THE STATIC AND DYNAMIC ANALYSIS OF THE FEED KINEMATICAL LINKAGES OF THE MACHINE TOOLS CNC

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Abstract: The feed kinematical linkages of the numerical control machine tools have a direct and decisive influence on the machining accuracy. Their research in closed loop (for the speed and position circuits) is definitely needed, as the transient duty of the linkage participates directly to the path error of the work piece being machined. This paper presents a theoretical research method of the kinematical linkage as a whole, based on patterns, offering important data on the system performance.

Keywords: kinematical linkages, machine tools, dynamic characteristic, servomechanism, accuracy.

1. INTRODUCTION

The characteristic of the kinematical linkage means the ratio between the output and input rates of the servomechanism or of the component element. When the input rate is constant in relation to time, then the static characteristic will result. Unlike the static characteristic, the dynamic characteristic can be obtained under the conditions of a rate variable in time.
The researches on kinematical linkages have had as form of expression a mathematical model of the form:

$$X(x_e, \frac{dx_e}{dt}, \frac{dx^2{}_e}{dt^2} ...) = Z(f, \frac{df}{dt}, \frac{df^2}{dt^2} .... Y, \frac{dy}{dt}, \frac{dy^2}{dt^2} ...) \tag{1}$$

The symbols mean:
- f(t) = external factors ;
- y(t) = variation of the input rate;
- x_e(t) = variation of the output rate.

These models are reduced at a more simple form by replacement with practical equivalent models whose equations are linear, and with constant coefficients.

$$X(x_e, \frac{dx_e}{dt}, ... \frac{dx^m{}_e}{dt^m}) = F(f, \frac{df}{dt}, ... \frac{df^n}{dt^n}) + Y(y, \frac{dy}{dt} ... \frac{dy^k}{dt^k}) \tag{2}$$

The study of these equations is much easier by using the Laplace transformation.
The researches on a feed kinematical linkage (fig. 1) are presented at two stages:
The first stage establishes the equations of calculation based on the systems theory and the second stage calculates an example.

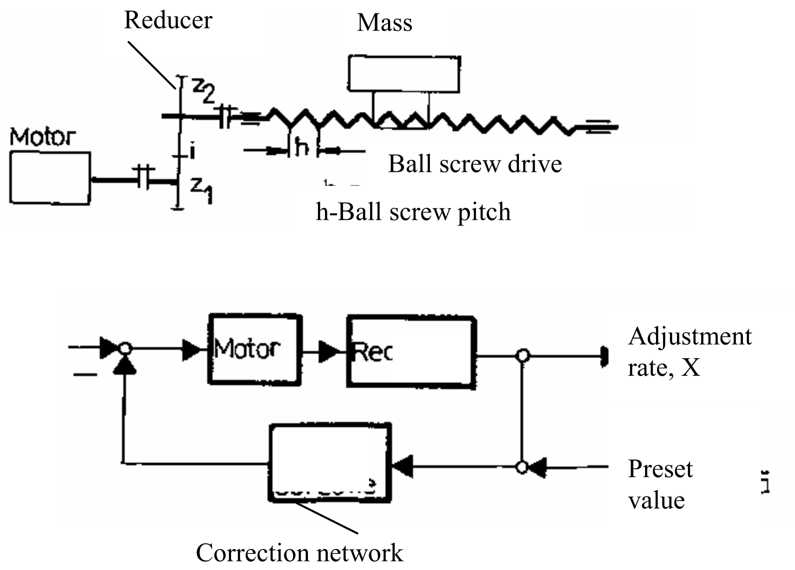
2. THEORETICAL RESEARCHES ON FEED KINEMATICAL LINKAGES

Several steps are attended, by beginning with deducing the substitute system (fig.2). The transformation is done based on the hypothesis that the mechanism is composed of concentrated inertia elements (motor, gears, and slide) and concentrated arc elements (shafts, ball screw). As such, the division is done into elements having an important elasticity treated as arcs and elements having an important inertia, treated as carrying mass. The reduction of the elastic deformations to the motor shaft will be:

$$C_{t1} = (1/i)^2 C_{t2} \tag{3}$$

where i means the transmission ratio.
The conversion of the axial elastic constant into rotary elastic constant will be:

