ON THE HEAT TRANSFER IN NANOFLUIDS

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Abstract. Using the fractal space-time theory (scale relativity theory), the dynamics of the fluid/nano-particle interface was analyzed. In the general case, the heat transfer through the interface reproduces a d.c. or an a.c. Josephson effects of thermal type, while in the linear approximation, the standard form of heat transfer is given. Consequently, a negative differential thermal conductance appears and an increasing of the heat transfer in nanofluids results.

Keywords: nanofluids, anomaly of the heat transfer, thermal conductance.

1. INTRODUCTION

Experiments over the past decade have revealed that thermal conductivity of the suspension can be significantly higher than that of the base medium [1-4]. Keblinski *et al* [4] have examined four possible mechanisms for the anomalous enhancement observed in nanofluids. The first is the transport of thermal energy by Brownian motion of the nano-particles. An order-of-magnitude calculation shows that the Brownian effects are not strong enough to explain the observed anomaly. A similar conclusion was also reached by Wang and Xu [1] with a somewhat different set of assumptions, and also later confirmed by a molecular dynamics simulation [4]. On the other hand, recent reports continue to attribute the enhanced conductivity to Brownian motion of the nano-particles [5-7]. Another mechanism, the formation of liquid layers around the particles, is considered by Keblinski *et al* [4]. The basic idea is that liquid molecules can form layers around the solid particles, thereby enhancing the local ordering. Since phonon transfer in crystalline solid is very effective, such local ordering in the liquid can lead to enhanced heat transport. A recent molecular dynamic simulation by Xue *et al* [8] confirmed the presence of short-ranged ordering of liquid molecules, but surprisingly observed little or no effect on the thermal conductivity. The third mechanism is related to the nature of heat transport in nano-particles. It is pointed out [4] that the generally accepted diffusive transport mechanism is not valid at the nano-scale. Instead of this, the ballistic transport is more realistic: if the ballistic 'phonons' initiated in a particle can persist in the liquid and get transmitted to another solid particle, the heat transport can significantly increase. The phonon mean free path in the liquid is typically small, because local ordering is limited to few atomic diameters. Since the particles have a constant Brownian motion, the coherent phonon transfer is possible even for low particle concentrations. Prasher et al [7] shows that the crystalline-like phonon modes account only for a fraction of the increased thermal conductivity observed in the experiments. Lastly, Keblinski et al [4] explore the possibility of nano-particles forming clusters and their effects on the thermal conductivity. This type of model may be appropriate if the particles are in the form of nanotubes or if they are not finely dispersed [9]. In contrast to the assessments of Keblinski et al [4] and Wang and Xu [1], other authors as Jang and Choi [5], Kumar et al [6] and Prasher et al [7]

hypothesized that Brownian motion of the nano-particles plays a leading role in energy transport of in nanofluids. While these models agree on the overall mechanism, they differ appreciably in how Brownian motion affects the thermal conductivity of the nanofluid.

Recently, the increasing of the heat transfer in nanofluids was related to the fractalic effects [10]. Moreover, Wang *et al.* [11] reported that the modified fractal model agreed well with the experimental data obtained for the SiO_2/ethanol nanofluid. This model has some disadvantages as difficult calculations and measurements on the nanoparticles cluster space-distribution. In such conjecture, the fractal space-time theory (scale relativity theory) is a new approach to understand quantum mechanics, and moreover physical domains involving scale laws, such as the nanosystems [12,13]. It is based on a generalization of Einstein's principle of relativity to scale transformations. Namely, one redefines space-time resolutions as characterizing the state of scale of reference systems, in the same way as velocity characterizes their state of motion. Then one requires that the laws of physics apply whatever the state of the reference system, of motion (principle of motion-relativity) and of scale (principle of scale-relativity). The principle of scale-relativity is mathematically achieved by the principle of scale-covariance, requiring that the equations of physics keep their simplest form under transformations of resolution. For example, considering that the motion of micro-particles take place on continuous but non-differentiable curves, *i.e.* on fractals [12-14], it was demonstrated that, in the topological dimension [14] D_T=2, the geodesics of the fractal space-time are given by a Schrödinger's type equation.

In the present paper, using the scale relativity theory, we propose a new mechanism capable to explain the experimentally observed enhanced thermal conductivity of nanofluid.

