THEORETICAL STUDY ABOUT DEFLECTION CURVE OF THE TELEPHONE CABLE

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Abstract: The present paper presents a theoretical study about the deflection curve, the length and the maximum tension in the uniform cable for the telephones. Over time the forces the cable exerts on telephone pole have caused it to tilt. Proper bracing should be required.

Keywords: Cable subjected to its own weight, deflection curve, length, tensions

1. THE PROBLEM

When the weight of the cable becomes important in the force analysis, the loading functions along the cable becomes a function of the arc length *s* rather than the projected length *x*.

It is kwon a telephone cable with the dimensions presented in figure 1. The cable weighs is ω_0 = 5N/m.

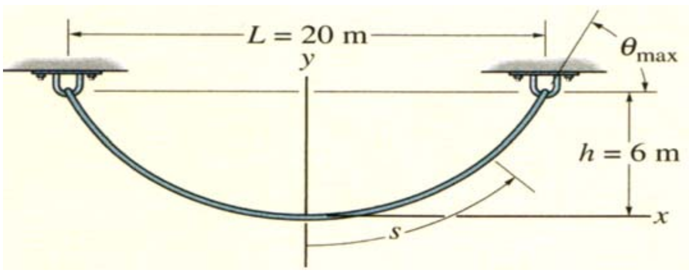

Fig. 1.

It must to determine the deflection curve, the length and the maximum tension in the cable.

2. SOLUTION

For reasons of symmetry, the origin of coordinates is located at the center of cable. Generalized loading function $\omega = \omega(s)$ acting along the cable is shown in figure 2.

The free body diagram for a segment of the cable is presented in figure 3.

$$T\cos\theta = F_H$$
$$T\sin\theta = \int \omega(s)ds$$
$$\frac{ds}{dx} = \frac{1}{F_H}\int \omega(s)ds$$

Since

$$ds = \sqrt{dx^2 + dy^2} \Rightarrow \frac{dy}{dx} = \sqrt{\left(\frac{ds}{dx}\right)^2 - 1}\ .$$

