KINEMATICAL ANALYSIS OF CYLINDRICAL GEAR MECHANISMS

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Abstract: The paper presents the method of independent cycles used for kinematical analysis of cylindrical gear mechanisms. This method needs firstly the graph elaboration and after it needs the equations of velocities for each independent cycle. The method can be usually used in computational algorithms and it is easily for planetary gears mechanisms calculus, in comparison with classical Kutzbach and Willis methods.
Keywords: Gear, mechanisms, graph, cycle

1. INTRODUCTION

The kinematical analysis of gear mechanisms represents the absolute or relative velocities determination for the gears which compose the mechanism, or the motion transmission report between two preestablished wheels of mechanism. The classical methods are very well known (Kutzbach method, Willis method) by didactic book s [1]. This paper presents the method of independent cycles used for kinematical analysis of cylindrical gear mechanisms. The method is preceded by the determination of the associated graph of mechanism. The method has two parts: **AICM** for written the absolute angular velocities of mechanism elements and **RICM** for written the absolute angular velocities **(A)** of mechanism elements as well as relative velocities **(R)** ones [3].

2. PROPERTIES

It is known that for the cylindrical gear mechanisms, the motion transmission report of the angular velocities is constant. This means that same report is true for between the displacements and the accelerations. This property allows that the velocity transmission report to be equal with the displacements and accelerations transmission reports.

3. THE CALCULUS METHOD

For each closed kinematical chain, it is true:

$$\sum \vec{v}_{Aji} + \sum \vec{\omega} \ x \ \overrightarrow{AB} = \vec{0} \tag{1}$$

It writes relation (1) for each cycle that composes the independent cycles base of mechanism graph. At the links of cylindrical gear mechanisms it is not the relative translation. This means that all v_{Aji} are null. All rotation axes (absolutes and relatives) of gear mechanism are parallel with a fix direction Ox and the vectors $\overrightarrow{AB}$ are perpendicular of this direction, the equation AICM is:

$$\sum \vec{\omega}_{i0} \, (\vec{y}_B - \vec{y}_A) = 0 \tag{2}$$

This relation represents a linear equations system. The number of equations is equal with the number of cycles which composes the independents cycles associated the mechanism. If it is known the positions of the links in a reference system in the plan of the mechanism and the input kinematical parameters, it is easily to determinate the unknown kinematical parameters (ω_{i0}). For each closed kinematical chain, it is true the following relations:

$$\sum \vec{\omega}_{i,i-1} = \vec{0} \tag{3}$$

$$\sum \vec{v}_{Aji} + \sum \overrightarrow{OA} \; x \; \vec{\omega}_{i,i-1} = \vec{0} \tag{4}$$

If we consider a reference system with Ox axis in the plane of $\vec{\omega}_{i,i-1}$, parallel with its common direction, the Oy axis in same plane, perpendicular on Ox and Oz axis perpendicular on xOz plane, it can observes that the two equations represents the equilibrium equations for an parallel and coplanar vectors system $\vec{\omega}_{i,i-1}$, included in same cycle: the sum of the vectors projections on Ox axis and the sum of its torque on Oz axis.

$$(\sum X_i = 0) \Rightarrow \sum \omega_{i,i-1} = 0 \, , \, (\sum M_{iz} = 0) \Rightarrow \sum y_A \cdot \omega_{i,i-1} = 0 \tag{5}$$

4. APPLICATION METHOD OF THE KINEMATICAL ANALYSIS OF GEAR MECHANISMS

We consider a planetary mechanism (fig. 1). We know the radius r_1, r_2, $r_{2'}$, r_3 and the angular velocity ω_{10} of input wheel. We must to calculate the angular velocities ω_{20} , ω_{30} .

