

A FEA APPROACH TO SIMULATE THE DYNAMICAL BEHAVIOUR OF A COMPLIANT MECHANICAL STRUCTURE

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Abstract: As opposed to rigid-body mechanisms, the structural elements of a compliant mechanism utilizes their elastic characteristic to generate a specifying mobility on the predetermined portion, or to delivers the desired motion and force by undergoing elastic deformation. Even the mechanical structure is designed and built as a rigid one, often the entire structure has a compliant comportment resulting from elastic deformations of links, elasticity of joints, dumping elements etc. Numerical simulation of mechanical response of such structure can be formulated using finite element method. In this paper a method to determine dynamical response of a compliant planar mechanism is proposed.

Keywords: finite element method, compliance, dynamical response,

1. INTRODUCTION

Dynamical response of the mechanical structure may be estimated using finite element method. Mashed structure are in the nodes longitudinal, transversal and rotation displacements as generalized unknown, which are the amplitudes of vibrational motion of nodes according to this directions.

For a dynamic system with more degree of freedom, as results from a finite element analysis of a mechanical mashed structure, dynamical motion equations form a differential equations system [3]:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\}, \quad (1)$$

with initial conditions

$$t = 0, \{U\} = \{U_0\}, \{\dot{U}\} = \{\dot{v}_0\}, \quad (2)$$

where

- $[M]$, $[C]$, $[K]$ are mass matrix, dumping matrix and stiffness matrix, named *characteristic matrix* of system,
- $\{U\}$, $\{\dot{U}\}$, $\{\ddot{U}\}$, $\{F\}$ are displacement, velocity, acceleration and external forces vectors, written in the global coordinates system.

The unknown values $\{U\}$ are time varying functions, $\{U\} = \{U(t)\}$, and generally represent longitudinal, transversal and rotations displacements.

2. NUMERICAL INTEGRATION

To determinate the values of unknowns there are some step by step methods, which integrate the motion equations at discrete time moments $t, t+\Delta t, t+2\Delta t, \dots, t+n\Delta t$, in considered interval. Recurrence relations determine the unknown values at the time step i , using the anterior values at steps $i-1, i-2, i-3$ etc.

For example, Hubolt integration method has the following recurrence relation [3]

$$\begin{aligned} \left(\frac{2}{h^2}[M] + \frac{11}{6h}[C] + [K] \right) \{U\}_i = \{F\}_i + \left(\frac{5}{h^2}[M] + \frac{3}{h}[C] \right) \{U\}_{i-1} - \\ - \left(\frac{4}{h^2}[M] + \frac{3}{2h}[C] \right) \{U\}_{i-2} + \left(\frac{1}{h^2}[M] + \frac{1}{3h}[C] \right) \{U\}_{i-3}, \end{aligned} \quad (3)$$

and allows displacements calculation at an certain time step i using the anterior values of displacements at three time steps, $i-1, i-2$ and $i-3$.

The elements of characteristic matrix of system, $[M], [C], [K]$, according to the dynamic model, may be in three distinct situations within study time interval : a) constants in time, linear systems, b) time varying or/and depending of unknowns and their derivatives, non-linear systems.

2.1. Linear systems

First situation is an ordinary one, do not involve major difficulties and is characteristic to rigid mechanical structures. Differentials equations system is linear and the solution is generated by recurrence relation (3) or others widely explained in technical literature [3].

For this kind of structure the characteristic matrix have following denominations : $[M]$ *mass matrix*, $[C]$ *damping matrix*, and $[K]$ *structural stiffness matrix*, and their elements are constants in time.

2.2. Non-linear systems

Nonlinear propriety occurs when supplementary hypothesis are admitted, for example compliance of structure, anthropomorphic mechanisms, present of dumping or large deflection elements etc. In this case characteristic matrix have behind constant components specifying perfect rigidity, time varying elements [4]. Formally on write

$$[M] = [\Phi(t)], [C] = [\Phi(t)], [K] = [\Phi(t)], \{F\} = \{\Phi(t)\}. \quad (4)$$

Integration of motion equations are completed at discrete time steps, within study time interval, and at each time step i characteristic matrix of structure and the external load forces vector must be recalculated :

$$[M]_i = [\Phi(t_i)], [C]_i = [\Phi(t_i)], [K]_i = [\Phi(t_i)], \{F\}_i = \{\Phi(t_i)\}. \quad (5)$$

In fact at each time step on integrate a different linear system of differential equations, because of the characteristic matrix are changed. For whole time interval of dynamic response the system has a pseudo-linear comportment.

