RESEARCH AND MODELLING OF THE EFFICIENCY FUNCTION IN A BUSINESS SYSTEM

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Abstract: The paper presents the concept of multidimensional research work on efficiency function in a real business system based on holistic-systemic method. Under real conditions the efficiency function (d) is non-homogeneous so the paper presents it in multiparameter form, i.e. d=d (x_i) based on parameters [1].

Keywords: coordinates, efficiency function (d), holistic-systemic method, multidimensional matrix, median, dispersion, regression, correlation

1. SELECTION OF POSSIBLE "INPUT" COORDINATES OF EFFICIENCY (GAIN) FUNCTION (D) AND MATRIX OF THE MODEL

Real objects of research work, complex business systems in particular, contain a large number of "input" coordinates due to complex and various transformation processes occurring in them. In order to make exact and reliable researches under such circumstances, it is necessary to include maximal number of coordinates, but it is very important to select significant coordinates. There are no general methods valid in research theory. It is recommended to use a priori information based on knowledge and experience obtained by preliminary researches and studies. However, an extra instruction is very important which directs to better solution: when selecting the coordinates it is desirable to select additional coordinates, even when there are no a priori information about their importance and influence on the system or process. The omission of at least one influential coordinate can cause huge mistakes when describing the state or process. By obeying the foregoing instruction, the selection of the following "input" coordinates has been done according to preliminary researches and analysis of parameters of real state in the business system:

N_{pr} - Number of production employees
N_r - Total number of employees
E_h, /h/emp/y/ - Possible fund of effective working hours
N_e, % - Efficiency of working hours
C_r, /din/h/ -Price of work
Z_b, /din/emp/y/- Employees' annual gross salary
$K_м$ - Coefficient of material costs in comparison to total costs
K_u - Coefficient of external services
K_E - Coefficient of energy costs in comparison to total costs
K_a - Coefficient of amortization costs in comparison to total costs
K_o - Coefficient of other costs in comparison to total costs
Kg - Coefficient of losses in comparison to total costs

The selection involves twelve coordinates of "input" which represent the efficiency (gain) function of the business system. On the basis of "input" structure, the efficiency function (d) has the following implicit form:

$$d=d\ (Npr, Nr, Eh, N_e, Cr, Z_b, K_м, K_u, K_E, K_a, Kg, K_o) \tag{1}$$

and the following coded form:

$$Y=Y\ (X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}) \tag{1'}$$

To achieve gain in a business system under the economic conditions is a complex and dynamic process, a challenge and a great suspense to management which consists of primary phases in decision making:

- identification, what to produce and for whom - marketing aspect?
- how, under which conditions to produce - technological aspect?
- how to collect - marketing aspect?
- how to distribute the effects for function continuity and development of a business system - management aspect?

Starting from model structure (1) and (1'), the efficiency function (d) can be explicitly expressed as:

$$d = Npr \cdot Eh \cdot \eta e \cdot Cr(1+Km)\cdot(2+Kn) - \frac{Nr \cdot Zb}{1-(Km+Ke+Ka+Kg+Ko)} + \frac{dR(Kgm-Kgi)}{Kgm} \tag{2}$$

where:

- dRm, the amount of identified loss [value unit],
- Kgi, coefficient of current loss in comparison to total costs,
- Kgm, coefficient of maximal loss in comparison to total costs.

In order to research the model (1) and (1') it is formed multidimensional matrix of the process by means of Plecet-Berman's saturated orthogonal plans [3,4], whose dimensions for the chosen model (i, j) = [N X K] = [15 X 13] according to table 1.

