THE ORIENTATION OF THE "RANGE MATRIX" IN MATHEMATICAL MODEL OF THE QUATERNIONS FOR A RTR ROBOT

SAVUC GABRIELA[1], FLORESCU DANIELA[2], STANCU MIHAI [1]

[1] *„Gh. Asachi" Tehnical Universityof Iaşi*
[2] *University of Bacau*

Abstract: The paper presented a stochastic model for the matrix used in determinations for the Denavit-Hartenberg parameters by means of quaternion method for an RTR slewing bracket.

Keywords: quaternion, matrix, hipercomplex numbers, Denavit-Hartenberg parameters

1. INTRODUCTION

Rotary quaternion is hipercomplex numbers witch are a special algebra that permitted the definitions of the space gyrations that the complex numbers express the plan rotations. The paper proposed an RTR robot witch is calculated the range matrix used two methods. Because the robot is two rotations it could be used the quaternion method, a hard methods of works comparative with another methods used in robot mechanisms [2],[3],[7].

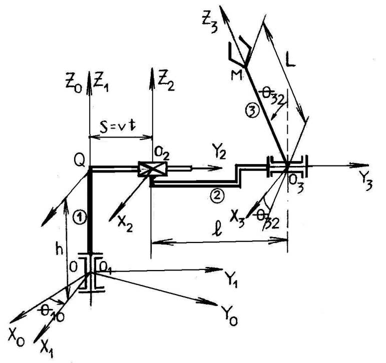

Fig. 1. The RTR robot

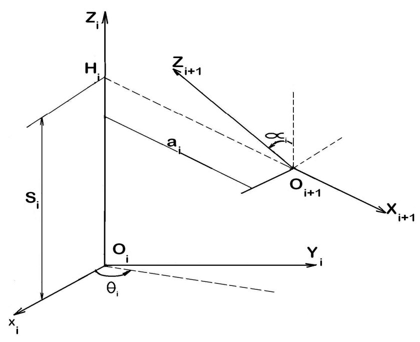

Fig. 2. The D-H parameters for two consecutive reference frame

2. DENAVIT-HARTENBERG PARAMETERS FOR TWO CONSECUTIVE REFERENCE FRAME

Figure 2 [11] presented the Denavit-Hartenberg parameters for two consecutive reference frame at general way, and after that figure 1 shows the RTR robot [10]. The reference frame $O_{i+1}x_{i+1}y_{i+1}z_{i+1}$ is obtaining with frame $O_ix_iy_iz_i$ used four consecutive movies: an rotations with θ_i angle around z_i axe; an translation by s_i distance in long of z_i axe; an translations by distance a_i in long of x_i axe; an rotations with α_i angle around x_i axe. The last position of the initial frame $O_ix_iy_iz_i$ is marked by $O_{i+1}x_{i+1}y_{i+1}z_{i+1}$ frame. The fourth quantities $\{\theta_i, s_i, a_i, \alpha_i\}$

are the Denavit-Hartenberg parameters. The homogeny linear mapping matrix A_i, rank 4x4, assert the inner product for four elementary homogeny linear matrix (rotation, translation, translation, rotation) show in figure 1:

$$A_i = R(\vec{k},\theta_i)\cdot T(\vec{k},s_i)\cdot T(\vec{i},s_i)\cdot R(\vec{i},\alpha_i) \quad (1)$$

$$R(\vec{k},\theta_i)=\begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0 & 0\\ \sin\theta_i & \cos\theta_i & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};\ R(\vec{i},\alpha_i)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta_i & -\sin\theta_i & 0\\ 0 & \sin\theta_i & \cos\theta_i & 0\\ 0 & 0 & 0 & 1\end{bmatrix};\ T(\vec{k},s_i)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & s_i\\ 0 & 0 & 0 & 1\end{bmatrix};\ T(\vec{i},a_i)=\begin{bmatrix}1 & 0 & 0 & a_i\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}; \quad (2)$$

By final effectors of the robot is attached a frame with variable size by three vectors with consecrated name in technical papers [11] (fig. 3):
$\bar{a}$ - Vicinity vector of the direction at characteristic line (Δ); $\vec{o}$ - orientation vector of the direction at auxiliary line (δ); $\vec{n}=\vec{a}\times\vec{o}$ - normally vector with is completed the tried.

