CERTAIN ASPECTS REGARDING THE THERMAL REGIMEN OF AXIAL-CYLINDRICAL NON-CONTACT SEALS, DESTINED FOR THE SEALING OF LIQUIDS

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Abstract: Non-contact seals eliminate the contact between the sealed surfaces and have the effect of avoiding all disadvantages related to friction, wear and lubrication, found at mobile-contact seals. Friction inside the non-contact seal clearances is a complex phenomenon, having as consequences the thermal process of energy loss (heat) and also the wear of the clearance active surfaces. It follows that the study of the tribological behavior of non-contact seals imposes their research to be done through the approach of at least the following aspects: the development, the evolution and the consequences of the thermal regimen, respectively, the appearance, the evolution and the effects of wear.

Keywords: non-contact seals, axial-cylindrical, thermal regimen.

1. THEORETICAL CONSIDERATIONS

The geometrical model of an axial-cylindrical non-contact seal is presented in fig. 1. In the case of these seals, the clearance is represented by a very narrow opening, having the thickness *h* (in a radial direction) and covering the length *l*.

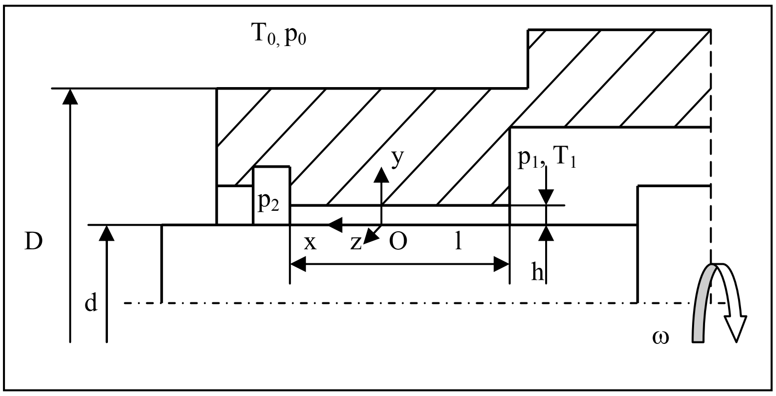

Fig. 1

The axial clearance is edged by two coaxial cylindrical surfaces, of which the interior one usually undergoes a rotation around its own symmetry axis. The seal between the mobile and the fixed surface is obtained through the introduction of a hydraulic resistance in the path of the main liquid's outflow or in the path of the exterior environment's penetration towards the interior, in a manner in which the outflow rate would be very small or null.

From the general balance of forces that act upon a certain liquid element of volume inside the clearance, which undertakes an incompressible flow, with friction, certain supplementary tensions emerge due to the presence of friction. In the case of Newtonian environments, there is a linear relation between tensions and the specific deformation rates. The dynamic viscosity, η,

$$\eta = \rho \cdot \nu \tag{1}$$

is the proportionality factor, representing the material property that characterizes the friction phenomenon in a moving fluid (ρ – the density and ν- the kinematic viscosity).

In general, in the movement of two layers of liquid situated at an infinitely small distance *dy*, sliding one relative to the other, with the relative velocity *du*, the value of the shear stress τ is proportional to the velocity gradient, *du*/*dy*, according to Newton's law:

$$\tau = \eta \cdot \frac{du}{dy} \tag{2}$$

The thermal properties of the liquid are given by the temperature coefficient, *k*,

$$k = \frac{\lambda}{\rho \cdot c_p}, \tag{3}$$

where λ is the liquid's thermal conductibility and c_p is its heat-absorption capacity [4].

For incompressible flows with $\rho = const$ and for constant material values η and k, the conservation equations for mass, impulse and energy are [4]:

$$divV = 0, \text{ (the equation of continuity);} \tag{4}$$

$$\partial V / \partial t + VgradV = f - (1/\rho)gradp + \nu\Delta V, \quad \text{(the Navier-Stokes equations);} \tag{5}$$

$$\partial e / \partial t + Vgrade = -(1/\rho)divq + \nu\Phi_V, \text{ (the energy conservation equation);} \tag{6}$$

where the external force fields are characterized by the specific mass force *f*. The internal energy is:

$$e = c_p \cdot T, \text{ (}T\text{–the liquid'stemperature)} \tag{7}$$

and the heat flow is given by the *Fourier law*:

$$q = -\lambda \cdot gradT. \tag{8}$$

In Cartesian coordinates these balance equations have the following forms:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

$$\frac{\partial u}{\partial t}+u\cdot\frac{\partial u}{\partial x}+v\cdot\frac{\partial u}{\partial y}+w\cdot\frac{\partial u}{\partial z}=f_x-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}+\nu\cdot\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{10}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=f_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{11}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=f_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\cdot\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{12}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\nu}{c_p}\Phi_v$$ with the dissipation function, Φ_v (13)

$$\Phi_v=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2. \tag{14}$$

These five non-linear equations with partial derivatives are sufficient for determining the velocity of the liquid particles, $V=V\ (u,v,w)$, its pressure and its temperature, p and T. In the following case of incompressible flows, the velocity field will be detached from the temperature field, and in the energy equation, the influence of friction appears through the dissipation function, Φ_v.