Matrix of gain function (d) of a business system. Table 1

No.	Coded and natural variables													Experimental values $Y_i=y_i$
	x_o	x_1 (N_{pr})	x_2 (N_r)	x_3 (E_h)	x_4 (η_E)	x_5 ($K_м$)	x_6 (K_u)	x_7 (C_r)	x_8 (Z_b)	x_9 (K_E)	x_{10} (K_A)	x_{11} (K_0)	x_{12} ($K_г$)	
1.	+1	+1	+1	+1	-1	-1	-1	+1	-1	+1	+1	-1	-1	$120,626\cdot10^6$
2.	+1	-1	-1	+1	-1	+1	+1	-1	+1	+1	+1	-1	-1	$-2,359\cdot10^6$
3.	+1	+1	+1	-1	-1	-1	+1	-1	+1	+1	-1	+1	+1	$-66,55\cdot10^6$
4.	+1	-1	+1	+1	-1	+1	+1	+1	-1	-1	-1	+1	+1	$2,61\cdot10^6$
5.	+1	+1	+1	-1	+1	+1	+1	-1	-1	-1	+1	-1	+1	$-25,057\cdot10^6$
6.	+1	+1	-1	-1	-1	+1	-1	+1	+1	-1	+1	+1	-1	$20,925\cdot10^6$
7.	+1	+1	+1	-1	+1	+1	+1	-1	-1	-1	+1	-1	-1	$20,574\cdot10^6$
8.	+1	+1	-1	+1	+1	+1	-1	-1	-1	+1	-1	+1	-1	$132,486\cdot10^6$
9.	+1	+1	-1	+1	+1	-1	+1	+1	+1	-1	-1	-1	-1	$182,307\cdot10^6$
10.	+1	-1	-1	-1	+1	-1	+1	+1	-1	+1	-1	+1	+1	$55,812\cdot10^6$
11.	+1	-1	+1	-1	+1	+1	-1	+1	+1	+1	-1	-1	+1	$-75,252\cdot10^6$
12.	+1	-1	+1	+1	+1	-1	-1	-1	+1	-1	+1	+1	+1	$-4,62\cdot10^6$
13.	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	$16,61\cdot10^6$
14.	+1	0	0	0	0	0	0	0	0	0	0	0	0	$43,536\cdot10^6$
15.	+1	0	0	0	0	0	0	0	0	0	0	0	0	$43,536\cdot10^6$

Range limits of variation of independent variables $X_{i,}$ i= 1,2 – 12 have been determined by analysis of real state in the business system in cooperation woth management.
In table 2, there are results of coding for independant variables $X_{i,}$ i= 1,2 – 12 which are being varied in the process of modelling.

. Results of factor coding. Table 2.

Factor level	X_1	N_{pr}	X_2	N_r	X_3	E_h	X_4	η_E	X_5	K_M	X_6	K_u
Upper level	+1	272	+1	560	+1	2066	+1	0,73	+1	0,36	+1	0,0101
Lower level	-1	245	-1	503	-1	1765	-1	0,66	-1	0,325	-1	0,0910
Basic level	0	258	0	530	0	1960	0	0,693	0	0,342	0	0,0960
Factor level	X_7	C_r	X_8	Z_b	X_9	K_E	X_{10}	K_A	X_{11}	K_O	X_{12}	K_g
Upper level	+1	980	+1	230.000	+1	0,0330	+1	0,0330	+1	0,0120	+1	0,3090
Lower level	-1	885	-1	207.600	-1	0,0295	-1	0,0295	-1	0,0108	-1	0,2788
Basic level	0	931	0	218.500	0	0,0310	0	0,0310	0	0,0114	0	0,2930

By means of structure and form of multidimensional matrix, table 1, the following is provided [4]:
- optimal arrangement of process results in experimental "hyper" space,
- minimal number of process results,
- maximal information on "state" in the model of gain function (d),
- minimal dispersion of results,
- efficiency of searching the structure of the "state" function (d).

The ranges stated in table 2 are objective organizational, technical and technological possibilities of a business system and they are determined according to the results and values from total income and expenditure opened in the business system for the year under research (2005).

2. COORDINATE VALUES OF EFFICIENCY FUNCTION

For each experiment, out of fifteen from "matrix plan", according to model (2), important values of "input" coordinates Y of gain function (d) have been determined in compliance to the levels of "input" coordinates X_i (-1,0,+1) based on natural numerical values of coordinates $X_{i,}$ i= 1,2 – 12.

Coordinate values Y_i have been defined by means of computers and TRS-SCAN program developed at the College of Mechanical Engineering in Trstenik.

