THEORETICAL STUDY ABOUT THE DETERMINATION OF THE ANGULAR SPEED FOR A RTRTR ROBOT USING THE QUATERNION METHOD

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Abstract: This present paper studies the determination of the rotational speed for an RTRTR robot using the quaternion method.

Keywords: quaternion method, rotational speed, reverses quaternion

1. INTRODUCTION

It is knowing [3] that the rotational speed is shows by the equation: $\omega_{k0} = -2\dot{q}_{ko} * q_{k0}^{-1} \quad (k = 1, 2, 3........., n)$, where ω_{k0} is the quaternion of the rotational speed for the T_k frame given by the T_0 frame and the projection of the ω_{k0} given by the T_k frame.

2. THE CALCULUS FOR THE ANGULAR SPEED

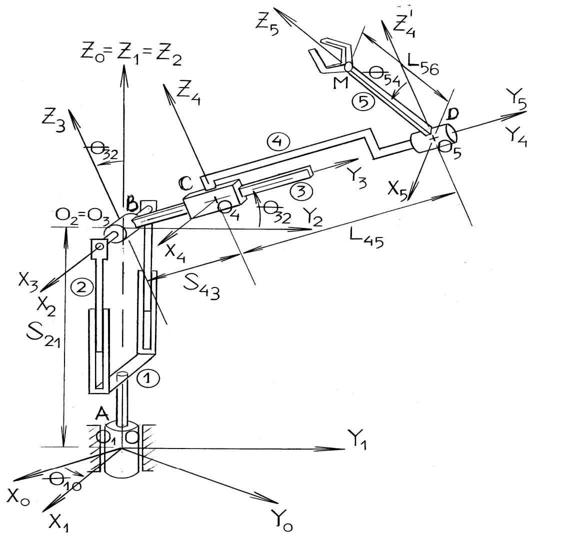

Fig.1. The constructive design for the RTRTR robot

Figure 1 shows the RTRTR robot, that it has in view to determinate the angular speed using the quaternion method.

Firstly, it is establishing a general equation [2], [3] for the right member of the equation (1); it knows rotational quaternion Q:

$$Q = Q_0e_0 + Q_1e_1 + Q_2e_2 + Q_3e_3 \qquad (1)$$

Using differentiation method by the time, equation (1) became:

$$\dot{Q} = \dot{Q}_0e_0 + \dot{Q}_1e_1 + \dot{Q}_2e_2 + \dot{Q}_3e_3, \qquad (2)$$

and the reverse will be:

$$Q^{-1} = Q_0e_0 - Q_1e_1 - Q_2e_2 - Q_3e_3 \qquad (3)$$

It is calculated $\dot{Q} * Q^{-1}$ using a matrix, rank 4x4:

$$\dot{Q}*Q^{-1}=\begin{bmatrix}\dot{Q}_0 & -\dot{Q}_1 & -\dot{Q}_2 & -\dot{Q}_3\\ \dot{Q}_1 & \dot{Q}_0 & -\dot{Q}_3 & \dot{Q}_2\\ \dot{Q}_2 & \dot{Q}_3 & \dot{Q}_0 & -\dot{Q}_1\\ \dot{Q}_3 & -\dot{Q}_2 & \dot{Q}_1 & \dot{Q}_0\end{bmatrix}\begin{bmatrix}Q_0\\ -Q_1\\ -Q_2\\ -Q_3\end{bmatrix}=\begin{bmatrix}0\\ -\dot{Q}_0Q_1+\dot{Q}_1Q_0-\dot{Q}_2Q_3+\dot{Q}_3Q_2\\ -\dot{Q}_0Q_2+\dot{Q}_1Q_3+\dot{Q}_2Q_0-\dot{Q}_3Q_1\\ -\dot{Q}_0Q_3-\dot{Q}_1Q_2+\dot{Q}_2Q_1+\dot{Q}_3Q_0\end{bmatrix}\begin{matrix}e_0\\ e_1\\ e_2\\ e_3\end{matrix} \tag{4}$$

The inner product diagram will be [4]:

$$\dot{Q}*Q^{-1}\Rightarrow \begin{matrix}\dot{Q}_0=\text{expresie}\\ \dot{Q}_1=\text{expresie}\\ \\ \dot{Q}_2=\text{expresie}\\ \dot{Q}_3=\text{expresie}\end{matrix} \tag{5}$$

e_1	e_2	e_3
$-Q_1$	$-Q_2$	$-Q_3$
Q_0	Q_3	$-Q_2$
$-Q_3$	Q_0	Q_1
Q_2	$-Q_1$	Q_0

Observation: The equation (5) is a linear combination from four components Q_0, Q_1, Q_2, Q_3. and for understanding the problem we give same paradigms.

