, came from Ernst Mach, who rejected Neumann's test as inconclusive. However, he wrote in 1872 in *History and Root of the Principle of the Conservation of Energy*: "*If we think of the Earth at rest and the other celestial bodies revolving around it, there is no flattening of the Earth ... at least according to our usual conception of the law of inertia. Now one can solve the difficulty in two ways; either all motion is absolute, or our law of inertia is wrongly expressed ... I [prefer] the second. The law of inertia must be so conceived that exactly the same thing results from the second supposition as from the first*".

We quote from an 1883 work by Mach on Newton's bucket: "*Newton's experiment with the rotating water bucket teaches us only that the rotation of water relative to the bucket walls does not stir any noticeable centrifugal forces; these are prompted, however, by its rotation relative to the mass of the Earth and the other celestial bodies. Nobody can say how the experiment would turn out, both quantitatively and qualitatively, if the bucket walls became increasingly thicker and more massive -- eventually several miles thick*".

Mach's argument is that Newton dismissed relative motion too readily. Certainly it was not rotation of the water relative to the bucket that should be considered but rotation of the water relative to all the matter in the universe.

If that matter wasn't there and all that there was in the universe was the bucket and water, then the surface of the water would never become concave. He disagreed with Newton's thought experiment based on two rocks tied together in completely empty space. If the experiment were carried out in a universe with no matter other than the rocks and the rope, then the conclusion one can deduce from Mach's idea is that one could not tell if the system was rotating. The rope would never become taut since rotation was meaningless. Clearly since this experiment cannot be performed it is impossible to test whether Mach or Newton is right.

After Einstein introduced the special theory of relativity in June 1905 the concept of absolute space was no longer tenable. Of course this theory only dealt with constant velocity motion, so did not apply to acceleration.

Newton's bucket involved circular motion, and so acceleration and the special theory did not apply. What about the two rocks joined by a rope in a universe without any other matter? Here Einstein's special theory would tend to support Newton rather than Mach, in other words the rope would go taut when the system rotated. But how can rotation mean anything once the notion of absolute space has been cast aside? Well the special theory of relativity still has absolutes. Absolute space-time is a feature of special relativity which, contrary to popular belief, does not claim that everything is relative. Although velocities, distances, and time intervals are relative, the theory still sits on a postulated absolute space-time. In special relativity observers moving at constant velocities relative to each other would not agree on the velocity of a bucket moving through space, nor would they agree about the time that has elapsed in the bucket experiment, but they would all agree on whether the bucket was accelerating or not.

After this Einstein began working on the theory of general relativity which incorporated acceleration and gravity. His theory was published in 1915. Even before this, however, he claimed that his theory would make Mach's view of Newton's bucket correct. He did so in a letter which he wrote to Mach in 1913 in which he told Mach that his view of Newton's bucket was correct and agreed with general relativity. Einstein even included "Mach's principle" into general relativity. The theory is based on the equivalence of gravity and acceleration (something which has been checked experimentally today to a high degree of accuracy). The behavior of Newton's spinning bucket is, as Mach claimed, determined by the gravitational forces of all the matter in the universe. There is, however, still a problem as Einstein came to understand. General relativity does not say that Newton's two rocks thought experiment in an empty universe agrees with Mach. Rather it still comes down on the side of Newton and the rope will become taut as the system spins. Why should that be? Well in simple terms, in a universe with no matter there is no gravity. Hence general relativity reduces to special relativity and now all observers agree when the rock system is spinning (i.e. accelerating). One could argue that there will be gravity produced by the mass of the rock system, but this is negligible and will not produce the necessary forces to make the rope become taut.

In 1918 Joseph Lense and Hans Thirring obtained an approximate solution of the equations of general relativity for rotating bodies. Their results show that a massive rotating body drags space-time round with it. This is now called 'frame dragging' or the “Lense-Thirring effect”. In 1966 Dieter Brill and Jeffrey Cohen showed that frame dragging should occur in a hollow sphere. In 1985 further progress by H Pfister and K Braun showed that sufficient centrifugal forces would be induced at the centre of the hollow massive sphere to cause water to form a concave surface in a bucket which is not rotating with respect to the distant stars. Here at last was a form of the symmetry that Mach was seeking.

