

MODELING AND OPTIMIZATION OF FRUIT YOGHURT MANUFACTURING PROCESS USING EXPERIMENTS DESIGN PROCEDURES

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Abstract:

This paper presents the application of the k^n experiment design procedure for simulation and optimization of the yogurt with fruit adding (cherries) manufacturing process. There were performed 27 experiments according to the experiment design procedure choose. The INPUT variables are the fat content of the raw milk (%), the amount of the added fruit (g/100 mL raw material) and the fermentation time (hours); in function of these variables was determined the titrable acidity of the final product.

Keywords: yoghurt, experiment design procedure, optimization, modelling

1. INTRODUCTION

The yoghurt is a product that has its origins in Small Asia and Balkan Peninsula and it is found in many countries [1,2]. The manufacturing of this product is based on using of selected lactic bacterium which are commercialized as mono- or mixed cultures which biotechnological properties especially for obtaining products of constant quality. According to the method of preparation, the literature mention different types of yoghurt: coagulated yoghurt (the milk is being put in packing immediately after being seeded and measured) or fluid yoghurt (the milk is being seeded and measured in tank, after coagulation being cooled and put into bottles) [3].

The fermentation of milk is the result of the two associated lactic bacterial action: *Lactobacillus bulgaricus* and *Streptococcus thermophilus* [1-3]. These two bacteria's grow inside the yogurt in a tight symbiosis, *Lactobacillus bulgaricus* enhancing *Streptococcus thermophilus* to evolve throughout a proteolytic activity that leads to freeing a series of casein amino-acids. There can also be used concentrated fruit juice to improve the yoghurt's colour or concentrated flavors using fruits in different forms. The acid character of the yoghurt gives a pleasant sensation of freshness, reducing in the same time the production of chloride acid in the human stomach.

Mathematical modeling through factorial design procedure are used by now in different domains for optimizing and modeling of different processes such: chocolate and Gruyere cheese obtaining [4], biogas production [5] etc. The advantages of these methods of optimization regarding others consist into the fact that these take into account not only the individual (simple) effect of each parameter but also the interaction and/or their possible synergy effects [6-8].

2. EXPERIMENTAL PART

The most frequently method for optimizing and modeling of processes is the design experiments of type k^n , k being the variation levels of parameters, and n the number of parameters. In this work, the experiments were programmed using the 3^3 design procedures and the acidity of the fruit yoghurt represent the response function [9]. The key-stage in the obtaining process of the fruit yoghurt is the fermentation. This key-stage is being influenced by there variables: quality of the percent of tot contained by the prime material (X_1), the quantity of added fruits (X_2), the fermentation time (X_3) – table 1.

Table 1. Variation level of the variables that influence the yoghurt manufacturing process

Nr. crt.	Variable (X_i)	Values min. ($X_i^{\min}$)	Values max. ($X_i^{\max}$)	ΔX_i	X_i^{med}
1.	Fat content of the raw material (cow milk) - % (X_1);	0,1	2,8	1,35	1,5
2.	Quantity of added fruits – g/100 mL (X_2);	2	25	11,5	13,5
3.	Fermentation time – hours (X_3)	1	6	2,5	3,3

3. RESULTS AND DISCUSSION

The measured values of the response function “acidity of the final product” are being displayed in table 2.

Table 2

No. experiments	The response function Y [$^{\circ}\text{T}$]
1	26
2	86
3	106
4	33
5	88
6	107
7	45
8	90
9	112
10	27
11	77
12	96
13	39
14	83
15	100
16	54
17	105
18	114
19	30
20	71
21	101
22	37
23	80
24	106
25	43
26	86
27	116

3.1. Elaboration of the mathematic model

The particular form of answer for a 3^3 design procedure type is:

$$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{123}x_1x_2x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 \quad (1)$$

The coefficients $a_0, a_1, a_2, a_3, \dots$ etc. from relation (1) are calculated according to the literature [6,7]. The results are displayed in table 4.

Table 4. Coefficients of the mathematic model

Coefficient	a_1	a_2	a_3	a_{12}	a_{13}	a_{23}	a_{123}	a_{11}	a_{22}	a_{33}	a_0
Adimensional value	-1,26	8,04	24,66	1,16	-0,66	-1,66	1,62	-1,66	2,16	-3,33	84,77

Replacing the results from table 4 in relation (1) is obtain the mathematical formula which describes the response function:

$$Y = 84,77 - 1,26x_1 + 8,04x_2 + 24,66x_3 + 1,16x_1x_2 - 0,66x_1x_3 - 1,66x_2x_3 + 1,62x_1x_2x_3 - 1,66x_1^2 + 2,16x_2^2 - 13,33x_3^2 \quad (2)$$

For simplifying the mathematical model (2), the literature mentions the application of the “t-student” test [7]. The t-student test values for each coefficient are displayed in table 5.

Table 5. Values of the t-student test coefficients

Coefficient	t_0	t_1	t_2	t_3	t_{12}	t_{13}	t_{23}	t_{123}	t_{11}	t_{22}	t_{33}
Value	101,14	1,50	9,60	29,46	1,38	0,78	1,98	1,93	1,99	2,58	15,92

From the t-student test results it can be observed that the terms that can be eliminated are: $x_1, x_{12}, x_{13}, x_{123}$. Hence, the mathematical formula that describes the answer functions of the optimizing criteria, after eliminating the insignificant terms, is:

$$Y = 84,77 + 8,04x_2 + 24,66x_3 - 1,66x_2x_3 - 1,66x_1^2 + 2,16x_2^2 - 13,33x_3^2 \quad (3)$$

The value of the coefficient a_0 (84,77) indicate that we have an optimized acidity to a value closer to this value. The coefficients a_2 and a_3 are positives, which results that both variables x_1 and x_2 have an individual positive influence, the positive effect of x_3 being a lot stronger that the positive effect of x_2 . Because the individual effect of x_1 has been determined by t-student test to be insignificant there will be no discussion. The coefficient of interaction a_{23} being negative we conclude the variables x_2 and x_3 , through their interaction, have a bad influence on the process.