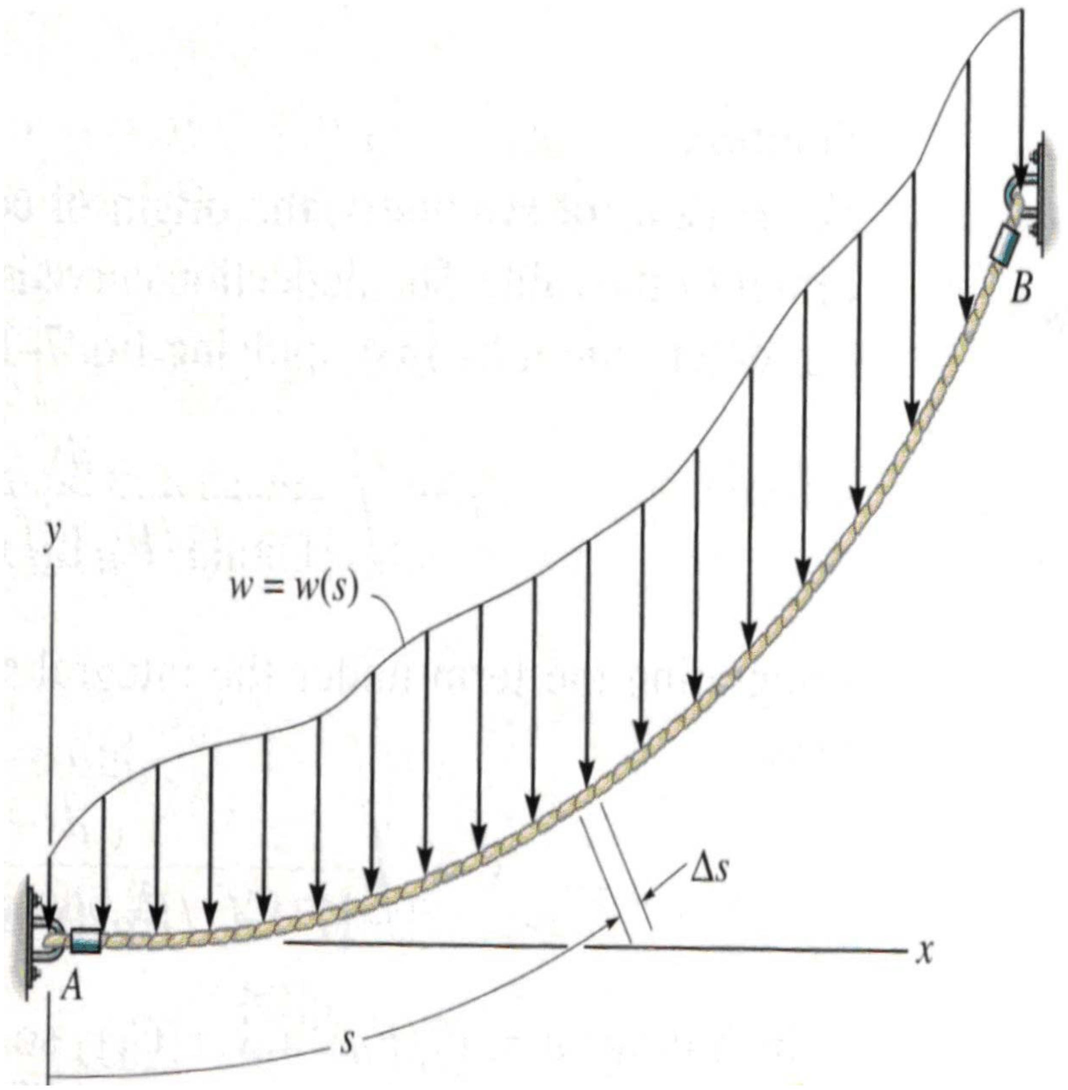

Fig. 2.

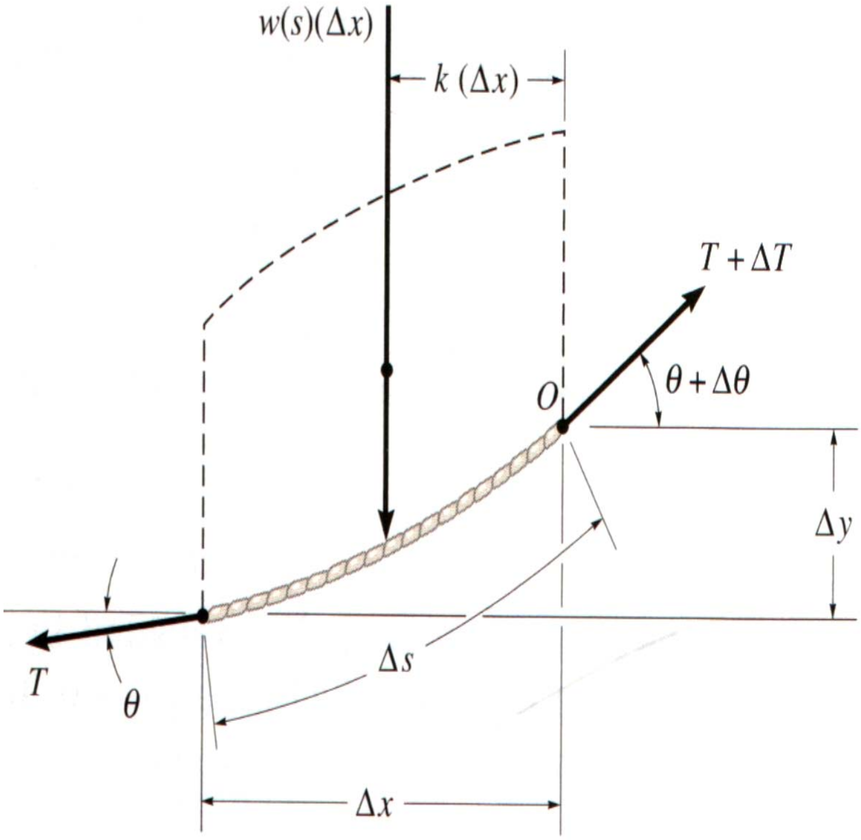

Fig. 3.

