PRACTICAL INTEREST OF ROTATING CONDENSATION

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Abstract: The problem of rotating condensation is formulated as an exact solution of the complete Navier-Stokes and energy equations. Numerical solutions are obtained for Prandtl numbers between 0.003 and 100 and for $c_p\Delta T/h_{fg}$ in the range 0.0001 to 1.0.

Keywords: heat-transfer

1. HEAT-TRANSFER COEFFICIENTS

To overcome the limitations inherent in natural condensation, a conventional alternative such as pumping or blowing might be used. But, a more intriguing idea is to create an artificial “gravity” by use of a centrifugal field, and this is the problem on which attention will be focused here. The prime results of this investigation are the heat-transfer characteristics of the system, heat-transfer coefficients are presented for fluids having Prandtl numbers in the range 0.003 to 100 [3]. Other results which are to be given include the film thickness, temperature profiles, and torque moment requirements. The most important results of practical interest are the heat-transfer characteristics of the system. The local heat flux to the disk may be computed by Fourier's Law:

$$q = \left(k \frac{\partial T}{\partial z} \right)_{z=0} \tag{1}$$

In terms of the transformed variables of the analysis, the expression for q becomes

$$q = -k(T_{sat} - T_w)\left(\frac{\omega}{\nu}\right)^{1/2} (d\theta / d\eta)_{\eta=0} \tag{2}$$

The derivative $(d\theta/d\eta)_{\eta=0}$, obtained from solutions of equations depends on the Prandtl number and on $c_p\Delta T/ h_{fg}$. For a particular liquid and a fixed temperature difference, equation (2) shows that

$$q \sim \omega^{1/2}$$

It is also worth remarking that q is uniform over the surface of the disk.
Introducing the definition of the local heat-transfer coefficient as follows

$$h \equiv \frac{q}{T_{sat} - T_w} \tag{3}$$

the heat-transfer results may be rephrased in the form

$$\frac{h\left(\frac{\nu}{\omega}\right)^{1/2}}{k} = -(d\theta/d\eta)_{\eta=0} \tag{4}$$

The quotient $(\nu/\omega)^{1/2}$ has the dimensions of a length, and so the left side of equation may be regarded as a Nusselt number [1]. Inasmuch as $(d\theta/d\eta)_{\eta=0}$ depends on Pr and $c_p\Delta T/h_{fg}$, so then do the Nusselt number results. .In presenting these results, it is convenient to look first at the higher Prandtl numbers (1, 10, 100) and then at the lower Prandtl numbers (0.003, 0.008, 0.03).

By evaluating equations (4) from the numerical solutions, heat-transfer results for the higher Prandtl numbers have been plotted on Fig. 1 for values of $c_p\Delta T/h_{fg}$ between 0.001 and 1.0. The choice of the ordinate variable is noteworthy. Rather than plot the Nusselt number alone, the group

$$\frac{h\left(\frac{\nu}{\omega}\right)^{1/2}}{k} = \left(\frac{c_p\Delta T/h_{fg}}{\Pr}\right)^{1/4}$$

has been used. By inspection of the figure, it is seen that for small values of $c_p\Delta T/h_{fg}$, all the results are represented by

$$\frac{h\left(\frac{\nu}{\omega}\right)^{1/2}}{k} = 0.904\left(\frac{\Pr}{c_p\Delta T/h_{fg}}\right)^{1/4} \tag{5}$$

Equation (5) corresponds precisely to the heat-transfer result for the situation where *energy convection and acceleration terms are negligible*. So, departures from this relationship represent the effects of convection and inertia, the former tending to increase the heat transfer and the latter tending to diminish it. With increasing Prandtl number, the effect of energy convection becomes relatively more important than the inertia effects. As a consequence, the Pr = 10 and Pr = 100 curves tend to exceed (or, in the limit, equal) the limiting value of 0.904. On the other hand, the inertia effects become relatively more important as the Prandtl number decreases, and so the Pr = 1 curve tends to drop slightly below the limiting value of 0.904. It is worth while noting that for a significant part of the technically important range of $c_p\Delta T/h_{fg}$, equation (5) is a good representation of the heat-transfer results of Fig. 1.

