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MATHEMATICAL MODELING AND OPTIMIZATION OF CREAM CHEESE MANUFACTURING USING FACTORIAL DESIGN PROCEDURES

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Abstract:
This paper presents a study regarding the optimization and mathematical simulation of the cream-cheese with adding's (dried parsley) manufacturing process using a factorial 3^3 experiment designs. The 27 experiment were performed in different conditions, the varying operating parameters for the 3^3 factorial experiment designs were: the melting temperature (x_1), the melting time (x_2) and the quantity of dried parsley added (x_3). This study provide the information's regarding the effect of melting temperature, melting time and the quantity of adding's when the melting salts amounts were fixed.

Keywords: factorial design procedures, cream cheese with adding's, modelling, optimized variables.

1. INTRODUCTION

Cream-cheese are products with minimum 30% fat in dry substance, obtained by one or more types of cheese or fresh cheese by melting or emulsification of them in thermal conditions and melting salts adding [5,6]. According to the consistence, creamed-cheese is a mellow-cheese that can be manufactured in a lot of types: creamed-cheese without other adding's, creamed- cheese with vegetable or animal adding's, desert creamed-cheese etc. According the fat content, creamed-cheese can be: double cream with 60% fat in d.s., cream with 50% fat in d.s., very fatty with 40% fat in d.s., three quarters fatty with 80% fat in d.s. For creamed-cheese with adding's used products (thermal treated) or properly performed: vegetable products (tomato paste, onion, olives), products based on meat or fish (bacon, salamis, fish paste etc.), dairy products (powder milk, butter etc.), spices (dill, pepper, pepperoni powder) [12]. The proportion of vegetable adding's or animal adding's is of 16% maximum in dry substance of final product. For desert cheese manufacturing is used sugar, natural fruit essences, fruit and cocoa. Mathematical modelling through factorial design procedure was used by now in different domains for optimizing and modelling of different process such: yoghourt manufacturing [8], chocolate and Gruyere cheese obtaining [7], water depollution using some unconventional procedures [9], biogas production [14] etc. The advantages of these method of optimization regarding others consist into the fact that these take into account not only the individual (simple) effect of each parameter but also the interaction and/or their possible synergy effects [1-4,11,13].

Hence, the most often used experimental programs are these of K^n factorial design type, in which case K is the number of variation of parameters level, and n is the number of parameters [1,2].

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2. EXPERIMENTAL PART

For the experimental part it has been adopted a factorial type design procedure of type 3^3, with three variables and three variation levels, that requires 27 experiments, for measuring the taste and the acidity of cream cheese with dried vegetables adding's. The key-step that influences the mixing process of the compound is the melting process, this step being influenced by three variables (Table 1): the melting temperature (x_1), the melting time (x_2) and the quantity of added vegetables (x_3) - dried parsley. The melting salt amount were maintained constant (0,3 g $NaHCO_3$/100 g raw food) in each of the 27 experiment.

Table 1. The variables that influence the cream cheese manufacturing process

Nr. crt.	Variables (X_i)	Minimal level (X_i^{min})	Maximal level (X_i^{max})	ΔX_i	X_i^{med}
1.	Melting temperature, ^0C (X_1)	65	90	12,5	75
2.	Melting time, min (X_2)	5	20	7,5	12,5
3.	Vegetables amount, % wt. (X_3)	0,1	1,2	0,55	0,65

3. RESULTS AND DISCUSSION

The response functions measured in the laboratory are displayed in Table 2.

Table 2. The values of the response functions

Number of experiments	Response function	
	Y_1 (taste)	Y_2(acidity)
1	9	23
2	9	11
3	9	28
4	10	30
5	8	17
6	8	18
7	9	8
8	8	7
9	9	17
10	10	14
11	9	20
12	10	25
13	9	24
14	9	16
15	10	20
16	10	10
17	9	20
18	9	15
19	8	10
20	9	9
21	8	14
22	9	16
23	10	7
24	9	11
25	9	16
26	8	9
27	8	19

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The values presented in Table 2 are noted subsequent: Y_1 - represent the mark given by the tasters to the final product in a scale from one to ten (one represent the minimum mark and ten the maximum mark) and Y_2 - the acidity of cream cheese with vegetable adding. For calculating of program significance there has been performed another three measures in the central point of the domain (0,0) obtaining the values presented in Table 3.

Table 3. The values of the measurements performed in the central point of the domain (0,0)

Y_K^0	Y_1^0	Y_2^0	Y_3^0
Y_{1k}^0	8	9	9
Y_{2k}^0	17	14	20

3.1. Elaboration of the mathematical model

The particular shape of response function for factorial program of type 3^3 is:

$$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{123}x_1x_2x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 \quad \textbf{(1)}$$

In Table 4 are displayed the calculus relations as well as the coefficients values of polynomial mathematical model.