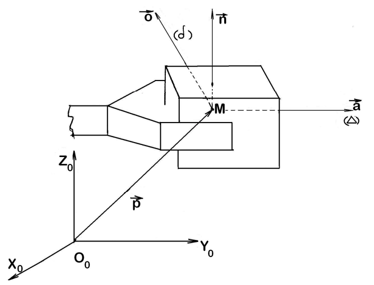

Fig. 3. The frames witches are attached to end-effectors

M is the specific point (the geometrical nucleus of the object) and the origin of the tried $(\vec{a},\vec{o},\vec{n})$. The position' vector for M point is marked with $\vec{p}$ and is show by equation $\overline{OM}=\vec{p}$ (3)
The fourth vectors, $(\vec{a},\vec{o},\vec{n},\vec{p})$, are configures the situation matrix, with the symbol T_M:

$$T_M=\begin{bmatrix}n_x & o_x & a_x & p_x\\ n_y & o_y & a_y & p_y\\ n_z & o_z & a_z & p_z\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (4)$$

For T_M we multiply the *n* matrixes $A_i (i=1,2,....,n)$,

$$T_n = A_1\cdot A_2\cdot ....\cdot A_{n-1}\cdot A_n \quad (5)$$

Because $T_m = T_n$, the position of the M is calculated with equation:

$$(r_M)_0 = T_n (r_M)_n \quad (6)$$

Where $(r_M)_0$ represented the space coordinate column for M in fix system $O_0x_0y_0z_0$; $(r_M)_n$ represented the space coordinate column for M in bound system by the final effectors. The table 1 shows the D-H parameters.

Table 1.

Coupling	Coupling type	The element numbers	Denavit-Hartenberg parameters θ_i	s_i	a_i	α_i
A(0,1)	Rotations	1	$\theta_{10}=\omega_1 t$	0	0	0
B(0,1)	Translation	2	0	$s_{21}=v_2 t$	0	0
C(2,3)	Rotations	3	$\theta_{32}=\omega_3 t$	0	0	0
D(3,4)	Translation	4	0	$s_{43}=v_4 t$	0	0
E(4,5)	Rotations	5	$\theta_{54}=\omega_5 t$	L_{45}	0	0

Using equation (1) results:

$$A_1 = R(\vec{k},\omega_1 t)\cdot T(\vec{k},0)\cdot T(\vec{i},0)\cdot R(\vec{i},0) \quad (7)$$

Equation (7) is reduce at $A_1 = R(\vec{k}, \omega_1 t)$, where $R\left(\bar{k}, \omega_1 t\right)$ are the homogeny transformation matrix with $R_{1,0}$ form.

$$R_{1,0} = \begin{bmatrix} \vec{i}_1 \cdot \vec{i}_0 & \vec{j}_1 \cdot \vec{i}_0 & \vec{k}_1 \cdot \vec{i}_0 & 0 \\ \vec{i}_1 \cdot \vec{j}_0 & \vec{j}_1 \cdot \vec{j}_0 & \vec{k}_1 \cdot \vec{j}_0 & 0 \\ \vec{i}_1 \cdot \vec{k}_0 & \vec{j}_1 \cdot \vec{k}_0 & \vec{k}_1 \cdot \vec{k}_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\omega_1 t & -\sin\omega_1 t & 0 & 0 \\ \sin\omega_1 t & \cos\omega_1 t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1 \tag{8}$$

$$A_2 = R(\vec{k}, 0) \cdot T(\vec{k}, v_2 t) \cdot T(\vec{i}, 0) \cdot R(\vec{i}, 0)$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & v_2 t \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Similar with A_1, the A_3 matrix will be: $A_3 = R(\vec{i}, \omega_3 t)$, where $R(\vec{i}, \omega_3 t)$ is identically with $R_{3,2}$:

$$R_{3,2} = \begin{bmatrix} \vec{i}_3 \cdot \vec{i}_2 & \vec{j}_3 \cdot \vec{i}_2 & \vec{k}_3 \cdot \vec{i}_2 & 0 \\ \vec{i}_3 \cdot \vec{j}_2 & \vec{j}_3 \cdot \vec{j}_2 & \vec{k}_3 \cdot \vec{j}_2 & 0 \\ \vec{i}_3 \cdot \vec{k}_2 & \vec{j}_3 \cdot \vec{k}_2 & \vec{k}_3 \cdot \vec{k}_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_3 t & -\sin\omega_3 t & 0 \\ 0 & \sin\omega_3 t & \cos\omega_3 t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_3 \tag{10}$$

The A_4 matrix is reduces at translation matrix $T(\vec{j}, v_4 t)$.

$$A_4 = T(\vec{j}, v_4 t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v_4 t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$A_5 = R(\vec{j}, \omega_5 t) \cdot T(\vec{j}, L_{45})$, where $R(\vec{j}, \omega_5 t)$ is:

$$R_{5,4} = \begin{bmatrix} \vec{i}_5 \cdot \vec{i}_4 & \vec{j}_5 \cdot \vec{i}_4 & \vec{k}_5 \cdot \vec{i}_4 & 0 \\ \vec{i}_5 \cdot \vec{j}_4 & \vec{j}_5 \cdot \vec{j}_4 & \vec{k}_5 \cdot \vec{j}_4 & 0 \\ \vec{i}_5 \cdot \vec{k}_4 & \vec{j}_5 \cdot \vec{k}_4 & \vec{k}_5 \cdot \vec{k}_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\omega_5 t & 0 & \sin\omega_5 t & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega_5 t & 0 & \cos\omega_5 t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = R(\vec{j}, \omega_5 t) \tag{11}$$

$T(\vec{j}, L_{45})$ is:

$$T(\vec{j}, L_{45}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_{45} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Multiplied equation (11) with equation (12) and A_5 is obtaining from equation (10):

$$A_5 = \begin{bmatrix} \cos\omega_5 t & 0 & \sin\omega_5 t & 0 \\ 0 & 1 & 0 & L_{45} \\ -\sin\omega_5 t & 0 & \cos\omega_5 t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

After that is calculated the $T_n\left(n=5\right)$ matrix used the (6) equation: $T_5 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5$, (14)

$$A_1 \cdot A_2 = \begin{bmatrix} \cos\omega_1 t & -\sin\omega_1 t & 0 & 0 \\ \sin\omega_1 t & \cos\omega_1 t & 0 & 0 \\ 0 & 0 & 1 & v_2 t \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad A_3 \cdot A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_3 t & -\sin\omega_3 t & v_4 t\cos\omega_3 t \\ 0 & \sin\omega_3 t & \cos\omega_3 t & v_4 t\sin\omega_3 t \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4 = \begin{bmatrix} \cos\omega_1 t & -\sin\omega_1 t\cdot\cos\omega_3 t & \sin\omega_1 t\cdot\sin\omega_3 t & -v_4 t\cos\omega_3 t\cdot\sin\omega_1 t \\ \sin\omega_1 t & \cos\omega_1 t\cdot\cos\omega_3 t & -\cos\omega_1 t\cdot\sin\omega_3 t & v_4 t\cos\omega_3 t\cdot\cos\omega_1 t \\ 0 & \sin\omega_3 t & \cos\omega_3 t & v_4 t\sin\omega_3 t + v_2 t \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