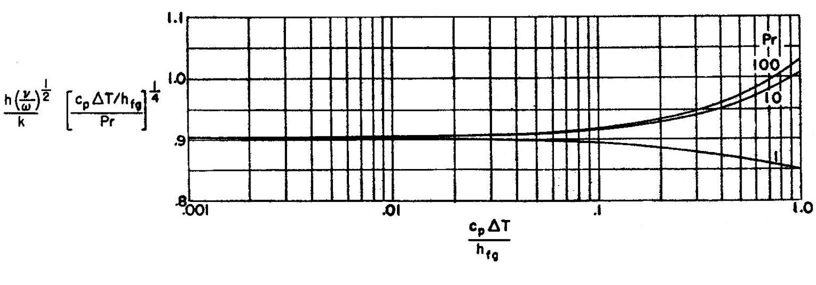

Fig. 1. Heat-transfer results for high Prandtl number fluids

Now, we turn to the low Prandtl number heat-transfer results which are plotted on Fig. 2. The ordinate and abscissa variables are the same as on the previous figure, but here we are concerned with smaller values of $c_p\Delta T/h_{fg}$ to correspond to the relatively small values of cp/h_{fg} for liquid metals. For the smallest values of

$c_p\Delta T/h_{fg}$ which are plotted on the figure, the curves tend to approach (but never quite reach) the limiting value of 0.904. The inertia terms play an important role for these low Prandtl number liquids, tending to substantially decrease the heat transfer "as the condensate layer thickens (increasing values of $c_p\Delta T/h_{fg}$).

2. CONDENSATE LAYER THICKNESS

The analysis predicts that the dimensionless condensate layer thickness $\delta(\omega/\nu)^{1/2}$ will be uniform over the disk and will vary with Prandtl number and $c_p\Delta T/h_{fg}$. This dependence may be determined by utilizing the numerical solutions of equations. For the limiting situation of negligible inertia and heat convection effects, the following limiting relationship has been derived.

$$\delta\left(\frac{\omega}{\nu}\right)^{1/2} = 1.107\left(\frac{c_p\Delta T/h_{fg}}{\text{Pr}}\right)^{1/4} \tag{6}$$

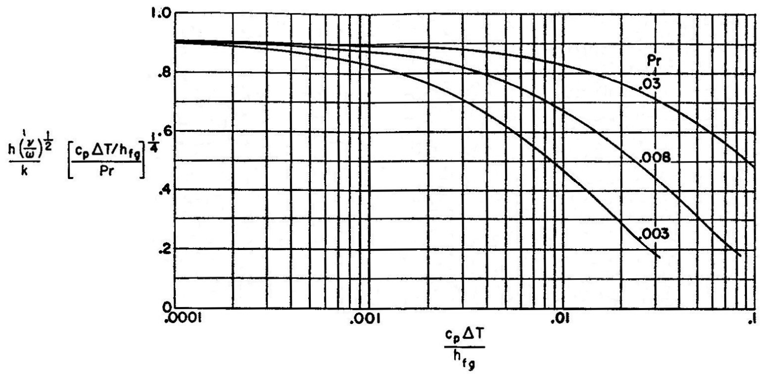

Fig. 2 Heat-transfer results for low Prandtl number fluids

Fig. 3 shows the condensate layer thickness results for the high Prandtl numbers. The limiting result of equation (6) is achieved for small values of $c_p\Delta T/h_{fg}$, while the deviations from this limit never appear too great (in common with the high Prandtl number heat transfer). The low Prandtl number results are plotted on Fig. 4. There, substantial upward deviations from the limit of equation (6) may be observed as a consequence of the inertia effects.