Evaluation of dispersion of efficiency function and intensity of "input" coordinates has been done by multidimensional dispersion analysis. The procedure has been done by median (Me) as a statistical parameter which reliably represents the dispersion center of measuring values of the process [4, 6]. Difference in medians points to the intensity of influence of each "input" coordinate (X_i) on efficiency function (d).

Identification and quantification of significance of "input" coordinates $X_{i,}$ i= 1.2 – 12 for efficiency function (d) have been done by application of multidimensional dispersion analysis [4, 6]. In geometric presentation, that is the process of searching the "hyper" space of function (d) in order to rank the influence of "input" coordinates on the value of function (d).

The process of defining the intensity of "input" acting on function (d) has been done by program whose result is shown in Figure 1:

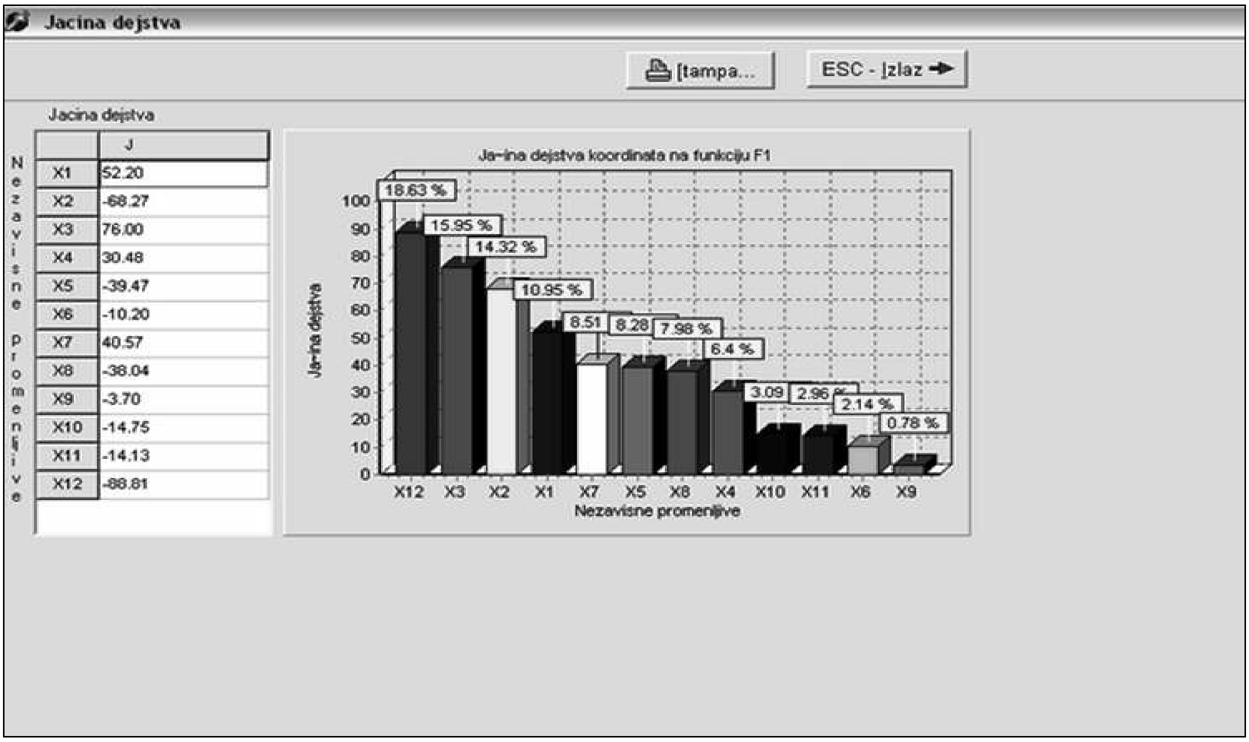

Fig. 1. Range and intensity of "input" coordinates acting on efficiency function (d)

3. MATHEMATICAL MODELING OF THE PROCESS

In general, when researching any process or effect, the mathematical (analytical) form of the equation describing the influence of factors, i.e. independent variables on function is as follows [4,6]:

- mathematical model of linear type, or non-linear type with line factor effects or,
- mathematical model of linear type, or non-linear type with interaction between some factors.