Multiplied equation (16) with A_5 from equation (13) result:

$$T_5 = \begin{bmatrix} (\cos\omega_1 t\cdot\cos\omega_5 t - \sin\omega_1 t\cdot\sin\omega_3 t\cdot\sin\omega_5 t) & -(\sin\omega_1 t\cdot\cos\omega_3 t) & (\cos\omega_1 t\cdot\sin\omega_5 t + \sin\omega_1 t\cdot\sin\omega_3 t\cdot\sin\omega_5 t) & E_{14} \\ (\sin\omega_1 t\cdot\cos\omega_5 t + \cos\omega_1 t\cdot\sin\omega_3 t\cdot\sin\omega_5 t) & (\cos\omega_1 t\cdot\cos\omega_3 t) & (\sin\omega_1 t\cdot\sin\omega_5 t - \cos\omega_1 t\cdot\sin\omega_3 t\cdot\cos\omega_5 t) & E_{24} \\ -(\cos\omega_3 t\cdot\sin\omega_5 t) & \sin\omega_3 t & (\cos\omega_3 t\cdot\cos\omega_5 t) & E_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$E_{14} = -L_{45}\sin\omega_1 t\cdot\cos\omega_3 t - v_4 t\cos\omega_3 t\cdot\sin\omega_1 t; \qquad E_{24} = L_{45}\cos\omega_1 t\cdot\cos\omega_3 t + v_4 t\cos\omega_3 t\cdot\cos\omega_1 t$$

$$E_{34} = L_{45}\sin\omega_3 t + v_4 t\cdot\sin\omega_3 t + v_2 t$$

It is used equation (8) where $(r_M)_n$ is $(r_M)_5$: $(r_M)_5 = \begin{bmatrix} 0 \\ 0 \\ L_{56} \\ 1 \end{bmatrix}$ (18)

$$(r_M)_0 = \begin{bmatrix} L_{56}(\cos\omega_1 t\cdot\sin\omega_5 t + \sin\omega_1 t\cdot\sin\omega_3 t\cdot\cos\omega_5 t + E_{14} \\ L_{56}(\sin_1 t\cdot\sin\omega_5 t - \cos\omega_1 t\cdot\sin\omega_3 t\cdot\cos\omega_5 t + E_{24} \\ L_{56}\cos\omega_3 t\cdot\cos\omega_5 t + E_{34} \\ 1 \end{bmatrix} \tag{19}$$

From equation (19) is obtaining the M space coordinate by the basic frame $O_0x_0y_0z_0$:

$$\begin{aligned} (x_M)_0 &= \sin\omega_1 t\cdot[L_{56}\sin\omega_3 t\cdot\cos\omega_5 t - \cos\omega_3 t(L_{45} + v_4 t)] + L_{56}\cos\omega_1 t\cdot\sin\omega_5 t \\ (y_M)_0 &= -\cos\omega_1 t\cdot[L_{56}\sin\omega_3 t\cdot\cos\omega_5 t - \cos\omega_3 t(L_{45} + v_4 t)] + L_{56}\sin\omega_1 t\cdot\sin\omega_5 t \\ (z_M)_0 &= L_{56}\cos\omega_3 t\cdot\cos\omega_5 t + (L_{45} + v_4 t)\cdot\sin\omega_3 t + v_2 t \end{aligned} \tag{20}$$

3. THE QUATERNION MODEL APPLICATIONS

The present problem is to determinate the implication of the range matrix (4) witch is T_5 matrix, shows by the (17) equation and where this matrix can be found in quaternion method. It is thought q_{50} rotation quaternion $q_{50} = N_{50}e_0 - N_{51}e_1 - N_{52}e_2 - N_{53}e_3$, which is internal attach with the final effectors of the robot.

$$q_{50}^{-1} * e_j * q_{50}, \qquad (j = 1, 2, 3) \tag{21}$$

It is used the equations [1],[2],[3][8]:

$$p^{-1} * e_1 * p = \begin{bmatrix} 0 \\ (p_0^2 + p_1^2 - p_2^2 - p_3^2) \\ 2(-p_0 p_3 + p_1 p_2) \\ 2(p_0 p_2 + p_1 p_3) \end{bmatrix}; \; p^{-1} * e_2 * p = \begin{bmatrix} 0 \\ 2(p_0 p_3 + p_1 p_2) \\ (p_0^2 - p_1^2 + p_2^2 - p_3^2) \\ 2(-p_0 p_1 + p_2 p_3) \end{bmatrix}; \; p^{-1} * e_3 * p = \begin{bmatrix} 0 \\ 2(-p_0 p_2 + p_1 p_3) \\ 2(p_0 p_1 + p_2 p_3) \\ (p_0^2 - p_1^2 - p_2^2 + p_3^2) \end{bmatrix}. \tag{22}$$