The analysis of quadratic terms a_{11}, a_{22} and a_{33} shows that the answer function is characterized by a maximum comparing to the variable x_1 , through a minimum compared to the variable x_2 and through a maximum compared to variable x_3 .

3.2. Determination of the optimum values

Determination of the optimum value of the response function is realized within the derivate method; this method consists in calculating the partial derivates of first order comparing which each variable and then equalizing to zero the resulted equations. The partial derivates of order I compared which each variable are:

$$\left\{ \begin{array}{l} \frac{\partial y}{\partial x_1} = -3,32x_1 = 0 \\ \frac{\partial y}{\partial x_2} = 8,04 - 1,66x_3 + 4,32x_2 = 0 \\ \frac{\partial y}{\partial x_3} = 24,66 - 1,66x_2 - 26,66x_3 = 0 \end{array} \right. \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = \frac{1,53}{-1,66} = -0,92 \\ x_3 = \frac{-116,87}{-117,92} = 0,99 \end{array}$$

Resolving the linear systems leads to the obtaining of the following dimensionless values of the parameters: $x_1 = 0$, $x_2 = -0,92$, $x_3 = 0,99$. Knowing the variation domains of variables x_1 and x_2 we obtain the real values of each variables using the relation:

$$X_i = \Delta X_i \cdot x_i + X_i^{med} \quad (4)$$

where: X_i – the real values of each variable, x_i – the dimensionless values of the parameters, ΔX_i – the step of each variation domain and X_i^{ed} – the medium real value of the parameters. Hence, the optimum values in dimensional coordinates are:

$$\left\{ \begin{array}{l} X_1 = 1,35 \cdot 0 + 1,45 = 1,45\% \text{ fat} \\ X_2 = 11,5 \cdot (-0,92) + 13,5 = 3\% \text{ fruits} \\ X_3 = 2,5 \cdot 0,99 + 3,5 = 5,97 \text{ hours} \end{array} \right.$$

For graphically representation of the mathematical relation (3) with MathLab soft, we need to equalize with zero each parameter, the model it will be figure depending on two parameters, the other parameter being constant to the value 0 and representing the centre of the chosen variation domain. It results, thee areas of response (fig. 1) characterized by the following mathematic formulas:

$$\begin{array}{ll} x_1=0 & Y=84,77+8,04x_2+24,66x_3-1,66x_2x_3+2,16x_2^2-13,33x_3^2 \\ x_2=0 & Y=84,77+24,66x_3-1,66x_1^2-13,33x_3^2 \\ x_3=0 & Y=84,77+8,04x_2-1,66x_1^2+2,16x_2^2 \end{array}$$

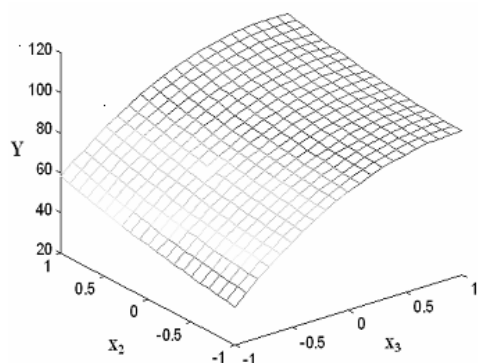


Fig.1. Effect of fruit quantity added and the fermentation time on the acidity when the fat content of the raw material is maintained in the central domain

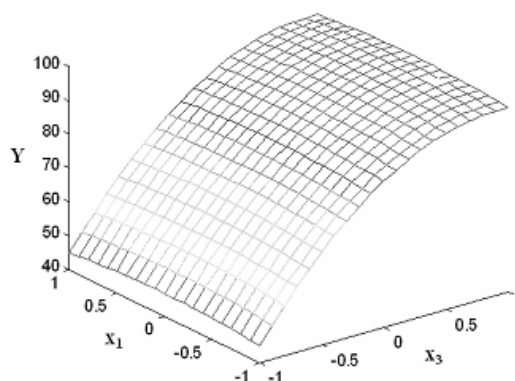


Fig. 2. Effect of fat content of the raw material and the fermentation time on the acidity when the fruit quantity is maintain in the central domain

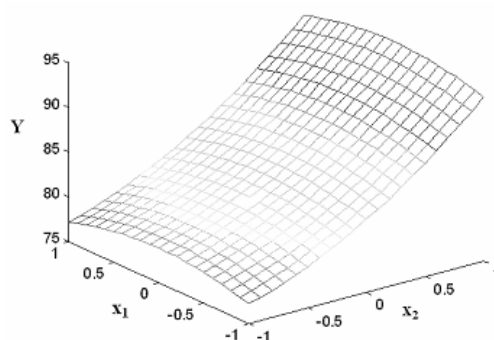


Fig. 3. Effect of the fat content of the raw material and the quantity of fruit added on the acidity when the fermentation time is maintained in the central domain.

4. CONCLUSIONS

As we observed from the real values of the variables, we can notice the following observations: the best percent of fat contained in the raw material doesn't cross the [0,1-2,85 %] domain initially chose and used in the experimental program, the best percent of the fermentation time doesn't cross the [1-6 hours] domain used in the experimental program, the best percent for the fruit quantity added doesn't cross the domain donned in table 1.

According to the observations below it results that the experimental program adopted has permitted to find the best values for the variables of the process, this fitting in the limits of the initial domain.

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