THEORETICAL RESEARCH CONCERNING OF DEFLECTIONS FOR HORIZONTAL SHAFT OF MIXING DEVICES

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Abstract: The classic calculation (for dynamic conditions) on deflections of mixing devices shafts (in horizontal position), is, generally, done with assumption that important factors (gyroscopic moment, axial force and torque) are neglected. For this reason, a certain discrepancy between theoretical and practical results exists.
The paper presents a methodology calculation of shaft deflection, considering the all these factors (gyroscopic moment, axial force, torque), which are usually neglected.

Keywords: mixing device, shaft, deflection

1. INTRODUCTION

The present paper concerns the deformations under various supports conditions of a flexible shaft, or beam with non-zero mass and constant cross section in free vibratory- motion supported at is ends. The reactions and moments produced it the support its may conform to ideal, linear or non-linear laws as regards the radial deformations and the slope angles formed at the supports owing to bending moments.

The general deformation equations, which are valid both respect of flexural vibration and various composite whirling motions, are derived by applying the Toshenko' expressions for the bending moment shearing force [51 In this connection the influences of centric axial force, which is assumed to be constant, of the own mass of the shaft of beam, o shear deformation and the case of the rotating shaft of the gyroscopic moments, and of rotatory inertia in flexural vibration, are taken into account.

The constant torque, however, is only taken into consideration in the stress condition equation, in the case which come into question in practice, than the equation, in the case which come into question in practice, than the effects of the afore-mentioned factors. Internal and external damping is neglected.

In the case of non-linear support conditions the general solution does not revert to a linear eigenvalue problem as in the ideal and linear cases. To each speed then correspond one single given amplitude of vibration of the shaft or beam, which theoretically may also be infinite.

In another paper, the author will consider as an application example a steel shaft symmetrically carried in single-row deep groove ball bearings. The lowest critical speed is determined in the non-linear case by means of the stress condition as well as the amplitude condition, employing theoretically derived radial and bending slope elasticity functions. The said functions account for the elastic deformations *o[* bearings themselves. For comparison, also the critical speed consistent with ideal boundary conditions arc presented

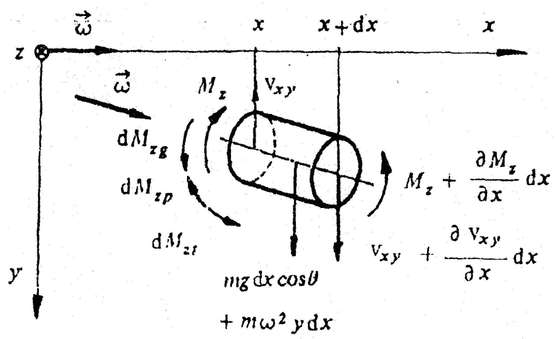
2. COMPUTING ASSUMPTIONS

In order to restrict the subject under consideration, the following assumptions are made;

1. Bernoulli - Navier's law is valid, i.e., the cross section planes of the unloaded shaft of beam remained planes even after deformation.
2. The shaft orbeam material obeys Hook's law also in the range above the proportional limit σ_{pr} up to the maximum stresses which are produced.
3. The homogeneous shaft or beam of uniform thickness supports at its ends is assumed to be ideally straight in unloaded conditions, i.e., the initial curvature is assumed to be zero.
4. The cross section of the rotating shaft is a circle or a symmetrical circular ring, whereas in respect of the transfer sally vibrating beam only the restriction is imposed that one of its two axes of symmetry lies in the plane of vibration x-y, Fig. 1
5. There are no flywheels or point masses on the shaft
6. No internal or external damping occurs in the vibratory system
7. The elasticity of bearing arrangement arc permanent and, in the case of rotating shaft, they arc permanent and, in the case of rotating shaft, they arc circular symmetric, i.e., independent of angle 8, and in association of transverse bending vibrations, they are symmetrical with reference to the x-z plane - Fig. 3
8. The masses of bearings, bearing bushings other associated supporting structures do not participate in the elastic properties of all this factors may be taken into account in the elasticity functions.

3. EQUILIBRUM CONDITIONS

In accordance with Fig 1 and Fig 2, the equilibrium conditions (for forces and moments), of the mass element *mdl* ≈ *mdx,* in the x-y plane are made.



