SOME CONSIDERATIONS REGARDING DYNAMIC STRESS INTENSITY FACTOR DETERMINATION USING A THREE POINTS BEND ANALITYCAL METHOD

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Abstract: This paper presents some considerations regarding dynamic stress intensity factor determination using a three point's bends analytical method. This method is based on a double mass-spring system. In this system the specimen and the striker are both represented as mass-spring system. The specimen stiffness is computed using the midspan deflection of the beam and the contact stiffness is computed using a linear force-deformation relation.

Keywords: dynamic stress intensity factor, contact stiffness, impact test, midspan deflection, three point bend test.

1. INTRODUCTION

Three point bend test under impact loading carried out on drop weight or swing pendulum machines has became an important tool in determination of dynamic fracture toughness. For a rate sensitive material, it permits thinner specimens to obtain plane strain fracture toughness due to elevation of yield stress with strain rate. However, due to specimen inertia, the load experienced by the specimen is different from that measured by the instrumented tup, and dynamic stress intensity factor has to be measured directly on the specimen. This is a difficult prescription to follow, especially in routine material testing, and for this reason we need to use analytical methods for the prediction of dynamic stress intensity factor from remote impact load measurements. Nash [1] carried out an analysis to obtain the mode shape and natural frequency of notched beams. Kishimoto et al. [2] simplified that to suit instrumented Charpy testing, and also formulated similar models for both elastic and viscoelastic materials by applying the boundary loads via contact spring. In this paper the dynamic stress intensity factor is analytical determinate using a double spring mass model.

2. THE ANALYTICAL MODEL

The model used here for determination of dynamic stress intensity factor is a double spring mass model [3]. This model, presented in Fig.1, considers only the fundamental mode of vibration, and neglect the shear and rotary inertia. Application of impulse-momentum consideration permits the computation of the rigid body motion of the tup, and the same is applied as boundary condition to the spring-mass system.
Consider the rigid body motion of the striker; the following equation can be obtained easily:

$$y = v_0 t - \frac{1}{M}\int_0^t F(\tau)(t-\tau)d\tau \qquad (1)$$

where v_0 is the initial impact velocity, M is the mass of the tup and F(τ) is the force measured by the instrumented tup.

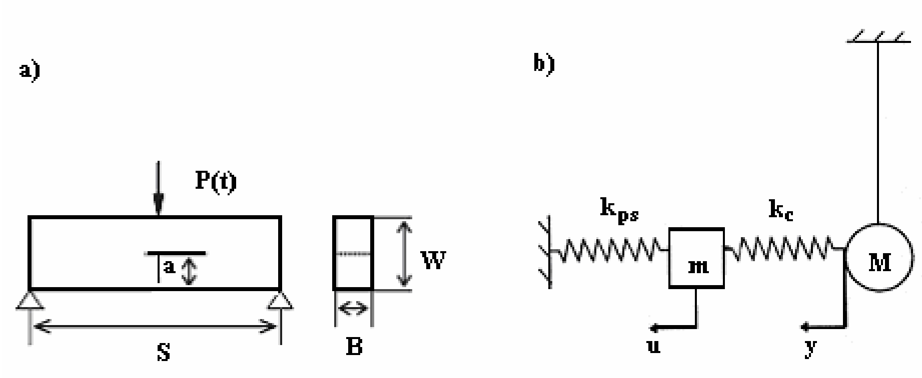

Fig.1. The double spring-mass system. a) Three points bend specimen geometry. b) Mathematical model

The equation (1) is solved numerically to obtain y, and then we can obtain the midspan deflection u(t) from equation (2), which is the equation of motion for the system without the damping:

$$m\ddot{u} + k_{ps}u = k_c(y - u) \tag{2}$$

where k_{ps} is the precracked specimen stiffness and k_c is the contact stiffness, u(t) is the mass displacement and m is the equivalent mass which is:

$$m = \frac{17}{35}\rho AS \tag{3}$$

where ρ is the density, S is the span and A is the area of gross section of the beam.