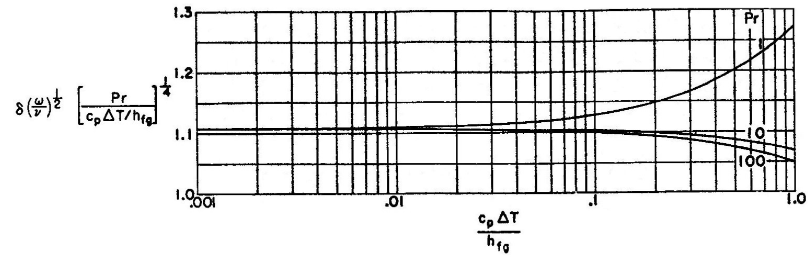

Fig. 3 Condensate layer thickness for high Prandtl number fluids

By comparing Figs. 3 and 4, it can be seen that for a given value of $c_p\Delta T/h_{fg}$, the magnitude of $\delta(\omega/\nu)^{1/2}$ increases with decreasing Prandtl number. Finally, we note that for a given fluid and a fixed temperature difference

$$\delta \sim \omega^{-1/2}$$

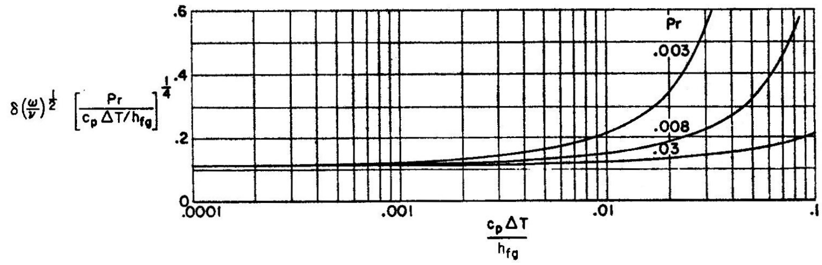

Fig. 4 Condensate layer thickness for low Prandtl number fluids

3. TEMPERATURE PROFILES

The temperature distribution across the condensate layer may also be of some interest. We will content ourselves with displaying representative results, focusing attention on Pr = 10 for the high Prandtl number group and Pr = 0.008 for the low Prandtl number group.

Dimensionless temperature profiles for Pr = 10 are plotted on Fig. 5. The profiles for $c_p\Delta T/h_{fg} < 0.1$ cannot be distinguished from a straight line on the scale of this figure. The deviations from the straight line profile increase with increasing $c_p\Delta T/h_{fg}$ (i.e., increasing layer thickness), but do not become decisive even for the highest value of $c_p\Delta T/h_{fg}$ plotted here. The effect upon the heat transfer of these deviations has already shown on Fig. 1 to be. small. Numerous other cases were available for plotting in the range $0.1 \le c_p\Delta T/h_{fg} \le 1$, but they have been omitted for the sake of clear reproduction [4].

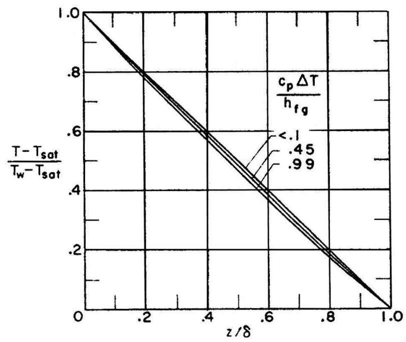

Fig. 5 Representative temperature distributions across condensate layer for Pr = 10

Fig. 6 shows temperature profiles for Pr = 0.008. For values of $c_p\Delta T/h_{fg} < 0.02$, the curves could not be distinguished from a straight line. Even the extreme case on Fig. 6, $c_p\Delta T/h_{fg} = 0.083$, so closely parallels the straight line profile that it was physically impossible to draw other intermediate curves [4].

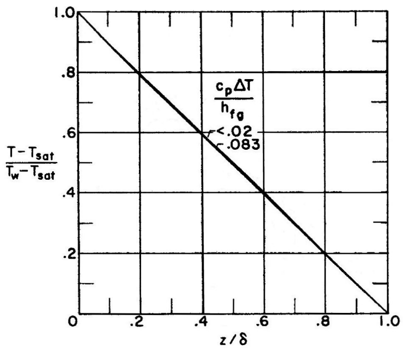

Fig. 6 Representative temperature distributions across condensate layer for Pr = 0.008

It would appear from both Figs. 5 and 6 that, for most technically interesting situations, the temperature profile across the condensate layer closely approximates a straight line.