$$C_t = C_A (h/2\pi)^2 \tag{4}$$



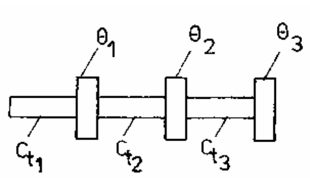


Fig.1.
Feed kinematical linkage and equivalent block diagram

Fig.2.
Equivalent system

Where: C_A = axial elastic constant;
C_t = rotary elastic constant;
h = the screw pitch.
The inertia torques of the rotation is brought to the motor shaft by means of the relation:

$$\theta_1 = (1/i)^2 \theta_2 \tag{5}$$

The mass conversion to the lead screw will be:

$$\theta_{red} = (h/2\pi)^2 m_T \tag{6}$$

where m_T means the slide mass (weight).
Through the relations of above, fig. 1 will be transformed into an equivalent scheme, as per fig. 2. Further on, the transfer function of the mechanism will be deduced through the relations of inter-dependence of the elements and the relation of below will result:

$$G(P)=\frac{\omega_a^L}{\omega_e^L}=\frac{1}{Ap^6+Bp^5+Cp^4+Dp^3+Ep^2+Fp^2+1} \quad (7)$$

which is known as transfer function for a sixth range member.

Table No. 1: The coefficients of the characteristic equation: A, B, C, D, E, F, G.

COEFICIENTUL	EXPRESIA
A	$\frac{J_1 J_2 J_3}{C_1 C_2 C_3}$
B	$\frac{J_1 J_2 k_3}{C_1 C_2 C_3}+\frac{J_1 J_3 k_2}{C_1 C_2 C_3}+\frac{J_2 J_3 k_1}{C_1 C_2 C_3}$
C	$\frac{J_1 J_2 C_3}{C_1 C_2 C_3}+\frac{J_1 J_3 C_3}{C_1 C_2 C_3}+\frac{J_1 J_3 C_2}{C_1 C_2 C_3}+\frac{J_1 k_2 k_3}{C_1 C_2 C_3}+\frac{J_2 J_3 C_1}{C_1 C_2 C_3}+$ $+\frac{J_2 J_3 C_2}{C_1 C_2 C_3}+\frac{J_2 k_1 k_3}{C_1 C_2 C_3}+\frac{J_3 k_1 k2}{C_1 C_2 C_3}$
D	$\frac{J_1 k_2 C_3}{C_1 C_2 C_3}+\frac{J_1 k_2 C_3}{C_1 C_2 C_3}+\frac{J_1 k_3 C_2}{C_1 C_2 C_3}+\frac{J_1 k_1 C_3}{C_1 C_2 C_3}+\frac{J_2 k_3 C_1}{C_1 C_2 C_3}+$ $\frac{J_2 k_3 C_2}{C_1 C_2 C_3}+\frac{J_3 k_2 C_1}{C_1 C_2 C_3}+\frac{J_3 k_1 C_3}{C_1 C_2 C_3}+\frac{J_3 k_1 C_2}{C_1 C_2 C_3}+\frac{J_3 k_2 C_2}{C_1 C_2 C_3}+$ $\frac{k_1 k_2 k_3}{C_1 C_2 C_3}$
E	$\frac{J_1 C_2 C_3}{C_1 C_2 C_3}+\frac{J_2 C_2 C_3}{C_1 C_2 C_3}+\frac{J_2 C_3 C_1}{C_1 C_2 C_3}+\frac{J_3 C_2 C_3}{C_1 C_2 C_3}+\frac{J_3 C_3 C_1}{C_1 C_2 C_3}+$ $\frac{J_3 C_2 C_1}{C_1 C_2 C_3}+\frac{k_1 k_2 C_3}{C_1 C_2 C_3}+\frac{k_1 k_3 C_3}{C_1 C_2 C_3}+\frac{k_2 k_3 C_2}{C_1 C_2 C_3}+\frac{k_2 k_3 C_1}{C_1 C_2 C_3}$
F	$\frac{k_1 C_2 C_3}{C_1 C_2 C_3}+\frac{k_2 C_2 C_3}{C_1 C_2 C_3}+\frac{k_2 C_1 C_3}{C_1 C_2 C_3}+\frac{k_3 C_2 C_3}{C_1 C_2 C_3}+\frac{k_3 C_1 C_3}{C_1 C_2 C_3}+$ $\frac{k_3 C_1 C_2}{C_1 C_2 C_3}$
G	$\frac{C_1 C_2 C_3}{C_1 C_2 C_3}=1$