Table 4. Mathematical relations of calculus for polynomial coefficients

Coefficient	Relations of calculus	Coefficients values	
		Y_1(for taste)	Y_2(for acidity)
Arithmetic average of the measured values (27 experiments)	$y_0^{\,i} = \dfrac{\sum_{k=1}^{27} Y_k}{27}$	$y_0^1 = 8{,}96$	$y_0^2=88{,}66$
• for simple taste	$a_1 = \dfrac{\sum_{k=1}^{27} x_{1k}\cdot Y_k}{\sum_{k=1}^{27} x_{1k}^2}$	a_1=-0,055	a_1=-2,66
		a_2=-0,11	a_2=-1,83
		a_3=-0,16	a_3=-1,44
• for interactions	$a_{ij} = \dfrac{\sum_{k=1}^{27} (x_{ij}x_2)\cdot Y_k}{\sum_{k=1}^{27} (x_i x_j)_k^2}$	a_{12}=1,25	a_{12}=3,41
		a_{13}=-0,38	a_{13}=0
		a_{23}=-0,16	a_{23}=-0,25
		a_{123}=-0,16	a_{123}=-0,83
• for quadratics terms	$a_{ii} = \dfrac{\sum_{k=1}^{27} (x_i^2 - 2/3)_k \cdot Y_k}{\sum_{k=1}^{27} (x_i^2 - 2/3)_k^2}$	a_{11}=-0,72	a_{11}=0,11
		a_{22}=0,22	a_{22}=-1,27
		a_{33}=0,26	a_{33}=0,26
• calculating the a_0 coefficient	$a_0 = y_{0^{\cdot}} - \dfrac{2}{3}(a_{11} + a_{22} + a_{33})$	a_0=27,33	a_0=88

According to the polynomial coefficients, the mathematical models which describe the response functions of the optimizing criteria are:

$$Y_1 = 27{,}33 - 0{,}055x_1 - 0{,}11x_2 - 0{,}16x_3 + 1{,}25x_1x_2 - 0{,}38x_1x_3 - 0{,}16x_2x_3 - 0{,}12x_1x_2x_3 - 0{,}72x_1^2 - 0{,}22x_2^2 + 0{,}26x_3^2 \quad \textbf{(2)}$$

$$Y_2 = 88{,}66 - 2{,}66x_1 - 1{,}43x_2 - 1{,}44x_3 + 3{,}41x_1x_2 - 0{,}25x_2x_3 - 0{,}83x_1x_2x_3 + 0{,}11x^2 - 1{,}27x_2^2 + 4{,}61x_3^2 \quad \textbf{(3)}$$

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3.2. Determination of the coefficients significance

In order to determine the significance of the polynomial coefficients there was used the **t student test**. For this is necessary first to calculate the average value of the three response functions and of the average value of the measurement errors according to the algorithm presented in the literature [1]. The t-student test results are used to determine the insignificant coefficients (Table 5).

Table 5. The value of the t-student for each coefficient

Function	t_0	t_1	t_2	t_3	t_{12}	t_{13}	t_{23}	t_{123}	t_{11}	t_{22}	t_{33}
Taste	260,28	0,52	1,04	1,52	11,90	3,61	1,52	1,142	6,85	2,09	2,47
Acidity	455,42	4,61	3,17	2,49	5,90	0	0,43	1,43	0,19	2,20	47,93

From Table 5 it can be observed that some terms that are insignificant can be eliminated; these terms are: **for taste -** x_1, x_2, x_3, x_{23},x_{123} and **for acidity -** x_{13}, x_{23}, x_{11},x_3^2.

After the elimination of the less important terms, the mathematical models that describe the response function of the optimizing criteria are:

$$Y_1 = 27{,}33 + 1{,}25x_1x_2 - 0{,}38x_1x_3 - 0{,}72x_1^2 - 0{,}22x_2^2 + 0{,}26x_3^2 \tag{6}$$

$$Y_2 = 88{,}66 - 2{,}66x_1 - 1{,}83x_2 - 1{,}44x_3 + 3{,}41x_1x_2 - 1{,}27x_2^2 + 4{,}61x_3^2 \tag{7}$$

3.3. Parameters effects discussion

A). Discussion regarding the parameters effects for Y_1 (taste)

The value of a_0 (27,33) indicates the fact that we have on optimum taste to a value close to this one. Remaining the fact that, the coefficients for the simple effects have negative values, it can be concluded that for each one of three parameters, taken individually, they have a negative effect on the taste of the cream cheese. It is noticed that the values of the interaction coefficients are positive, so the three variables, through their interaction, have a positive effect over the "taste" response function.