For the theoretical and experimental study of non-contact seals, along with the already mentioned equations and laws, *Newton*'s law of thermal convection plays an important part: the density of the thermal flow transmitted from a surface A of a solid body to a fluid environment (with which it is in contact) is determined by the relation [1, 2]:

$$q=\alpha\cdot\left(t_A-t_f\right) \tag{15}$$

where t_A and t_f represent the body surface temperature, respectively the average fluid temperature, expressed in °C, and α is the superficial thermal exchange coefficient, or the thermal convection coefficient.

2. THE DETERMINATION OF THE TEMPERATURE FIELD EQUATION

The integration of the energy conservation equation, presented in relation (13), requires establishing certain simplifying hypotheses and performing certain preliminary calculations:
- inside the clearance the thermal regimen is stationary:

$$\frac{\partial T}{\partial t}=0$$

- the liquid's temperature inside the seal is constant in reference to the *oz* axis:

$$\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} = 0$$

- the solution for the energy equation will be analyzed when the unitary heat flow at the seal bushing wall is known [1, 2]; if one considers that the unitary heat flow, calculated with the *Fourier* relation (8), is constant and the physical characteristics of the fluid do not change with temperature in a transversal section, then the determining or volumetric average temperature T_f will increase linearly in the liquid flow direction; thus, it follows that:

$$\frac{\partial T}{\partial x} = const = c; \frac{\partial^2 T}{\partial x^2} = 0 \tag{16}$$

- one will define, for the transversal section of the clearance, the determining temperature of the liquid using the relation:

$$T_f = \frac{T_a + T_p}{2} \tag{17}$$

where: T_a is the temperature on the shaft surface, and T_P is the seal bushing interior wall temperature;
- because the maximum liquid velocity is on the surface of the shaft, it follows that the temperature will be maximum on the same surface and thus the uniqueness conditions will be:

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0$$

$$(T)_{y=h} = T_p \text{ or } -\lambda_f \cdot \left(\frac{\partial T}{\partial y}\right)_{y=h} = q_p \tag{18}$$

In these conditions, the differential equation of the temperature field for the axial-cylindrical clearance will have the following form (one will use the calculus formulae for the liquid flow velocity components when the sealed liquid's pressure is low – they are invariable respective to the *ox* coordinate):

$$u \cdot \frac{\partial T}{\partial x} = k \cdot \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \Phi_v \tag{19}$$

Knowing that:

$$v = 0;\ u = u(y);\ w = w(y) \Rightarrow \frac{\partial u}{\partial y} = \frac{\Delta p}{2 \cdot \eta \cdot l} \cdot (h - 2y); \frac{\partial w}{\partial y} = -\frac{U}{h} = -\frac{d \cdot \omega}{2 \cdot h} \text{ [5]};$$

and with the kinematic viscosity and thermal diffusivity values given by (1) and (3), respectively, equation (19) becomes:

$$\frac{\partial^2 T}{\partial y^2} = A \cdot \frac{\partial T}{\partial x}(h - y) \cdot y - B \cdot (h - 2 \cdot y)^2 - C, \tag{20}$$

where:

$$A = \frac{\Delta p \cdot \rho \cdot c_p}{2 \cdot \eta \cdot l \cdot \lambda_f};\ B = \frac{\Delta p^2}{4 \cdot \eta \cdot l^2 \cdot \lambda_f};\ C = \frac{\eta \cdot d^2 \cdot \omega^2}{4 \cdot \lambda_f \cdot h^2}. \tag{21}$$

In these conditions, the energy equation can be easily integrated:

$$T = A \cdot \frac{\partial T}{\partial x}\left(\frac{h \cdot y^3}{6} - \frac{y^4}{12}\right) - B \cdot \left(\frac{h^2 \cdot y^2}{2} - \frac{2 \cdot h \cdot y^3}{3} - \frac{y^4}{3}\right) - C \cdot \frac{y^2}{2} + c_1 \cdot y + c_2 \tag{22}$$

With the aid of the uniqueness conditions, one calculates the integration constants, which have the following values:

$$c_1 = 0;\ c_2 = T_p - \frac{A \cdot h^4}{12} \cdot \frac{\partial T}{\partial x} + \frac{B \cdot h^4}{6} + \frac{C \cdot h^2}{2}.$$

