FUNCTION OPTIMIZATION OF A BUSINESS SYSTEM EFFICIENCY

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Abstract: The paper presents analytical and experimental method of optimization (Box-Wilson's gradient method) obtained during researching the function parameters of a business system efficiency.

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1. ANALYTICAL AND EXPERIMENTAL METHOD OF OPTIMIZATION

Fundamental feature of all methods, also known as adaptive or stochastic, is their application which does not cause previously known mathematical modulus of a "goal" function. The optimum of optimization object in such circumstances is defined by direct experimental testing.

By application of these methods it is possible to describe the object by experiments in a rather reliable manner and to optimize it even if the mechanism of interactions in the processes and systems is not entirely known.

Real systems and processes in practice are followed by a large number of coordinates (variables) so we deal with complex multidimensional researches. Under such conditions it is unlikely to use analytical research methods. In such circumstances only analytical-experimental methods provide proper description and optimization of real process or research system. It is vital to emphasize that application of these methods does not require the knowledge on mathematical model of "goal" function or topography of its "hyper" surface, or optimum position accordingly. It is very important to quantitative determine the "output", that is the goal according to which the optimization is being done as well as to vary the sum of relevant coordinates of input variables within allowed domain (matrices of the process).

Out of all foregoing analytical-experimental methods, Box-Wilson's gradient method [1,2,3] is remarkable for its efficient application. This method is widely applied in optimization of complex systems and processes; therefore, it is used in this paper. By general theory of Box-Wilson's gradient methods it is possible to successfully solve three optimization assignments [4]:

1. Identification of unknown parameters or effects in assumed linear or nonlinear mathematical modulus of the process whose time is short, testing costs are low and whose level of reliable results is high.
2. Significance analysis of some controlled factors of the process, selection by groups which is very important in optimization methodology and procedure.
3. Definition of the most suitable working conditions of the system through identification of gradient lines or optimal routes of process control (experimental-statistic optimization).

2. OPTIMIZATION PROCEDURE

This method is applied on the basis of successive steps which help to move along the gradient line towards the optimal area in allowable domain of goal function of the system or process.

Optimization of this method follows the final phase of mathematical modeling stated in [1]. The procedure is done in the following manner:

At the beginning of optimization procedure, we identify the directions of gradient routes at unknown "hyper" function of the state d=d ($X_{i,}$ i= 1,2 – 12) and after that we move along the route by defined number of steps until we reach optimal or extreme value of the function d=d (X_i), i.e. maximum d_{max}. Directions of gradient routes are determined by gradients of efficiency function which is optimized. Starting points (positions) of the routes are most often presented by the points at which the extreme values were previously identified.

In these researches [1] the extreme values of the function is obtained $d_{max} = 182.306841 * 10^6$, experiment E-9, for values of "input" coordinates given in Table 1.

Starting points at gradient route (M_1). Table 1

No.	Coordinates		Value
	Inputs	Sign	
1.	Total number of production employees in a business system	N_{npr}	272
2.	Total number of employees in a business-production system	N_r	503
3.	Annual level of effective hours	E_h	2066
4.	Efficiency of effective hours	η_E	0,73
5.	Coefficient of material costs in comparison to total costs	$K_м$	0,325
6.	Coefficient of service costs in comparison to total costs	K_u	0,0101
7.	Price of work	C_r	980
8.	Employees' gross earnings	Z_b	230.000
9.	Energy costs (participation coefficient in total costs)	K_E	0,0295
10.	Coefficient of amortization costs	K_A	0,0295
11.	Coefficient of other costs in a business system	K_o	0,0108
12.	Coefficient of unnecessary costs	K_g	0,280

Complete procedure of Box-Wilson's gradient method is done by standard Table 2, and geometric presentation is shown in figure 1. Starting basis at gradient route is defined by point $M_p=M_1$ at gradient "hyper" surface of efficiency function. Values of "input" coordinates which make maximal value of function d=d (X_1) are shown in Table 2.

Direction of gradient route necessary for movement towards optimum (point M_0) is determined by regression coefficients $b_{o,}$ $b_{i,}$ i=1,2–12, i.e. by tangencies of inclination angle of "hyper" surface of function (d) towards the coordinate planes. These coefficients have been calculated in [1] and checked by means of coefficient of multiple correlation R=0.842634. Calculated values of regression coefficients are:

K_0 = 31012251,13 ; K_1 = 30193137, 31 ; K_2 = -33342230,09
K_3 = 37229721,00 ; K_4 = 14953008,38 ; K_5 = -17712089,00
K_6 = -3341402,54 ; K_7 = 18149564,54 ; K_8 = -20708525,31
K_9 = -3737566,85 ; K_{10} = -9071716,54 ; K_{11} = -7445061,31
K_{12} = -46478975,00

Figure 1 shows efficiency function for two most influential coordinates (X_{12} and X_{13}), of shape d=d(Kg, Eh), [1].