A strong non-linearity appears when motion equations (1) are nonlinear differential second order equations, with unknown coefficients depending of unknowns, their derivatives and eventually the time :

$$[M] = [\Phi(u, \dot{u}, \ddot{u}, t)], [C] = [\Phi(u, \dot{u}, \ddot{u}, t)], [K] = [\Phi(u, \dot{u}, \ddot{u}, t)], \{F\} = \{\Phi(t)\}. \quad (6)$$

For numerical integration of these systems there are not a direct method like for linear systems and the solution results after a successive iteration process.

Integration will be made at discrete time steps. At each time step i would evaluate characteristic matrix of structure $[M]$, $[K]$ and $[C]$, but these depend of unknowns and their derivatives would be known at this time.

The exit of this situation is a successive iterations process, supposing for start an arbitrary set of values for variables.

A numerical procedure proposed in [9] is :

- the interval of time is divided in equal subinterval with h length ;
- at each time step $t_0 + ih, i = 0, n$, on iterate operations :
- at $j := 1$ iteration :
 - on initialize unknowns with an arbitrary set of values

$$\{U\}_{0i} := [U_{01}, U_{02}, \dots, U_{0n}]^T; \quad (7)$$

- on calculate the first and second order derivatives of unknowns using the initial values $\{U\}_{0i}$ and the values of three anterior time steps determinated with regressive differential method [3]

$$\begin{aligned} \{U\}_{-1} &= \{U\}_0 - h\{\dot{U}\}_0 + \frac{h^2}{2}\{\ddot{U}\}_0, \\ \{\ddot{U}\}_0 &= [M]^{-1}(\{F\}_0 - [C]\{\dot{U}\}_0 - [K]\{U\}_0), \\ \{U\}_{-2} &= \{U\}_0 - 2h\{\dot{U}\}_0 + 2h^2\{\ddot{U}\}_0. \end{aligned} \quad (8)$$

- on determine characteristic matrix of system $[M]$, $[K]$ and $[C]$ using (7) and (8) ;
- on integrate resulting linear system using Hubolt method (3) [3,7,9] ;
- on compare the resulted solution, $\{U\}_{1i}$, with iteration starting values $\{U\}_{0i}$; if the errors are grater than admitted tolerance on proceed an another iteration $j := j+1$, using start values the solution determinated at anterior iteration, $\{U\}_{0i} := \{U\}_{1i}$;

On observe that the number of iterations is unknown to reach at an approximate solution with an accepted error.

Generally the iterative process is convergent, but there is not a theory to analyze or guarantee this aspect.

3. CASE STUDY

One of typical structure with compliant behavior is the mechanical structure of a manipulator or industrial robot. Dynamical response estimation of a mechanical structure formed by flexible parts like links and joints involve additional factors to be considered versus a classical analysis considering rigid body elements.

Even the parts are over dimensioned to increase rigidity and assure the repeatedly of motion, the link are flexible under the task and the actuators are not the same comportment in different situations, the position, velocity and acceleration depending on resistant couple.

The compliance of entire structure results from elastic deformations of links and compliance joints.

For motion equations formulation finite element method are adopted, leading to a mashed structure with n degree of freedom in nodes, Figure 1.

In [1,9] the explicit dynamic model of a manipulator is presented, using an isoparametric three nodes finite element, Figure 2.