It follows that, for the temperature profile inside the clearance, which remains constant for any x value, one can obtain the expression (the equation for the sealed liquid's temperature field inside the axial-cylindrical clearance):

$$T - T_p = A \cdot \frac{\partial T}{\partial x}\left(\frac{h \cdot y^3}{6} - \frac{y^4}{12} - \frac{h^4}{12}\right) - B \cdot \left(\frac{h^2 \cdot y^2}{2} - \frac{2 \cdot h \cdot y^3}{3} + \frac{y^4}{3} - \frac{h^4}{6}\right) + \frac{C}{2} \cdot \left(h^2 - y^2\right) \tag{23}$$

According to the laws of *Fourier* and *Newton* one defines the unitary heat flow near the seal bushing wall, using the relation:

$$q_p = -\lambda_f \cdot \left(\frac{\partial T}{\partial y}\right)_{y=h} = \alpha_i \cdot \left(T_f - T_p\right) \tag{24}$$

where λ_f is the thermal conductivity of the sealed liquid, considered at the T_f temperature, and α_i is the thermal convection coefficient for the sealed liquid – seal bushing contact.

From relations (23) and (24) it follows that:

$$q_p = -A \cdot \lambda_f \cdot \frac{\partial T}{\partial x} \cdot \frac{h^3}{6} + B \cdot \lambda_f \cdot \frac{h^3}{3} + C \cdot \lambda_f \cdot h \tag{25}$$

If in relation (25) one replaces constants A, B and C with relations (21) and groups the terms of the equation in a convenient way, one obtains the relation that expresses the analytical connection between the thermal energies that are found inside the axial-cylindrical clearance (with P one refers to the dissipated energy inside the seal because of the liquid friction produced by the shaft's rotation and with W one refers to the thermal energy dissipated in the surrounding environment through the seal body):

$$q_p = -\frac{\partial T}{\partial x} \cdot \frac{Q \cdot \rho \cdot c_p}{\pi \cdot (d + h)} + \frac{Q \cdot \Delta p}{\pi \cdot l \cdot (d + h)} + \frac{P}{\pi \cdot d \cdot l} = \frac{W}{\pi \cdot l \cdot (d + 2 \cdot h)} \tag{26}$$

Relation (26) emphasizes the following energy-related aspects of axial-cylindrical seals:

- inside the clearance there are four unitary thermal flows: the one created by viscous friction due to the shaft's rotation (q_P); the one produced by the axial pressure fall inside the seal, due to the hydraulic resistance introduced by the clearance ($q_{\Delta p}$); the one produced axially by the energy transport through the liquid outflow (q_Q); the one produced radially by the heat evacuation from inside the seal through its body (q_p);
- for the thermal balance established after a certain functioning period of the seal device, one can write a connecting relation between these thermal flows;

$$q_p + q_Q = q_P + q_{\Delta p} \tag{27}$$

- similarly to the relation for the unitary thermal flows, one ca write the relation between the thermal energies created inside the seal;

$$W + Q \cdot \rho \cdot c_p \cdot l \cdot \frac{\partial T}{\partial x} = P + Q \cdot \Delta p \tag{28}$$

Taking in consideration the fact that the main objective in the designing of a non-contact seal device is to obtain an outflow rate as small as possible, and even null, if possible, one can admit the hypothesis that the entire quantity of heat produced inside the seal, due to viscous friction, is eliminated in the exterior only through the seal walls; in these conditions, one can admit the equality *W=P* and, considering the surfaces through which the unitary thermal flows are transmitted, one obtains, after certain calculations, the following relations [5]:

$$\frac{\partial T}{\partial x} = \frac{\Delta p}{\rho \cdot c_p \cdot l} + \frac{2P}{Q} \cdot \frac{h \cdot (d+h)}{\rho \cdot c_p \cdot l \cdot d \cdot (d + 2 \cdot h)} \tag{29}$$

$$q_p = \frac{W}{\pi \cdot l \cdot (d+2h)} = \frac{P}{\pi \cdot l \cdot (d + 2 \cdot h)} = \frac{\eta \cdot d^3 \cdot \omega^2}{4 \cdot h \cdot (d + 2 \cdot h)} \tag{30}$$

$$T_a - T_p = (T - T_p)_{y=0} = \Delta T_{\max} = \frac{\eta \cdot d^3 \cdot \omega^2}{8 \cdot \lambda_f \cdot (d + 2 \cdot h)} \tag{31}$$

$$T_f - T_p = \frac{T_a + T_p}{2} - T_p = \frac{T_a + T_p - 2 \cdot T_p}{2} = \frac{\Delta T_{\max}}{2} = \frac{\eta \cdot d^3 \cdot \omega^2}{16 \cdot \lambda_f \cdot (d + 2 \cdot h)} \tag{32}$$

From equalities (24) and relations (30-32) one obtains the *Nusselt* criterion:

$$\alpha_i = 4 \cdot \frac{\lambda_f}{h} \tag{33}$$

$$Nu_h = \frac{\alpha_i \cdot h}{\lambda_f} = 4 \tag{34}$$

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