B). Discussion regarding the parameters effects for Y_2 (acidity)

The value of a_0 (88) coefficient indicate the fact that we have a optimum acidity to a value close to this value. Reminding the fact that the coefficients for simple effects have negative values, we conclude that each one of the three parameters, taken individually, have a negative effect on the acidity of the cream cheese. The values of the interaction coefficients are positive, hence the three variables, through their interaction have a positive effect on the response function „ acidity of cream cheese with dehydrated vegetables adding".

3.4. Optimization of the process

To determine the response function in this paper was used the derivation method. This method consist into the first order partial derivates calculating reported to each variable and then equalizing the obtained equations with zero. It can be obtained a system of three equations with three unknown terms for each of the two response functions (Y_1 and Y_2) subsequent:

For Y_1 $\begin{cases} \dfrac{\partial y}{\partial x_1} = 1{,}25x_1 - 0{,}38x_3 - 1{,}44 = 0 \\ \dfrac{\partial y}{\partial x_2} = 1{,}25x_2 - 0{,}44 = 0 \\ \dfrac{\partial y}{\partial x_3} = -0{,}38x_1 + 0{,}52 = 0 \end{cases}$ and for Y_2 $\begin{cases} \dfrac{\partial y}{\partial x_1} = -2{,}66 + 3{,}41x_2 = 0 \\ \dfrac{\partial y}{\partial x_2} = -1{,}83 + 3{,}41x_1 - 2{,}54x_2 = 0 \\ \dfrac{\partial y}{\partial x_3} = -1{,}44 + 9{,}22x_3 = 0 \end{cases}$

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The solutions of the linear systems are the following adimensional values: for taste: x_1 =1,36; x_2 =0,35; x_3 =0,31, and for acidity : x_1 = - 0,02: x_2 =0,35; x_3=0,15. As a result, the optimum searched point is: (1,36; 0,35; 0,31) for taste, and for acidity (0,02; 0,35; 0,15) represented by adimensional coordinates. It can be observed that the optimum values are not further than the limit's of the admitted domain [-1, +1].

3.4. Determination of real values of the optimum

Knowing the adimensional variation domains of the variables x_1,x_2, x_3, it can be calculated the real values of the optimum point using the mathematic relation:

$$Xi = \Delta X_i \cdot x_i + X_i^{med} \quad \textbf{(8)}$$

where: x_i – the real values of the optimum, X_i – the adimensional values of the optimum, Δx_i – the step of each variation domain, X_i^{med}- the medium real value of the parameters.

For the experiments realized in the laboratory the results are:

$$\begin{cases} x_1 = 12,5 \cdot 1,36 + 77,5 = 94,5^0\,C \\ x_2 = 7,5 \cdot 0,35 + 12,5 = 15,12\,\text{min} \\ x_3 = 0,55 \cdot 0,31 + 0,65 = 0,82\% \end{cases} \quad \text{and} \quad \begin{cases} x_1' = 12,5 \cdot (-0,02) + 77,5 = 77,2^0\,C \\ x_2' = 7,5 \cdot 0,78 + 12,5 = 18,35\,\text{min} \\ x_3' = 0,55 \cdot 0,15 + 0,65 = 0,73\% \end{cases}$$

An optimum taste is obtained for a melting temperature of 94,5^{0}C, a melting time of 15,12 min and with 0,82% vegetable adding's and an optimum acidity is obtained at a melting temperature of 77,2^{0}C, a time melting of 18,35 min and with 0,73% dried sparsely.

4. CONCLUSIONS

As it can be noticed from the real values of optimum for taste it can be concludes the followings: the optimum temperature of melting goes further than the domain of 65-90^{0}C, used in the experimental program, so the variation domain has to be reconsidered; the optimum time of melting doesn't go further than the ranger previously choose (5-20 minutes) used in the experimental program; the optimum amount of the added parsley doesn't go further than the domain [0,1-1,2 %] used in the experimental program (Table 1). From the real value of the optimum for acidity it can be observed that: the optimum temperature of the melting process doesn't go further than the values choose in Table 1; the optimum time of melting correspond to the domain choose for the experiments and the optimum amount of added parsley doesn't go further than the domain [0,1-1,2 %] used in the experimental program.

The previous conclusions show that the optimization models are valid and the factorial design of type k^n can be utilized successfully in cream cheese manufacturing process.

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