2. MATHEMATICAL MODEL

Let us consider the interaction between two fractal structures, *e.g*. the fluid and the nano-particle [10,11], and the corresponding interface. According with the scale relativity model, the interface dynamics is described by the coupled equations set (for details see Appendix A and [12,13]),

$$2i\mathcal{D}\partial_t\Psi_f = h_f\Psi_f + \Gamma\Psi_{np}; \qquad 2i\mathcal{D}\partial_t\Psi_{np} = h_{np}\Psi_{np} + \Gamma\Psi_f, \tag{1a, b}$$

with Ψ_f, Ψ_{np} the wave functions, h_f, h_{np} the specific "Hamiltonians" (per mass unit) on either side of the interface, Γ a coupling constant characterizing the interface, $\mathcal{D}$ the fractal/non-fractal transition coefficient [12,13], and the indices (*f*, *np*) refers to the fluid and nanoparticle, respectively. Generally, the "Hamiltonians" $H_i, i=1,2$ correspond to the free energy of a particle with the rest mass m_0 localized in *i* region. In the Appendix A it is exemplified such a situation for $\mathcal{D}=\hbar/2m$, *i.e.* for the standard Josephson effect [14].

Expliciting the wave functions by the following relations:

$$\psi_f = \sqrt{\rho_f}e^{i\theta_f}, \ \psi_{np} = \sqrt{\rho_{np}}e^{i\theta_{np}} \tag{2a, b}$$

and separating in (1a, b) the real parts from the imaginary ones, we obtain:

$$\partial_t\rho_f = -\partial_t\rho_{np} = \frac{\Gamma}{\mathcal{D}}\sqrt{\rho_f\rho_{np}}\sin(\theta_{np}-\theta_f), \quad \partial_t\theta_f = -\frac{h_f}{2\mathcal{D}} - \frac{\Gamma}{2\mathcal{D}}\sqrt{\frac{\rho_{np}}{\rho_f}}\cos(\theta_{np}-\theta_f)$$
$$\partial_t\theta_{np} = -\frac{h_{np}}{2\mathcal{D}} - \frac{\Gamma}{2\mathcal{D}}\sqrt{\frac{\rho_f}{\rho_{np}}}\cos(\theta_{np}-\theta_p) \tag{3a-c}$$

where $\sqrt{\rho_f}, \sqrt{\rho_{np}}$ are the amplitudes and θ_f, θ_{np} the phases. From here, with [4,8]

$$\rho_n = \rho_{np} = \rho\ ,\ h_f = \alpha T_f\ , h_{np} = \alpha T_{np}\ ,\ T = T_f - T_{np}\ , \theta = \theta_{np} - \theta_f\ ,\ \alpha = const. \tag{4a-f}$$

it results the heat flux:

$$J_Q = \varepsilon(\partial_t \rho_f - \partial_t \rho_{np}) = J_{Q,M} \sin\theta \tag{5}$$

of amplitude $J_{Q,M}$

$$J_{Q,M} = \frac{2\varepsilon\Gamma}{\mathcal{D}}\rho \tag{6}$$

and phase difference θ

$$\theta = \theta_0 + \frac{\alpha}{\mathcal{D}}\int T dt\ ,\ \theta_0 = const. \tag{7a, b}$$

with T_f ,T_{np} the absolute temperatures, T temperature difference on the interface, ε the elementary amount of energy transferred through the interface [9].

For *T*=0, relation (7a, b) reproduces a d.c. Josephson effect of thermal type, while for $T \neq 0$ an a.c. Josephson effect of thermal type, *i.e.* oscillations of the heat flux with the pulsation,

$$\omega = \frac{\alpha T}{\mathcal{D}} \tag{8}$$

In this last case, let us consider the dependency $T = T(t)$ in the form

$$T = T_0 + \overline{T}_0 \cos(\Omega t + \varphi_0)\ ,\ \varphi_0 = const. \tag{9a, b}$$

We notice that any time-dependent signal, for exemple (9a,b) admits locally a Fourier discrete decomposition [16]. This means that the previous results are of maximum generality.