Example 1^0.

It is consideration that:

$$Q_0=C_1,\ Q_1=Q_2=0,\ Q_3=-S_1 \tag{6}$$

Using differentiation method by the time, equation (6) became:

$$\dot{Q}_0=-\frac{\omega_1}{2}S_1;\qquad \dot{Q}_1=\dot{Q}_2=0;\qquad \dot{Q}_3=-\frac{\omega_1}{2}C_1 \tag{7}$$

In this way it is demonstrated the truth. The (6) equation is inner product with (7) equation, using the (5) diagram. It shows that the e_1 and e_2 coefficients are zero and the component from e_3 is:

$$-\frac{\omega_1}{2}(C_1^2+S_1^2)e_3=-\frac{\omega_1}{2}e_3\,.$$

Example 2^0.

$$Q=q_{30}=P_{30}e_0-P_{31}e_1-P_{32}e_2-P_{33}e_3 \tag{8}$$

where:

$$\begin{matrix}P_{30}=C_1C_3 & P_{32}=S_1S_3\\ P_{31}=C_1S_3 & P_{33}=S_1C_3\end{matrix} \tag{9}$$

Results

$$Q_0=P_{30},\qquad Q_1=-P_{31},\qquad Q_2=-P_{32},\qquad Q_3=-P_{33}, \tag{10}$$

$$\begin{aligned}
\dot{Q}_0 &= \dot{P}_{30} = -\frac{\omega_1}{2}S_1C_3 - \frac{\omega_3}{2}C_1S_3 = -\frac{\omega_1}{2}P_{33} - \frac{\omega_3}{2}P_{31} = \frac{\omega_1}{2}Q_3 + \frac{\omega_3}{2}Q_1 \\
\dot{Q}_1 &= -\dot{P}_{31} = -(-\frac{\omega_1}{2}S_1S_3 + \frac{\omega_3}{2}C_1C_3) = \frac{\omega_1}{2}P_{32} - \frac{\omega_3}{2}P_{30} = -\frac{\omega_1}{2}Q_2 - \frac{\omega_3}{2}Q_0 \\
\dot{Q}_2 &= -\dot{P}_{32} = -(\frac{\omega_1}{2}C_1S_3 + \frac{\omega_3}{2}S_1C_3) = -(\frac{\omega_1}{2}P_{31} + \frac{\omega_3}{2}P_{33}) = \frac{\omega_1}{2}Q_1 + \frac{\omega_3}{2}Q_3 \\
\dot{Q}_3 &= -\dot{P}_{33} = -(\frac{\omega_1}{2}C_1C_3 - \frac{\omega_3}{2}S_1S_3) = -\frac{\omega_1}{2}P_{30} + \frac{\omega_3}{2}P_{32} = -\frac{\omega_1}{2}Q_0 - \frac{\omega_3}{2}Q_2
\end{aligned} \quad (11)$$

e_1
$-Q_1$
Q_0
$-Q_3$
Q_2

With equation (11) and (5) it is evaluate the coefficients for quaternion unit's e_1, e_2, e_3, arrangement the terms by ω_1 and ω_3:

$$\begin{aligned}
e_1 &\Rightarrow \frac{\omega_1}{2}(-Q_1Q_3 - Q_0Q_2 + Q_1Q_3 - Q_0Q_2) + \frac{\omega_3}{2}(-Q_1^2 - Q_0^2 - Q_3^2 - Q_2^2) = -\frac{\omega_3}{2} \\
e_2 &\Rightarrow \frac{\omega_1}{2}(-2Q_2Q_3 + 2Q_0Q_1) + \frac{\omega_3}{2}(-Q_1Q_2 - Q_0Q_3 + Q_0Q_3 + Q_1Q_2) = -\frac{\omega_1}{2}\sin\omega_3 t \\
e_3 &\Rightarrow \frac{\omega_1}{2}(-Q_3^2 + Q_2^2 + Q_1^2 - Q_0^2) + \frac{\omega_3}{2}(-Q_1Q_3 + Q_0Q_2 + Q_1Q_3 - Q_0Q_2) = -\frac{\omega_1}{2}\cos\omega_3 t
\end{aligned} \quad (12)$$