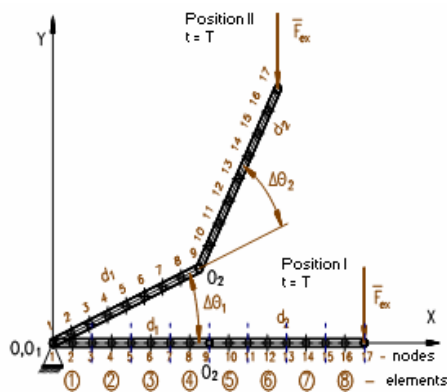


Fig. 1 Mashed structure

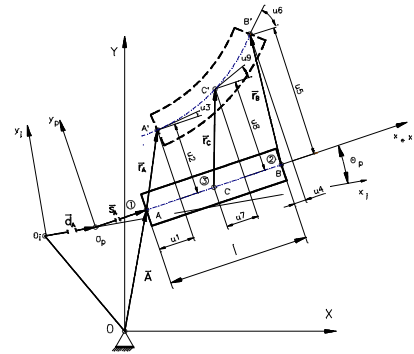


Fig. 2 Finite element definition

Generalized coordination of finite element nodes are longitudinal displacements u_1, u_4, u_7 , transversal displacements u_2, u_5, u_8 and rotations u_3, u_6, u_9 , forming displacements vector

$$\{u\}^e = [u_1, u_2, \dots, u_9]^T. \quad (9)$$

After characteristic matrix determination at finite element level, on transfer to global coordinates system, using transfer matrix and assembling the whole structure finally results the motion equations, the system (1). Detailed calculi are presented in [7,9].

The compliant behavior of whole structure is coming from two accepted hypothesis. In the first, on considers the links with a great length/transversal section ratio are flexible, so they are elastic deformed, and also the joints have a compliant comportment. That means over a transport motion of links are superposed a relative motion resulting from elastic deflection of links and compliance of joints.

These hypotheses lead to characteristic matrix with more components [1,7,9] as follow :

$$\begin{aligned} [K] &= [K^c] + [K^s] + [K^j], \\ [M] &= [M^s] + [M^e], \\ [C] &= [C^s] + [C^s] + [C^e], \end{aligned} \quad (10)$$

where $[K^c]$ is structural stiffness matrix, $[K^c]$ is centrifugal stiffness matrix, $[M^e]$ is mass matrix, and $[C^e]$ is gyroscopic damping matrix. The matrix $[K^j]$, $[M^e]$ and $[C^e]$ are equivalent stiffness of joints, mass of actuators and the external damping. $[C^s]$ is structural damping matrix determinated with Raleigh proportional damping method [1,9].

Centrifugal stiffness and gyroscopic damping are generated by transport velocity and Coriolis acceleration, generated by elastic deformations of links [1,7,9], so they are time varying.

That means at each time step of integration, numerical values of elements of characteristic matrix $[K]$, $[C]$ and $[M]$ are different, depending only by time and the motion equations are (5).

According to the second hypothesis on admits the angular deformations modify the position of local coordinates system of finite element with an additional rotation.

That involve the transfer matrix to global coordinates system depend on generalized coordination referring rotations, u_3, u_6, u_9 and finally the elements of characteristic matrix $[K]$, $[C]$ and $[M]$ depend on unknowns $\{U\}$ and their derivatives, system (6) [1,9].

To integrate systems (5) and (6) a Matlab toolbox was created. Numerical simulation of endtool movement of generic RRR industrial robot is done for each of three cases above mentioned.

Figure 5 shows the amplitude variation of longitudinal and transversal movement u_1 , u_2 . Figure 4 presents the graphics of transversal displacements u_2 of end tool of manipulator for case a) and nonlinear systems, case b). [4].

On observe that even the forms of evolution in time of dynamical responses are similar, there are significant difference between two models. Nonlinear system describes a more *compliant* structure.