Substituting the relation (9a, b) into (7a,b) and integrating it, we obtain the time dependence of the phase difference:

$$\theta = \theta_0 + \frac{\alpha T_0}{\mathcal{D}} t + \frac{\alpha \overline{T}_0}{\mathcal{D}\Omega}\sin(\Omega t + \varphi_0) \tag{10}$$

With relation (10), the expression (5) of the heat flux becomes

$$\begin{aligned} J_Q(t) &= J_{Q,M} \sin\left[\theta_0 + \frac{\alpha T_0}{\mathcal{D}} t + \frac{\alpha \overline{T}_0}{\mathcal{D}\Omega}\sin(\Omega t + \varphi_0)\right] = \\ &= J_{Q,M} \sum_{n=-\infty}^{+\infty} (-1)^n J_n\left(\frac{\alpha \overline{T}_0}{\mathcal{D}\Omega}\right) \sin\left[\left(\frac{\alpha T_0}{\mathcal{D}} - n\Omega\right)t + \phi_0\right] \end{aligned} \tag{11}$$

where J_n is the n-order Bessel function[16] and ϕ_0 a constant of integration.

When the pulsation $\Omega_n = \alpha T_0 / \mathcal{D}$ satisfies the relation $\Omega_n = n\Omega,\ n = 1,2,3,...$, the time-average of $J_Q \approx \left\langle J_Q(t) \right\rangle$ differs from zero, *i.e.* there is a continuous component of the heat flux of the form:

$$J_{Q,C} = (-1)^n J_{Q,M} J_n\left(\frac{\alpha \overline{T}_0}{\mathcal{D}\Omega}\right)\sin\phi_0 \tag{12}$$

From relation (12) it results peaks of the continuous heat flux for

$$T_n = nT_S,\quad T_n = \frac{\mathcal{D}\Omega_n}{\alpha},\quad T_S = \frac{\mathcal{D}\Omega}{\alpha},\quad n = 1,2,3,... \tag{13a-d}$$

and consequently a negative differential thermical conductance ($dJ_{Q,C} \big/ d\overline{T}_0 < 0$) – see Fig. 1. Moreover, from equation (12) the heat flux of the pick *n* can varies continuously in the range $[-J_{Q,M} J_n(\alpha\overline{T}_0 / \mathcal{D}\Omega), +J_{Q,M} J_n(\alpha\overline{T}_0 / \mathcal{D}\Omega)]$ at constant temperature T_n, and the phase varies in the range $[-\pi/2,\ +\pi/2]$. This means that in the interface, the heat can be generated or absorbed.

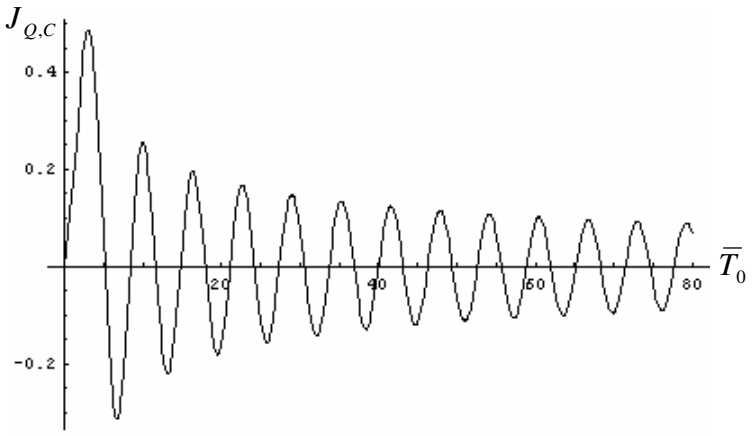

Fig. 1 – The negative differential thermal conductance, by means of the negative slope of the Bessel J_3 function.

3. CONCLUSIONS

A mathematical model of the heat transfer in nanofluids, is established. Then, the main conclusions are as follow:

i) Using the scale relativity theory, the dynamics of the fluid/nano-particle interface was analyzed. Then, the heat transfer through the interface reproduces a d.c. or an a.c. Josephson effects of thermal type;

ii) In the linear approximation ($\sin\theta \approx \theta$), the standard form of heat transfer results,

$$J_Q \approx GT,\quad G = J_{Q,M}\frac{\alpha\tau}{\mathcal{D}}$$

with *G* the thermal conductance and τ a characteristic time [1-4], while in the general cases [4], the heat transfer is non-linear;

iii) The interface is self-structuring and consequently, a negative differential thermal conductance appears. Since through self-structuring the interface generates heat to the environment, it explains the anomaly of the heat transfer in nanofluids.