4. VELOCITY PROFILE

A view of some of the hydrodynamic aspects of the problem may be gained by inspection of the velocity distributions. We confine ourselves to representative situations, selecting results for a layer thickness of $\delta(\omega/\nu)^{1/2} = 5$ to typify relatively thick films and those for $\delta(\omega/\nu)^{1/2} = 0.5$ to represent relatively thin films.

Turning first to the relatively thicker films, we present on Fig. 7 the distribution of each of the velocity components across the condensate film. It is seen that the tangential velocity decreases significantly across the layer, and that there is a substantial axial velocity carrying mass inward toward the disk surface. The general shape of the curves is not significantly different from those for von Karman's problem of a rotating disk in an infinite domain of single phase fluid [2].

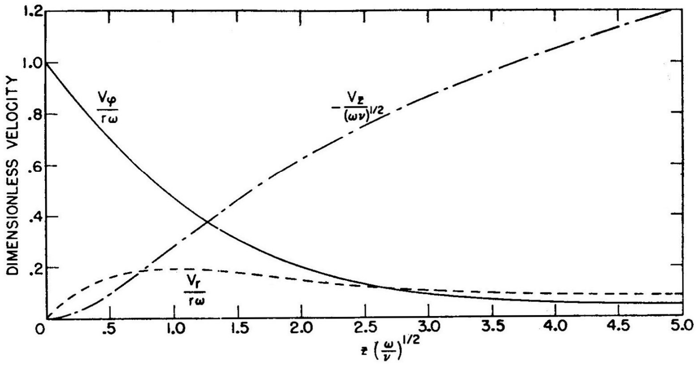

Fig. 7 Velocity distributions for $\delta(\omega/\nu)^{1/2} = 5.0$

Velocity profiles corresponding to $\delta(\omega/\nu)^{1/2} = 0.5$ (relatively thin films) are shown in Fig. 8. Here it may be noted that the tangential velocity changes very little across the layer, while the axial velocity is appreciably smaller than that for the thicker layers. This reduction in axial velocity in turn leads to a decrease in the effect of energy convection.

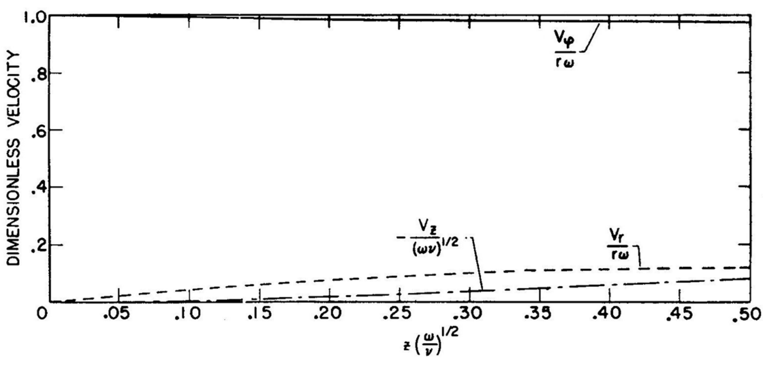

Fig. 8 Velocity distributions for $\delta(\omega/\nu)^{1/2} = 0.50$

5. LAMINAR-TURBULENT TRANSITION

If the results of the present analysis were to be applied, the question would immediately arise as to the conditions under which the laminar flow assumption would be valid. In the absence of experimental data on rotating condensation, we must rely on the findings for the somewhat similar system of a rotating disk in an infinite domain of single phase fluid. For that situation, it is observed [2] that the laminar results are usable up to a value of

$$\mathrm{Re} = \frac{r^2\omega}{\nu} = 3\times10^5$$

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