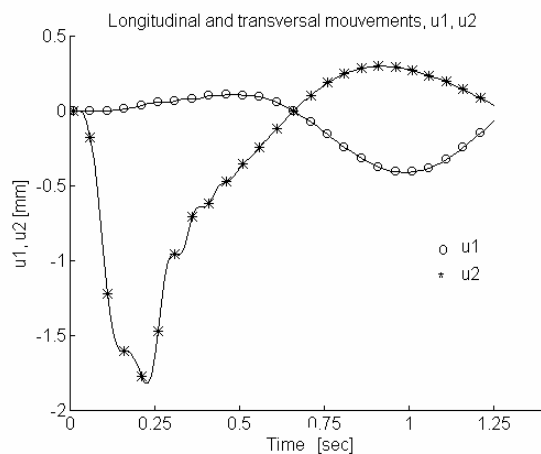


Fig. 4. Movements of endtool

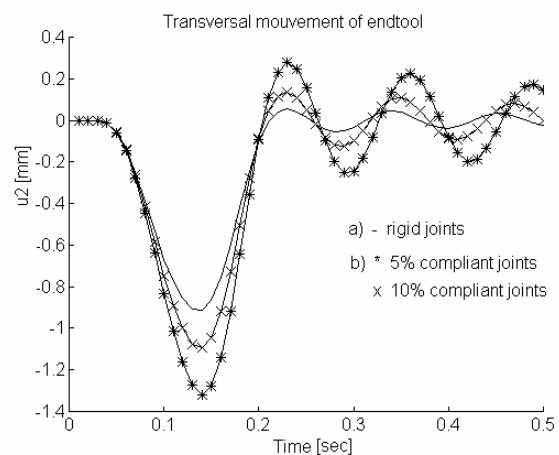


Fig. 5. Numerical/Experimental data

Experimental determinations [8] to validate theoretical model were done on RRR sequence of an actual robot.

The three links of manipulator, executing motions in a vertical plane, are finding in a work space horizontal position. Accelerometers are mounted in the critical points of structure, as end tooling or on the links near joints.

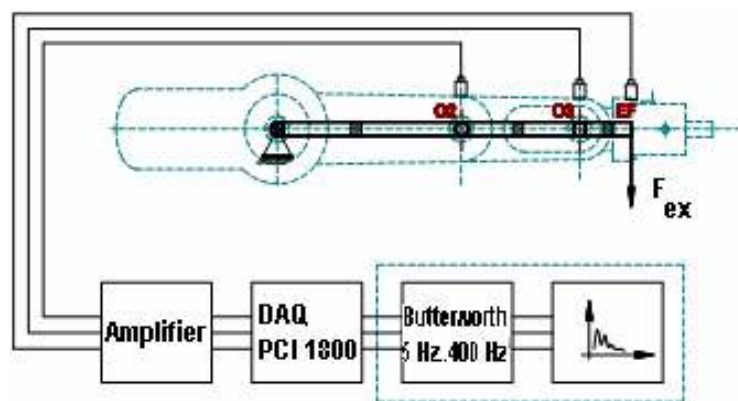


Fig. 5. Experimental data acquisition

Mechanical structure is excited with a vertical force F_{ex} applied in the end tooling, Figure 5.

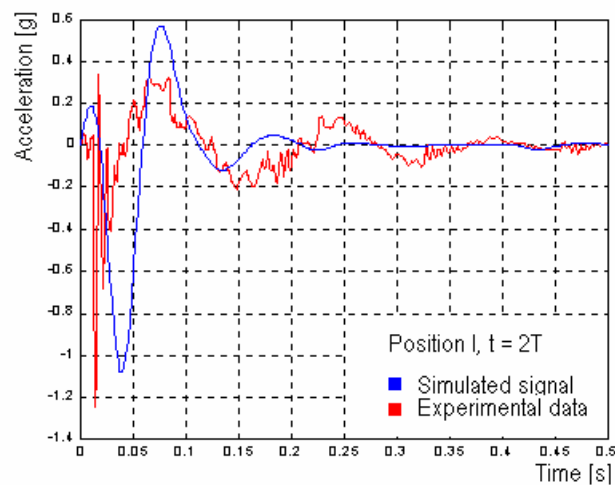


Fig. 6. Experimental/Numerical data

Numerical results for accelerations amplitude obtained after the run of simulation software are compared with experimental data and drawn in Figure 6, for endtool point of structure. Acceleration amplitude and dumping time of vibration are suitable parameters to characterize numerical - experimental correspondence [6,8].

Numerical results estimate the dynamic response estimation of mechanical structures of industrial robots and are useful for corrections in command system.

4. CONCLUSIONS

Dynamical analysis of mechanical structure with flexible elements involves combined numerical techniques to determine motion equations of critical points of structure. Finite element method may be used to take in account the special proprieties of compliant structure like compliance of joint, links flexibility, damping elements etc.

To solve the differential equation system of motion an algorithm and a toolbox of procedures were created, under the mathematical environment Matlab. Numerical results, consisting of longitudinal, transversal and rotation displacements of structure nodes, are important to predict the dynamical response of critical points of mashed structure.

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