From (4), (8), (12) results:

$$\dot{q}_{30} * \dot{q}_{30}^{-1} = -\frac{\omega_3}{2}e_1 - \frac{\omega_1}{2}\sin\omega_3 t\cdot e_2 - \frac{\omega_1}{2}\cos\omega_3 t\cdot e_3 \quad (13)$$

From (13) results:

$$\omega_{30} : \omega_{30} = -2\dot{q}_{30} * q_{30}^{-1} = \omega_3 e_1 + \omega_1 \sin\omega_3 \cdot t e_2 + \omega_1 \cos\omega_3 \cdot t e_3 \quad (14)$$

<u>Example 3^0</u>

$$Q = q_{50} = N_{50}e_0 - N_{51}e_1 - N_{52}e_2 - N_{53}e_3 \quad (15)$$

where [4]:

$$\begin{aligned}
N_{50} &= P_{30}C_5 - P_{32}S_5 \qquad & N_{52} &= P_{32}C_5 + P_{30}S_5 \\
N_{51} &= P_{31}C_5 - P_{33}S_5 \qquad & N_{53} &= P_{30}C_5 + P_{31}S_5
\end{aligned} \quad (16)$$

$P_{30}, P_{31}, P_{32}, P_{33}$ are made in equation (9). From (15) results the Q components:

$$Q_0 = N_{50},\ Q_1 = -N_{51},\ Q_2 = -N_{52}, Q_3 = -N_{53} \quad (17)$$

It is calculated $\dot{N}_{50}$ using (16) and (11):

$$\dot{N}_{50} = -\frac{\omega_5}{2}P_{30}S_5 - \frac{\omega_5}{2}P_{32}C_5 + \dot{P}_{30}C_5 - \dot{P}_{32}S_5 = -\frac{\omega_5}{2}N_{52} + (-\frac{\omega_1}{2}P_{33} - \frac{\omega_3}{2}P_{31})C_5 + (-\frac{\omega_1}{2}P_{31} - \frac{\omega_3}{2}P_{33})S_5$$

$$\dot{N}_{50} = -\frac{\omega_5}{2}N_{52} - \frac{\omega_1}{2}(P_{33}C_5 + P_{31}S_5) - \frac{\omega_3}{2}(P_{31}C_5 + P_{33}S_5) \Rightarrow \dot{N}_{50} = -\frac{\omega_5}{2}N_{52} - \frac{\omega_1}{2}N_{53} - \frac{\omega_3}{2}(P_{31}C_5 + P_{33}S_5) \quad (18)$$

The last expression ($P_{31}C_5 + P_{33}S_5$) don't work with anyone ($N_{50}, N_{51}, N_{52}, N_{53}$) shows in equation (16).

It is summated member with member:

$$N_{51}C_5 + N_{53}S_5 = P_{31}C_5^2 - P_{33}S_5C_5 + P_{33}C_5S_5 + P_{31}S_5^2 = P_{31} \tag{19}$$

$$N_{53}C_5 = N_{51}S_5 = P_{31}C_5^2 + P_{31}S_5C_5 - P_{31}C_5S_5 + P_{33}S_5^2 = P_{33} \tag{20}$$

Using P_{31} from (19) and P_{33} from (20) will be defining the expression $P_{31}C_5 + P_{33}S_5$:

$$\begin{aligned} P_{31}C_5 + P_{33}S_5 &= (N_{51}C_5 + N_{53}S_5)\cdot C_5 + (N_{53}C_5 - N_{51}S_5)\cdot S_5 = \\ &= N_{51}(C_5^2 - S_5^2) + N_{53}\cdot 2S_5C_5 = N_{51}\cos\omega_5 t + N_{53}\sin\omega_5 t \end{aligned} \tag{21}$$