Fi.g.1 The forces and moments acting on a mass element of the shaft

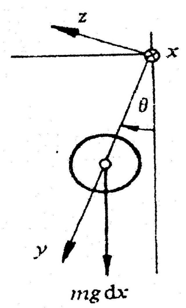


Fig-2Deformations of the mass element of the shaft

Neglecting the small terms, and calculating, the equations are transformed into:

$$V_{xy} = \frac{d}{dx}\left(M_z + M_{zg} - M_{zp} + M_{zt}\right) \tag{1}$$

where : v - shear force
Mg - gyroscopic moment
Mp - bending moment due to axial force
Mt - torque.

Relied on Fig. 2 and gyroscopic moment and bending moment expressions, it is obtained:

$$M_{zp} = -F_y \ \ (\mathrm{F} > 0) \tag{2}$$

F - axial tension force

In accordance with Fig 3 and with calculation assumption the torque from y-z plane is determined:

$$M_l \approx M_t \equiv \cancel{M}_x \tag{3}$$

and torque from x-z and x-y, also:

$$M_{yt} \approx -M_t \frac{dy}{dx} \tag{4}$$

$$M_{zt} \approx +M_t \frac{dz}{dx} \tag{5}$$

4. EXPRESSION OF SHAFT DEFLECTION

When the Expression of shear force, bending moment and torque components derived in the foregoing are respected, the dimensionless coordinates are introduced:

$$X = \frac{x}{L}, Y = \frac{y}{L}, Z = \frac{z}{L},$$

Calculating, it is obtained :

$$D^4Y - M^* D^3 Z + c_1 D^2 Y - c_2 Y = 0 \tag{6}$$

where:

$$c_1 = d^2\left(h^2 - \lambda k^2\right) - F^*$$
$$c_2 = d^2\left(1 + -dh^2 \lambda k^2\right)$$

$d_n \sqrt{\frac{mL^3}{EI}} \omega_n$ - dimensionless critical speed of order n

$M^* = \frac{M_T L}{EI}$ - dimensionless torque

$F^* = \frac{FL}{EI} (F > 0)$ - dimensionless axial force

$k^2 = \frac{I}{AL^2}$ - effect of gyroscopic moment and of rotator inertia

$\lambda = 2\left(\frac{\Omega}{\omega} - 1\right)$ - gyroscopic parameter

Ω - angular velocity of main spindle

ω – angular velocity of whirl about x axis

In X-Z plane we find analogously, considering that the sing of M* changes in equation

$$D^4Z + M^* D^3 Y + c_1 D^2 Z - c_2 Z = 0 \tag{7}$$

We denote:

$$w = Y + iZ \tag{8}$$

and multiplied (7) with "i" and add it together with (6), whereby we obtained:

$$D^2 w + M^* i D^3 w + c_1 D^2 w - c_2 w = 0 \tag{9}$$

The linear, homogeneous differential equation of the fourth order has the general solution:

$$w = \sum_{n=1}^{4} A_n e^{iqX} \tag{10}$$

where the values q_n are the roots of the auxiliary equation :

$$q^4 + M^* q^3 - c_1 q^2 - c_2 = 0 \tag{11}$$

If M*=0, there is from Equations (11) and (10)

$$q_{1,2} = \pm i \sqrt{+\sqrt{\frac{c_1}{4} + c_2} - \frac{c_1}{2}} = \pm i t_1 \tag{12}$$

$$q_{3,4} = \pm \sqrt{+\sqrt{\frac{c_1}{4}} + c_2 + \frac{c_2}{2}} = \pm t_2 \tag{13}$$

$w = Y = C_1 \cosh .t_1 X + C_2 \sinh t_1 X + C_3 \cosh t_2 X + C_4 \sinh t_2 X$

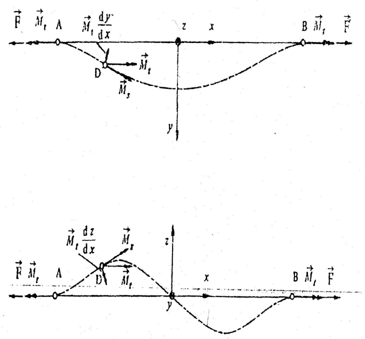


Fig.3 Shaft loaded by axial force and torsion moment

5. CONCLUSIONS

Analyzing the significance of the expression the deflection, the following conclusions could be said. The deflection depends on the length L, and the rigidity El of the main spindle. It also is very crested to notice that the deflection w depends on the dimensionless critical speed d_n therefore on ω and Ω.

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