3. DYNAMIC STRESS INTENSITY FACTOR DETERMINATION

The stress intensity factor solution for three point bend specimen given in ASTM E399-83 standard is:

$$K_I^d = \frac{S}{BW^{3/2}}F(t)f\left(\frac{a}{W}\right) \tag{4}$$

where S is the span length, B is the width, W is the thickness and

$$f\left(\frac{a}{W}\right) = \frac{3\left(\frac{a}{W}\right)^{1/2}\left\{1{,}99 - \frac{a}{W}\left(1 - \frac{a}{W}\right)\left[2{,}15 - 3{,}93\frac{a}{W} + 2{,}7\left(\frac{a}{W}\right)^2\right]\right\}}{\left[2\left(1 + 2\frac{a}{W}\right)\left(1 - \frac{a}{W}\right)^{3/2}\right]} \tag{5}$$

(the above equation is valid only if plane strain condition prevails in the specimen).
F(t) is the force experienced by the beam and is obtained from the midspan beam deflection:

$$F(t) = k_{ps} u(t) \tag{6}$$

4. EVALUATION OF STIFFNESS

4.1 Notched beam stiffness evaluation

Presence of a crack in a beam reduces its flexural rigidity. Hence the notched beam can be represented by an equivalent spring of reduced stiffness depending on the precrack length. For an unnotched simply supported beam loaded at its midpoint, Timoshenko and Goodier [4] give the displacement as:

$$u_{nc} = \frac{FL^3}{48EI} + \left[1 + 2{,}85\left(\frac{W}{L}\right)^2 - 0{,}84\left(\frac{W}{L}\right)^3\right] \tag{7}$$

and the displacement due to a crack is given by Tada et.al. [5] as:

$$u_c = \frac{PL^3}{48EI}\left[6\left(\frac{W}{L}\right)V\left(\frac{a}{W}\right)\right] \tag{8}$$

where

$$V\left(\frac{a}{W}\right) = \left[\frac{a/W}{1-(a/W)}\right]^2 \left(5{,}58 - 19{,}57\left(\frac{a}{W}\right) + 36{,}82\left(\frac{a}{W}\right)^2 - 34{,}94\left(\frac{a}{W}\right)^3 + 12{,}77\left(\frac{a}{W}\right)^4\right) \tag{9}$$

So the stiffness of the precracked specimen is:

$$k_{ps} = \frac{F}{u} = \frac{48EI}{L^3}\left(1 + 2{,}85\left(\frac{W}{L}\right)^2 - 0{,}84\left(\frac{W}{L}\right)^3 + \frac{6W}{L}V\left(\frac{a}{W}\right)\right)^{-1} \tag{10}$$

where F is the impact load applied to the specimen.

4.2 The contact stiffness evaluation

Using the Hertz theory, the contact stiffness is compute using the specimen geometry, profile of the striker tip, and elastic constants of the tup and beam materials [6]:

$$k_c = \frac{(1 - 0{,}15r/W)\left[3{,}038 + (1{,}071B/W + 0{,}361)\nu^2\right]EB}{\ln\left(\frac{\pi BW^2 E}{rF_{\max}}\right) + \frac{1}{2}} \tag{11}$$

where r is the radius of curvature of the striker, F_{max} is the maximum contact force, and υ is the Poisson coefficient.

5. COMPARISON WITH THE EXPERIMENT

For comparison we used the experimental data obtained in paper [3] in a low-blow impact test. The test were carried on a Al 6061 T6 alloy specimen with: L= 172 mm, W=25,4 mm, B=6,35 mm and a= 15,5 mm. The specimen was impacted with an initial velocity of 5 m/s.
Figure 2 shows the computed and measured dynamic stress intensity factor versus time. It can be observed that the measured data agrees with the computed dynamic factor intensity.

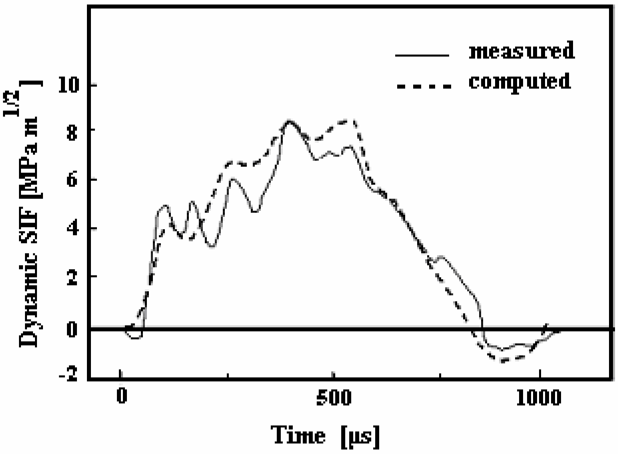

Fig.2 Computed and measured dynamic stress intensity factor versus time

6. CONCLUSION

In this paper is presented a three point's bend analytical model to compute dynamic stress intensity factor. This model used a double spring – mass system because the force applied to the specimen by the striker is also represented as a spring-mass system using a contact spring and the tup mass. The contact stiffness is compute using a linear force-deformation law and the specimen stiffness is computed using the midspan deflection relation of a simply supported beam. The model agrees well with the experimental data used.

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