Therefore,

$$\frac{ds}{dx} = \left[1 + \frac{1}{F_H^2} \cdot \left(\int \omega(s) ds\right)^2\right]^{1/2}.$$

The deflection curve is expressed as $y = f(x)$. We can determine it by first applying equation

$$x = \int \frac{ds}{\left[1 + \frac{1}{F_H^2}\left(\int w(s) ds\right)^2\right]^{\frac{1}{2}}}$$

obtained by separating the variables and integrating yields, where $w(s) = w_0$

$$x = \int \frac{ds}{\left[1 + \frac{1}{F_H^2}\left(\int w_0 ds\right)^2\right]^{\frac{1}{2}}}$$

Integrating the term under the integral sign in the denominator, we have:

$$x = \int \frac{ds}{\left[1 + \frac{1}{F_H^2}\left(w_0 s + C_1\right)^2\right]^{\frac{1}{2}}}$$

Substituting

$$u = \frac{1}{F_H^2}\left(w_0 s + C_1\right)$$

so that $du = \left(\frac{w_0}{F_H}\right) ds$ a second integration yields

$$x = \frac{F_H}{w_0}\left\{\sinh^{-1}\left[\frac{1}{F_H}\left(w_0 s + C_1\right)\right] + C_2\right\} \tag{1}$$

The evaluate the constants note that, from equation

$$\frac{dy}{dx} = \frac{1}{F_H}\int w(s) ds \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{F_H}\left(w_0 s + C_1\right)$$

Since $\frac{dy}{dx} = 0$ at $s = 0$, then $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_0 s}{F_H} \qquad \text{â} \tag{2}$$

The constant C_2 may be evaluated by using the condition $s = 0$ at $x = 0$ in (1), in which case $C_2 = 0$. To obtain the deflection curve solve for *s* in (1), which yields

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right) \tag{3}$$

Now substitute into (2), in which case

$$\frac{dy}{dx} = \sinh\left(\frac{w_0}{F_H}x\right)$$

Hence

$$y = \frac{F_H}{w_0}\cosh\left(\frac{w_0}{F_H}x\right) + C_3 \tag{4}$$

If the boundary condition $y = 0$ at $x = 0$ is applied, the constant $C_3 = -\frac{F_H}{w_0}$, and therefore the deflection curve becomes

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

This equation defines the shape of a *catenary's curve*. The constant F_H is obtained by using the boundary condition that $y = h$ at $x = \frac{L}{2}$, in which case

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2F_H}\right) - 1\right] \tag{5}$$

Since $w_0 = 5N/m$, $h = 6m$, and $L = 20m$, equations (4) and (5) become

$$y = \frac{F_H}{5}\left[\cosh\left(\frac{5}{F_H}x\right) - 1\right] \tag{6}$$

$$6 = \frac{F_H}{5}\left[\cosh\left(\frac{50}{2F_H}\right) - 1\right] \tag{7}$$

Equation (7) can be solved for F_H by using a trial-and-error procedure. The result is

$$F_H = 45{,}9N$$

And therefore the deflection curve, (6), become

$$y = 9{,}19\left[\cosh(0{,}109x) - 1\right] \text{ m}$$

Using equation (3), with $x = 10m$, the half-length of the cable is

$$\frac{L}{2} = \frac{45{,}9}{5}\sinh\left[\frac{5}{45{,}9}(10)\right]$$

Hence,

$$L = 24{,}2 \text{ m}$$

Since $T = \frac{F_H}{\cos\theta}$, equation $T\sin\theta = \int w(s)ds$, the maximum tension occurs when θ is maximum, At $s = \frac{l}{2} = 12{,}1$ m. Using equation (2) yields

$$\left.\frac{dy}{dx}\right|_{s=12{,}1m} = \tan\theta_{\max} = 1{,}32$$

$$\theta_{\max} = 52{,}8^\circ$$

Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{45{,}9}{\cos 52{,}8^\circ} = 75{,}9 \text{ N}$$

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