With (21) it is writing equation (18):

$$\dot{N}_{50} = -\frac{\omega_1}{2}N_{53} - \frac{\omega_3}{2}(N_{51}\cos\omega_5 + N_{53}\sin\omega_5 t) - \frac{\omega_5}{2}N_{52} \tag{22}$$

It is observing that the equation (22) contains in right member a linear combination with variable coefficients, by the N_{51}, N_{52}, N_{53} components. The $\dot{N}_{50}$'derivation it is linear made with three components of the q_{50} quaternion. If it is made similarly for $\dot{N}_{51}, \dot{N}_{52}, \dot{N}_{53}$ results:

$$\dot{N}_{51}: \quad \dot{N}_{50} = -\frac{\omega_5}{2}N_{53} - \frac{\omega_1}{2}N_{52} + \frac{\omega_3}{2}(P_{30}C_5 + P_{32}C_5) \tag{23}$$

$$\dot{N}_{51} = -\frac{\omega_1}{2}N_{52} + \frac{\omega_3}{2}(N_{50}\cos\omega_5 t + N_{52}\sin\omega_5 t) - \frac{\omega_5}{2}N_{53} \tag{24}$$

$$\dot{N}_{52} = \frac{\omega_1}{2}N_{50} + \frac{\omega_1}{2}N_{51} + \frac{\omega_3}{2}(P_{33}C_5 - P_{31}S_5) \tag{25}$$

$$\dot{N}_{52} = \frac{\omega_1}{2}N_{51} + \frac{\omega_3}{2}(N_{53}\cos\omega_5 t - N_{51}\sin\omega_5 t) + \frac{\omega_5}{2}N_{50} \tag{26}$$

$$\dot{N}_{53} = \frac{\omega_5}{2}N_{51} + \frac{\omega_1}{2}N_{50} - \frac{\omega_3}{2}(P_{32}C_5 - P_{30}S_5) \tag{27}$$

$$\dot{N}_{53} = \frac{\omega_1}{2}N_{50} - \frac{\omega_3}{2}(N_{52}\cos\omega_5 t - N_{50}\sin\omega_5 t) + \frac{\omega_5}{2}N_{51} \tag{28}$$

e_1	e_2	e_3
N_{51}	N_{52}	N_{53}
$-N_{50}$	N_{53}	$-N_{52}$
$-N_{53}$	$-N_{50}$	N_{51}
N_{52}	$-N_{51}$	$-N_{50}$

The (5) diagram is made directly for $q_{50} = N_{50}e_0 - N_{51}e_1 - N_{52}e_2 - N_{53}e_3$:

$$\begin{aligned} \dot{N}_{50} &= -\frac{\omega_1}{2}N_{53} + \frac{\omega_3}{2}(-N_{51}\cos\omega_5 t - N_{53}\sin\omega_5 t) - \frac{\omega_5}{2}N_{52} \\ \dot{N}_{51} &= -\frac{\omega_1}{2}N_{52} + \frac{\omega_3}{2}(N_{50}\cos\omega_5 t + N_{52}\sin\omega_5 t) - \frac{\omega_5}{2}N_{53} \\ \dot{N}_{52} &= \frac{\omega_1}{2}N_{51} + \frac{\omega_3}{2}(N_{53}\cos\omega_5 t - N_{51}\sin\omega_5 t) + \frac{\omega_5}{2}N_{50} \\ \dot{N}_{53} &= \frac{\omega_1}{2}N_{50} + \frac{\omega_3}{2}(-N_{52}\cos\omega_5 t + N_{50}\sin\omega_5 t) + \frac{\omega_5}{2}N_{51} \end{aligned} \tag{29}$$

Using the (29) equations, the e_1 coefficient is:

$$e_1 = \frac{\omega_1}{2}(2N_{50}N_{52} - 2N_{51}N_{53}) + \frac{\omega_3}{2}\cos\omega_5 t(-N_{50}^2 - N_{51}^2 - N_{52}^2 - N_{53}^2) + \\ + \frac{\omega_3}{2}\sin\omega_5 t(N_{50}N_{52} + N_{51}N_{53} - N_{50}N_{52} - N_{51}N_{53}) + \frac{\omega_5}{2}(N_{51}N_{52} - N_{50}N_{53} + N_{50}N_{53} - N_{51}N_{52}) \tag{30}$$