where p is equal with q_{50}, [4] $p_0 = N_{50}, \; p_1 = -N_{51}, \; p_2 = -N_{52}, \; p_3 = -N_{53},$ (23)

$$q_{50}^{-1} * e_1 * q_{50} = \begin{bmatrix} \cos\omega_1 t \cdot \cos\omega_5 t - \sin\omega_1 t \cdot \sin\omega_3 t \cdot \sin\omega_5 t \\ \sin\omega_1 t \cdot \cos\omega_5 t + \cos\omega_1 t \cdot \sin\omega_3 t \cdot \sin\omega_5 t \\ -\cos\omega_3 t \cdot \sin\omega_5 t \end{bmatrix} \begin{vmatrix} e_1 \\ e_2 \\ e_3 \end{vmatrix} \tag{24}$$

$$q_{50}^{-1} * e_2 * q_{50} = \begin{bmatrix} \sin\omega_1 t \cdot \cos\omega_3 t \\ \cos\omega_1 t \cdot \cos\omega_3 t \\ \sin\omega_3 t \end{bmatrix} \begin{vmatrix} e_1 \\ e_2 \\ e_3 \end{vmatrix} \tag{25}$$

$$q_{50}^{-1} * e_3 * q_{50} = \begin{bmatrix} \cos\omega_1 t \cdot \sin\omega_5 t + \sin\omega_1 t \cdot \sin\omega_3 t \cdot \cos\omega_5 t \\ \sin\omega_1 t \cdot \sin\omega_5 t - \cos\omega_1 t \cdot \sin\omega_3 t \cdot \cos\omega_5 t \\ \cos\omega_3 t \cdot \cos\omega_5 t \end{bmatrix} \begin{vmatrix} e_1 \\ e_2 \\ e_3 \end{vmatrix} \tag{26}$$

4. CONCLUSIONS

It is easy to see that the equations (24), (25) and (26) are similar with the first three column of the range matrix $T_{5,}$ equation (17). In practically problems it is possible that same of the Denavit-Hartenberg parameters will be zero (maximum three parameters), dependent by the consecutive frame. The element of the (1) equation will be basic matrix with rank four.

REFERENCES

[1] ALECU A - *Aplicații ale cuaternionilor în cinematica rigidului* - Studii şi Cercetări de Mecanică Aplicată, 3- 4 mai-august 1997, pag. 155-166.

[2] ALECU A. - *Aplicații ale cuaternionilor în dinamica rigidului* - Studii şi cercetări de Mecanică aplicată , 5-6 septembrie-decembrie 1997, pag. 277-283.

[3] ALECU A, BURACU V. - *Mecanica Solidului*, Editura Printech, 2005

[4] BURACU V, ALECU A.- *On the use of a theorem by V. Valcovici in planar motion dynamics***,** Proceedings of the Romanian Academy, vol. 6, no. 1, 2005.

[4] IACOB C.- *Mecanica Teoretică***,** Editura Didactică şi Pedagogică, Bucureşti, 1980

[5] LANDAU L.D., LIFSCHITZ E.M. **-** *Mecanică*, Editura Tehnică, Bucureşti 1966.

[6] LOIȚIANSKII L. G., LURIE A. I. - *Kurs teoreticeskoi mehaniki torn vtoroi*, Moskva, 1955.

[7] MANGERON D., IRIMICIUC N. - *Mecanica rigidelor cu aplicații în inginerie*, vol l-ll, Editura Tehnică, Bucureşti, 1978-1980.

[8] NIȚĂ C., NĂSTĂSESCU C. - *Bazele algebrei* , vol. I, Editura Tehnică, Bucureşti, 1985

[9] OPRIŞAN C, DOROFTEI I. - *Introducere în ciematica şi dinamica roboților şi manipulatoarelor*, Editura Cermi, Iaşi 1998.