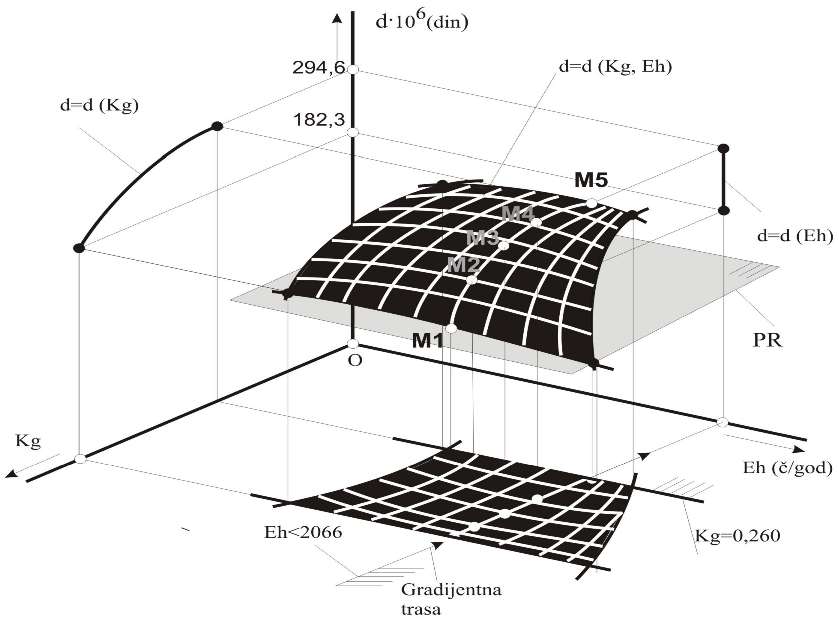

Fig. 1. Schematic of optimisation function d=d (Kg, Eh)
(Nr=450, Npr=280, Cr=980, Km=0,305, Zb=230.000,η_e=0,75, K_A=0,0295, Ko=0,010, Ku=0.0089, Ke=0,0091)

The coordinates of point $M_1^{(M)}$ at gradient route are determined according to [3,4]:

$$X_1^{(M)} = X_1^{(Ma)} \pm h_r = X_1^{(Ma)} \pm \lambda \text{ Ki Wi......} \quad (1)$$

Where:

$X_1^{(Ma)}$ - values of "input" coordinates at the beginning of the route,

h_r - calculating step along the route,

K_i - regression coefficients ,

W_i - variation interval of "input" coordinates (X_i)

$\lambda = \dfrac{\mu}{\left|K_i^*\right|}$ - parameter of route step

$\left|K_i^*\right|;(b_i^*)$ - absolute value of regression coefficient which meets the condition::

$K_i \cdot W_i = (K_i \cdot W_i)_{max}$, i.e. $b_i \cdot W_i = (b_i \cdot W_i)_{max}$ $0 < \mu < 1$- route parameter.

Upon final movement along the gradient route in each point (M_i) from starting point (M_1) to optimum (M_0), for calculated "input" coordinate (X_i), the value of function (d) is calculated. This process lasts until the area of optimum is reached which is recognized by lower and lower change of function value (d).

In mentioned example the optimal value of function is reached at point M_5 and its amount is d=d_{Max}=294.7 * 10^6 dinars.

Gradient method. Table 2

		HYPER SPACE OF "INPUT"												HYPER SPACE OF THE "STATE"
natural coordinates of "INPUT"		K_g	E_h	N_r	N_{pr}	C_r	K_M	Z_h	η_E	K_A	K_o	K_u	K_E	
coded coordinates of "INPUT"		X_{12}	X_3	X_2	X_1	X_7	X_5	X_8	X_4	X_{10}	X_{11}	X_6	X_9	
PARAMETERS OF GRADIENT ROUTE	$0 \le \mu \le 1$	0,30												
	$\frac{K_i}{(K_i W_i)_{MAX}}$	$231{,}935 \cdot 10^9$												
	$\lambda = \mu / K_i$	$0{,}14 \cdot 10^7$												
	calculating step $h_r = \lambda \cdot K_i \cdot W_i$	— 0,010167	+ 80,9	— 14	+ 6,12	+ 12,7	— 0,00449	— 3.360	+ 0,0076	— 0,00023	— 0,000065	— 0,000024	— 0,000095	
	rounded step h_z	0,01	const.	14	6	const.	0,005	const.	0,01	const.	0,0001	0,0003	0,0001	
EXPERIMENTS AT GRADIENT ROUTE	location of experiment	VALUES OF "INPUT" COORDINATES AT GRADIENT ROUTE												Efficiency function $d(10^6)$
1	M_1	0,280	2066	503	272	980	0,325	230.000	0,73	0,0295	0,0108	0,0101	0,0295	182,306
2	M_2	0,270	2066	189	278	980	0,320	230.000	0,74	0,0295	0,0107	0,0098	0,0094	242,2
3	M_3	0,2635	2066	175	280	980	0,315	230.000	0,75	0,0295	0,0106	0,0095	0,0093	274,5
4	M_4	0,263	2066	461	280	980	0,31	230.000	0,75	0,0295	0,0105	0,0092	0,0092	285,7
5	M_5	02635	2066	450	280	980	0,305	230.000	0,75	0,0295	0,0104	0,0089	0,0091	294,7
6	M_6	0,2635	2066	450	280	980	0,305	230.000	0,75	0,0295	0,0103	0,0089	0,0090	294,56

3. ANALYSIS OF RESULTS

The results of optimization shows the following:

1. The highest value of efficiency function (d) is obtained at point (M_5) in gradient route and its amount is: $d=d_{Max}=294.7 * 10^6$ dinars for the following "input" values:

K_{go}=0,2635, E_{ho}=2066 h/y, N_{ro}=450emps, N_{pro}=280empls, C_{ro}=980 din/h, K_{Mo}=0,305, Z_{bo}=230.000 din/h, $\eta_{eo} = 0{,}75$, K_{Ao}=0,0295, K_{oo}=0,0104, K_{uo}=0,0089, K_{Eo}=0,0091.

2. Evaluation of accuracy of optimum obtained:

Optimization is ended at the first gradient route due to limitation functions so there is no reason for further research of its flow.

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