The table No. 1 presents the expressions of the coefficients A, B, C, D, E, F. As in practice, the elasticity of the ball screw and nut represents 70% of the total elasticity, C_1 and $C_2 \approx \infty$ can be approximated with a good accuracy that leads to:

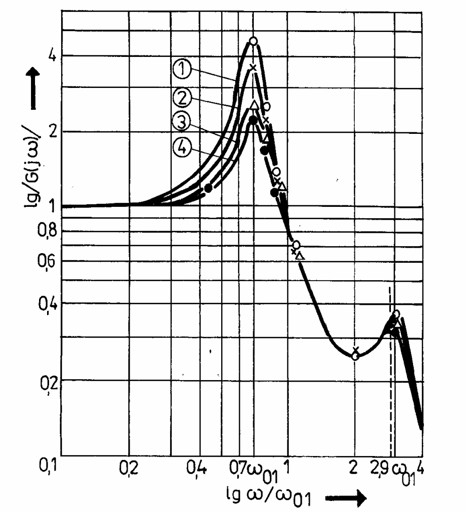
$$G_1(p)=\frac{1}{A_1 p^2+B_1 p+1} \quad (8)$$

and if $C_1 \rightarrow \infty$, it will result:

$$G_2(p)=\frac{1}{A_2 p^4+B_2 p^3+C_2 p^2+D_2 p+1} \quad (9)$$

With the models simplified as such, several harmonics will be lost, especially the ones of high frequency and low amplitude. The characteristic curve of amplitude of an oscillating system with two masses is presented in fig. 3. In consequence, the researches carried out by different authors have reduced the whole assembly to a linear system of one degree of freedom, as opinions are differing in terms of the mechanisms determining the characteristic of rigidity, mass, dumping. They may be grouped as follows:

- Total torsion rigidity of the kinematical linkage;
- Torsion rigidity of the lead screw;
- Axial rigidity of the lead screw;
- Mass of the slide along with work piece, reduced mass moment of inertia of all mechanisms composing the linkage.



	C_{t1}	θ_1	D_1	C_{t2}	θ_2	D
①	1	1	0,05	4	1	0,05
②	1	1	0,1	4	1	0,05
③	1	1	0,05	4	1	0,1
④	1	1	0,1	4	1	0,1

$\omega_{01}=\sqrt{C_{t1}/\theta}-1$

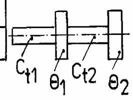


Fig. 3. Characteristic curve of amplitude of the oscillating system with two axes

The non-linear behavior of the lead screw-nut is known, but with a view to simplifying the analysis, it can be approximated to a linear element.
Most of the theoretical and experimental dynamic studies of the feed kinematical linkages have been finished at the level of the mechanical part.
The integration of the transfer function to the assembly structure of the respective axis loop and, afterwards, the application of the methods of the automatic adjustment theory has been less studied. This manner of approach of the theoretical research of the feed kinematical linkage in its entirety can be extended up to the level of construction parameters.

3. CONCLUSIONS

The usage of this method of theoretical research of the kinematical linkage as whole gives important data on the performances of the kinematical linkage. Through this research it can be known, even from the design stage, the behavior of the kinematical linkage in dynamic and static duties. At the same time several parameters can be adjusted, with a view to optimizing the static and dynamic behavior of the linkage.
This method of analysis and research, though featuring several simplifying hypotheses, gives good results on the steps to de done even from the design stage of the kinematical linkage. The method itself is easily usable through the computer and represents a useful guide to any builder of numerical control machine tools.
The results of the researches are answering the issues of the transient duty and linkage stability, as well.
Knowing the transient duty both while accelerating and decelerating, gives important data for sequent analysis of the path error of machining through interpolation. This topic is more complex and is not approached by this work.

REFERENCES

[1] Pressman, R., (1999), *Numerical Control and Computer Aided Manufacturing*, New York.
[2] Cervenka. N., Reimbun. U., (2001), *Analyse des dynamischen Verhaltens einen Hochleitun*, Maschinemarkt, nr. 84.
[3] Marusche, H.,(2000), *Probleme bein Walzschranbtreib als Antriebs und MeBelement în Werkzeugmaschinen,* Maschinenbautechnik, nr 2,1990.