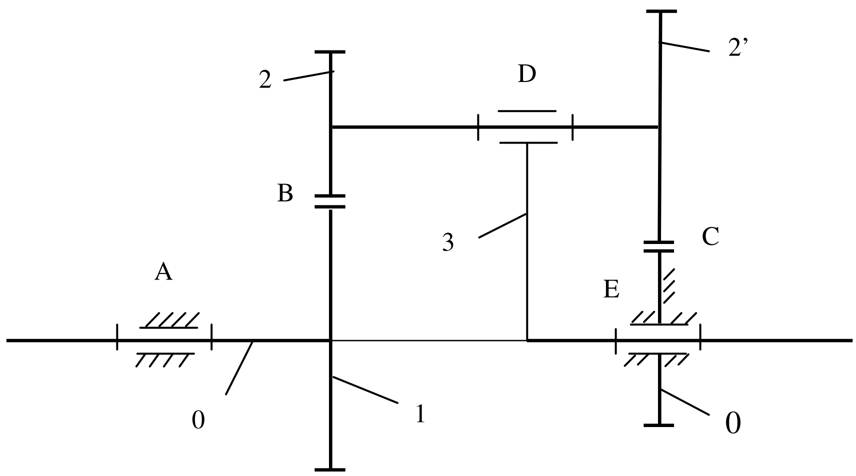

Fig. 1 The gear mechanism

The associated graph is represented in fig. 2. The two cycles which compose the independents cycles base are:

Cycle I: 0-A-1-B-2-C-0

Cycle II: 0-C-2-D-3-E-0

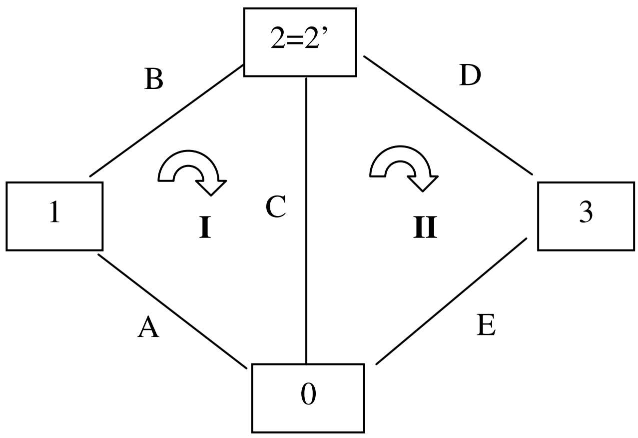

Fig. 2 The associated graph

Application of AICM
The equations of velocities are:

Cycle I: $$\omega_{10}(y_B - y_A) + \omega_{20}(y_C - y_B) = 0$$

Cycle II: $$\omega_{20}(y_D - y_C) + \omega_{30}(y_E - y_D) = 0$$

The values of y coordinations are:

$$y_A = 0,\ y_B = r_1,\ y_A = r_1 + r_2 - r_{2'},\ y_D = r_3\ .$$

We obtain:

$$\omega_{30} = \frac{r_{2'} \cdot r_1 \cdot \omega_{10}}{(r_{2'} - r_2)\, r_3} \text{ and } \omega_{20} = \frac{r_1 \cdot \omega_{10}}{r_{2'} - r_2}$$

Application of RICM
The equations of velocities are:

Cycle I: $\omega_{10} + \omega_{21} + \omega_{02} = 0, \; y_A \cdot \omega_{10} + y_B \cdot \omega_{21} + y_C \cdot \omega_{02} = 0$

Cycle II: $\omega_{20} + \omega_{32} + \omega_{03} = 0, \; y_C \cdot \omega_{20} + y_D \cdot \omega_{32} + y_E \cdot \omega_{03} = 0$

But, $\omega_{02} = -\omega_{20}, \; \omega_{03} = -\omega_{30}, y_A = 0, y_B = r_1, y_C = r_3 - r_{2'}, y_D = r_3, y_E = 0$

We obtain:

$$\omega_{21} = \frac{(r_3 - r_{2'}) \cdot \omega_{10}}{r_{2'} - r_2} \text{ and } \omega_{20} = \frac{r_1 \cdot \omega_{10}}{r_{2'} - r_2}$$

$$\omega_{32} = \frac{(r_{2'} - r_3) \cdot r_1 \cdot \omega_{10}}{(r_{2'} - r_2)\, r_3} \text{ and } \omega_{20} = \frac{r_{2'} .\; r_1 \cdot \omega_{10}}{(r_{2'} - r_2)\, r_3}$$

5. CONCLUSIONS

The independent cycles method can be used in gear mechanism calculus because its advantage is the simplicity of the equations. The method can also be utilized in numerical algorithms and programming and in the symbolic calculus.

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