It is knows that $N_{50}^2 + N_{51}^2 + N_{52}^2 + N_{53}^2 = 1$, because it is works with unitary quaternion. Using equation (16) with (9) results:

$$2(N_{50}N_{52} - N_{51}N_{53}) = \cos\omega_3 t \cdot \sin\omega_5 t \tag{31}$$

Equation (30) became:

$$e_1 \Rightarrow \frac{\omega_1}{2}\cos\omega_3 t \cdot \sin\omega_5 t - \frac{\omega_3}{2}\cos\omega_5 t \tag{32}$$

Using (29) results the e_2 coefficient:

$$e_1 \Rightarrow \frac{\omega_1}{2}(-N_{52}N_{53} - N_{50}N_{51}) \cdot 2 + \frac{\omega_3}{2}\cos\omega_5 t(-N_{51}N_{52} + N_{50}N_{53} - N_{50}N_{53} + N_{51}N_{52}) + \\ + \frac{\omega_3}{2}\sin\omega_5 t(-N_{52}N_{53} + N_{52}N_{53} + N_{50}N_{51} - N_{50}N_{51}) - \frac{\omega_5}{2}(N_{52}^2 + N_{53}^2 + N_{50}^2 + N_{51}^2). \tag{33}$$

From (16) and (9) results:

$$2(N_{50}N_{51} + N_{52}N_{53}) = \sin\omega_3 t \tag{34}$$

Equation (33) became:

$$e_2 = -\frac{\omega_1}{2}\sin\omega_3 t - \frac{\omega_5}{2} \tag{35}$$

$$e_3 \Rightarrow \frac{\omega_1}{2}(-N_{53}^2 + N_{52}^2 + N_{51}^2 - N_{50}^2) + \frac{\omega_3}{2}\cos\omega_5 t(-N_{51}N_{53} - N_{50}N_{52} + N_{51}N_{53} + N_{50}N_{52}) + \\ + \frac{\omega_3}{2}\sin\omega_5 t(-N_{53}^2 - N_{52}^2 - N_{51}^2 - N_{50}^2) + \frac{\omega_5}{2}(-N_{52}N_{53} + N_{52}N_{53} + N_{50}N_{51} - N_{50}N_{51}). \tag{36}$$

From (16) and (9) results:

$$N_{51}^2 + N_{52}^2 - N_{50}^2 - N_{53}^2 = -\cos\omega_3 t \cdot \cos\omega_5 t \tag{37}$$

Equation (36) became:

$$e_3 \Rightarrow -\frac{\omega_1}{2}\cos\omega_3 t \cdot \cos\omega_5 t - \frac{\omega_3}{2}\sin\omega_5 t \tag{38}$$

Using (32), (35) and (38) will be finding the result:

$$\dot{q}_{50} * q_{50}^{-1} = (\frac{\omega_1}{2}\cos\omega_3 t \cdot \sin\omega_5 t - \frac{\omega_3}{2}\cos\omega_5 t)e_1 - (\frac{\omega_1}{2}\sin\omega_3 t + \frac{\omega_5}{2})e_2 - (\frac{\omega_1}{2}\cos\omega_3 t \cdot \cos\omega_5 t + \frac{\omega_3}{2}\sin\omega_5 t)e_3 \quad (39)$$

From (39) results the angular speed ω_{50} :

$$\begin{aligned}\omega_{50} = -2\dot{q}_{50} * q_{50}^{-1} &= (-\omega_1 \cos\omega_3 t \cdot \sin\omega_5 t + \omega_3 \cos\omega_5 t)e_1 + (\omega_1 \sin\omega_3 t + \omega_5)e_2 + \\ &+ (\omega_1 \cos\omega_3 t \cdot \sin\omega_5 t + \omega_3 \sin\omega_5 t)e_3\end{aligned} \quad (40)$$

3. CONCLUSIONS

The determination of the angular speed using the quaternion method it is necessary because the versatility of the calculus is accessible like a work way, comparative with classically methods using today.

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