Our opinion are in according to Kebliski et al. [4] results given by means of molecular level simulations.

APPENDIX A

The theory of fractal space-time is the scale relativity (SR) theory [12,13]. A non-differentiable continuum is necessarily fractal and the trajectories in such a space (or space-time) own (at least) the following three properties: i) The test particle can follow an infinity of potential trajectories: this leads one to use a fluid-like description; ii) The geometry of each trajectory is fractal [15] (of dimension 2). Each elementary displacement is then described in terms of the sum, $d\mathbf{X} = d\mathbf{x} + d\xi$, of a mean classical displacement $d\mathbf{x} = \mathbf{v}dt$ and of a fractal fluctuation $d\xi$, whose behavior satisfies the principle of SR (in its simplest Galilean version). It is such that $\langle d\xi \rangle = 0$ and $\langle d\xi^2 \rangle = 2\mathcal{D}dt$. The existence of this fluctuation implies introducing new second order terms in the differential equation of motion; iii) Time reversibility is broken at the infinitesimal level: this can be described in terms of a two-valuedness of the velocity vector for which we use a complex representation, $\mathbf{V} = (\mathbf{v}_+ + \mathbf{v}_-)/2 - i(\mathbf{v}_+ + \mathbf{v}_-)/2$ (we denoted by $\mathbf{v}_+$ the "forward" speed and by $\mathbf{v}_-$ the "backward" speed). These three effects can be combined to construct a complex time-derivative operator,

$$\bar{\partial}_t = \partial_t + \mathbf{V} \cdot \nabla - i\mathcal{D}\Delta \tag{A.1}$$

with $\mathbf{V}$ the complex speed, $\mathbf{V} = -2i\mathcal{D}\nabla ln\psi$. Using the Newton's equation in its covariant form $\bar{\partial}_t \mathbf{V} = -m_0^{-1}\nabla\phi$, we obtain the Schrödinger type equation

$$2m_0\mathcal{D}^2\Delta\psi + 2im_0\mathcal{D}\partial_t\psi - (m_0)^{-1}\phi\psi = 0 \tag{A.2}$$

In the general case [12-14], the Schrödinger type equation take the standard form $2im_0\mathcal{D}\partial_t\psi = H\psi$, with *H* the "Hamiltonians" of the particle. If, in such conjecture, we consider the "interaction" between two physical systems which have a fractal structure, then the dynamics of the interface is described by the coupled equations set (for details see [12,13])

$$2i\mathcal{D}\partial_t\Psi_1 = h_1\Psi_1 + \Gamma\Psi_2, \qquad 2i\mathcal{D}\partial_t\Psi_2 = h_2\Psi_2 + \Gamma\Psi_1 \tag{A3}$$

with ψ_1, ψ_2 the wave function, h_1, h_2 the specific "Hamiltonians" (per mass unit) on either side of the interface and Γ a coupling constant characterizing the interface. Generally, the "Hamiltonians" $m_0 h_i = H_i$, $i = 1,2$ correspond to the free energy of a particle with the rest mass m_0 localized in *i* region. Particularly, for $\mathcal{D} = \hbar/2m$, with $\hbar$ the Planck reduced constant and *m* the effective mass of the Cooper pair (for details see [14]), *i.e.* at the quantum scale, the system (A3), with substitutions $\psi_{f,np} = \psi_{1,2}$, $mh_{f,np} = H_{1,2} = \mu_{1,2}$ take the standard form of the Josephson effect:

$$i\hbar\frac{\partial\psi_1}{\partial t}=\mu_1\psi_1+\Gamma\psi_2,\qquad i\hbar\frac{\partial\psi_2}{\partial t}=\mu_2\psi_2+\Gamma\psi_1 \tag{A4}$$

In these relations, $H_{1,2}$ represent the free energy $\mu_{1,2}$ of a Cooper pair in the mentioned region, and the difference $\Delta\mu=\mu_2-\mu_1=qV$, with *q* the effective charge of the Cooper pair gives the potential *V* applied on the interface.

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