3. NASA EXPERIMENTS

Frame dragging has recently been verified experimentally. This involved using the rotating earth as the massive body and putting a satellite into orbit with a gyroscope which kept it pointing in a fixed direction. Although the Earth has only a tiny frame dragging effect it was possible to detect the extremely small precession of the gyroscope which was caused. A report of the experiment was presented by NASA. Figure 1 shows the orbit track of the twin GRACE satellites. In one month its track will densely cover the entire globe. The shaded part of the image is night time. The un-shaded is daytime. The names on the image are ground stations from where scientists talk to the satellite.

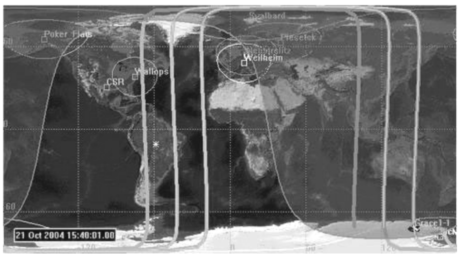

Fig. 1. The Grace Satellite's Orbit

An international team of NASA and university researchers has dramatically improved the accuracy of the first direct evidence that the Earth drags space and time around itself as it rotates. The measurements used the latest gravity models obtained from NASA's GRACE mission.
The researchers first measured the "Lense-Thirring Effect," predicted in 1918 using Einstein's theory of general relativity, in 1998 by precisely observing shifts in the orbits of two Earth-orbiting laser-ranging satellites. The team used additional observations of the same satellites, combined with a more accurate model of the Earth's gravity field, to yield the new measurement of the Lense-Thirring Effect. The team was led by dr. Ignazio Ciufolini of the University of Lecce, Italy and dr. Erricos C. Pavlis of the Joint Center for Earth System Technology, a research collaboration between NASA's Goddard Space Flight Center, Greenbelt, Md., and the University of Maryland Baltimore County. *"Our measurement agrees 99 percent with what is predicted by general relativity, which is within our margin of error of plus or minus five percent,"* said Pavlis. *"This is a significant improvement over our 1998 measurement, which had an error margin of plus or minus 20 percent using the best gravitational model available at the time."*

The research, reported in the journal Nature, is the most accurate direct measurement to date of the Lense-Thirring Effect - a bizarre effect of general relativity, which predicts a rotating mass will drag space around it. The Lense-Thirring Effect is also known as frame dragging.

Frame dragging is like what happens if a bowling ball spins in a thick fluid such as molasses. As the ball spins, it pulls the molasses around itself. Anything stuck in the molasses will also move around the ball. Similarly, as the Earth rotates, it pulls space-time in its vicinity around itself. This will shift the orbits of satellites near Earth.

The measurement is extremely challenging to make because other factors, like changes in the density of the Earth's upper atmosphere, have a much greater effect on satellite orbits than the Lense-Thirring Effect. Additionally, the Earth's gravitational field is not uniform. For example, the mass of a mountain range can tug on a satellite, slightly altering its orbit. The previous measurement was much less accurate than the current work, due to inaccuracies in the gravitational model available at the time. In figure 2 the relief indicates the deviation of the true gravitational field from that of a perfect spheroid with uniform mass distribution. Red is higher deviation, blue is lower deviation.

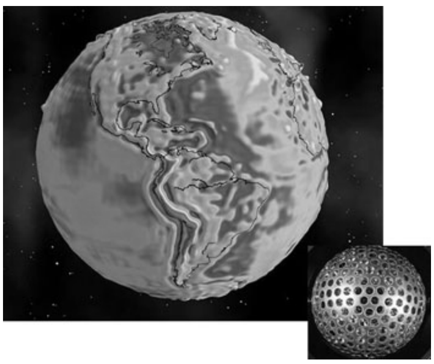

Fig. 2. Gravity's Complex Picture and Lageos Satelllite

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