Generally, mathematical modulus with line factor effects for the influence of independent variables has the following form [4, 6]:

$$Y = f(Xo, X_1, X_2, X_3 \,{}_{........} Xn). \, . \, . \, . \, . \qquad (3)$$

In this case the equation of linear type is as follows:

$$y = b_o + \sum_{i=1}^{k} biXi \; .... \qquad (4)$$

where:
X_i – coded factors
b_o - free polynominal member
b_i - coefficient of line effects

The form of linear function (2) in coded coordinates is:

$$y = b_o + b_1 X_1 + b_2 X_2 + ...b_k X_k = b_o + \sum_{i=1}^{12} b_i X_i ..... \qquad (5)$$

Coefficient of model regression (3) b_o, b_i, i = 1,2 – 12 is determined by means of matrix equation:

$$B = (X^T X)^{-1} XY..... \quad (6)$$

where:
B - matrix of coefficients
X - plan of experiment matrix (table 1)
X^T - transposed matrix of X matrix
$(X^T X)^{-1}$ - inverse matrix of (X^T) matrix
Y - matrix of coordinates of "output" vector Y_i.

By using the above mentioned program, the regression coefficients b_o, b_1, $b_2 – b_{12}$ are determined automatically by computer.

On the basis of regression coefficients calculated, mathematical model with factor effects is given in coded form as follows:

$$\begin{aligned} y = {} & 31{,}012267 \cdot 10^6 + 30{,}193077 \cdot 10^6 \cdot X_1 - 33{,}342308 \cdot 10^6 \cdot X_2 + 37{,}229846 \cdot 10^6 \cdot X_3 + 14{,}952923 \cdot 10^6 \cdot \\ & \cdot X_4 - 17{,}712154 \cdot 10^6 \cdot X_5 - 3{,}341385 \cdot 10^6 \cdot X_6 + 18{,}149538 \cdot 10^6 \cdot X_7 - 20{,}708462 \cdot 10^6 \cdot X_8 - 3{,}737385 \cdot \\ & \cdot 10^6 \cdot X_9 - 9{,}071846 \cdot 10^6 \cdot X_{10} - 7{,}445077 \cdot 10^6 \cdot X_{11} - 19{,}177407 \cdot 10^6 \cdot X_{12} \end{aligned} \quad (7)$$

Analysis of mathematical model is done on the basis of coefficients of multiple correlation R [6]. According to [5,6] the correlation coefficient for regression model (3) without interaction is:

$$R = \sqrt{1 - \frac{\sum_{u=1}^{n} \left(y_{iu} - \hat{y}_{iu}\right)^2}{\sum_{u=1}^{n} \left(y_{iu} - \bar{y}_i\right)^2}} ..... \quad (8)$$

By mathematical model (3) and (6), the efficiency function (d) in coded coordinates, it has been calculated the coefficient values of multiple correlation:

$$R = 0{,}842634....... \quad (9)$$

There are various ways to evaluate the interdependence between variables X and Y, e.g. R and t test. In table 3 there is an example of recommendation [6] for evaluation the relation between variables X and Y based on the values of correlation coefficients.

. Relation between X and Y based on values of correlation coefficients. Table 3

Values of correlation coefficients	**Relation between variables X and Y**
$0 \le R < 0{,}2$	No dependence
$0{,}2 \le R < 0{,}5$	Weak dependence
$0{,}5 \le R < 0{,}75$	Mean dependence which is important in practice
$0{,}75 \le R < 0{,}95$	Very strong dependence
$0{,}95 \le R \le 1$	Functional dependence

Concrete value of multiple correlation coefficients in equation (8) for model (4) is R=0,842634, and according to table 3 it ranges from $0{,}75 \le R < 0{,}95$, which is **very strong** dependence between variables X and Y.

4. ANALYSIS OF RESULTS

- Presented method of multidimensional researches and modeling of efficiency function is original contribution to development of econometric methods applied in holistic-systemic concept when researching the complex systems and processes;
- Successful application of foregoing concept and reliability of research results are conditioned by adequate information system in a business system
- Stated result of research is the basis for strategy management of planning, controlling and managing in complex business systems.

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