SENSORLESS CONTROL OF THE INDUCTION MACHINE BY USING A KALMAN OBSERVER

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Abstract. This paper work presents a method to receive in a sensorless way the rotor speed and the rotor flux of an induction machine by using a Kalman observer. There is presented the Kalman filter general theory and its applying way for the implementation of an optimal recursive observer, named Extended Kalman Filter (EKF). The observer with EKF for the rotor speed and flux estimation is simulated by using MATLAB\SIMULINK files, and the conclusions of the simulation study are presented.

Keywords: Kalman filter, Extended Kalman Filter (EKF), induction machine.

1. INTRODUCTION

The modern systems of adaptive-optimal control of the induction machine use the machine model for the implementation of the control lows and for their readjustment at the operate conditions. As the machine model is powerfully nonlinear, variable with the speed and the loading, the model parameters have to be precisely known and continuously updated to obtain the best performances of the control system. In many cases it is impossible to use sensors for velocity or flux measurement, either because it is technically impossible, or too expensive. The Kalman observer has a good dynamic behaviour, disturbance resistance, and it can work even in standstill.

2. ALGORITHM OF THE EXTENDED KALMAN FILTER

It considers the nonlinear system's model in the form:

$$\begin{cases} x_{k+1} = F_k(x_k, u_k) + w_k \\ y_k = H \cdot x_k + v_k \end{cases} \tag{1}$$

in which the state equation non-linearity is characterized by the nonlinear function $F_k(x_k,u_k)$. In the case of a time-variable and non-linear dynamic system, like the induction machine is, in which the state-variables non-linear vary at the speed variation, the state estimation on the Kalman filter base is applied after the system is linearized and with on-line model updating at every sampling step. In this form, the state estimator based on Kalman filter is known as the extended Kalman filter, shortly abbreviated EKF. The extended Kalman filter algorithm is applied through the system previous linearizing around the last estimated value:

$$F_k(x_k, u_k) \approx F_k(\hat{x}_k, u_k) + \left(\frac{\partial F_k(x_k, u_k)}{\partial x_k} \right)_{x_k = \hat{x}_k} (x_k - \hat{x}_k)\ . \tag{2}$$

To simplify the writing the so-called gradient matrix is written down with f_k:

$$f_k(\hat{x}_k) = \left(\frac{\partial F_k(x_k, u_k)}{\partial x_k} \right)_{x_k = \hat{x}_k} \tag{3}$$

The linearized model of the system has the following form:

$$\begin{aligned} x_{k+1} &= F_k(\hat{x}_k, u_k) + f(\hat{x}_k)\cdot(x_k - \hat{x}_k) + w_k \\ y_k &= H\cdot x_k + v_k \end{aligned} \tag{4}$$

and the extended Kalman filter is described by the equations:

I. The prediction

$$\begin{aligned} \tilde{x}_{k+1} &= F_k(\hat{x}_k, u_k) \\ \tilde{y}_{k+1} &= H\cdot \tilde{x}_{k+1} \end{aligned} \tag{5}$$

II. The filtering (correction)

$$\begin{aligned} \hat{x}_{k+1} &= \tilde{x}_{k+1} + K_{k+1}(y_{k+1} - u_k\ \tilde{y}_{k+1}) \\ \hat{y}_{k+1} &= H\cdot \hat{x}_{k+1} \end{aligned} \tag{6}$$

The overall structure of the extended Kalman filter is presented in figure 1.

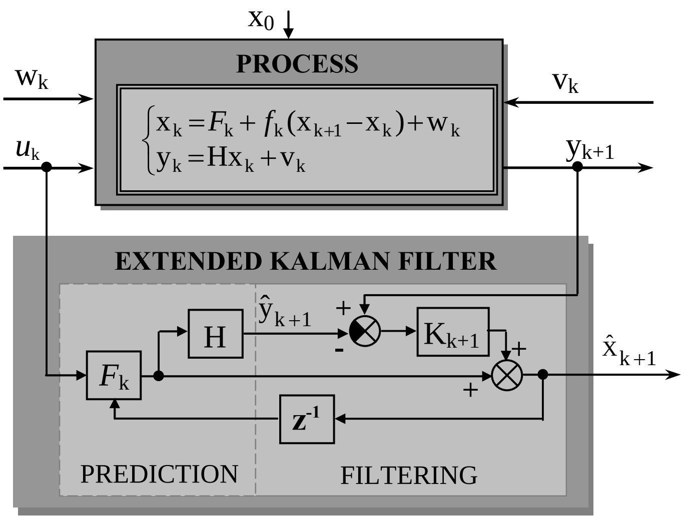

Fig. 1. Block diagram of EKF state observer.

The correction matrix K_{k+1} of the estimated state is on-line calculated, with a decreasing recursive algorithm of the estimation error. The observer estimates the state through the real time simulation of the system behavior and the estimated value is iteratively corrected (filtered) up to the nullity of the error between the estimated output and the real system output. By annulling this error, the estimation model is optimized and the estimated state represents the real state of the process. The covariance matrices of the prediction and estimation errors are calculated by using the linearized model's equations (4) and the extended Kalman filter equations (5) and (6). As to these relations, the prediction error is

$$\tilde{e}_{k+1} = \tilde{x}_{k+1} - x_{k+1} = F_k - F_k - f_k\cdot(x_k - \hat{x}_k) + w_k = f_k\cdot(\hat{x}_k - x_k) + w_k\,, \tag{7}$$

and the estimation error is

$$\begin{aligned} \hat{e}_{k+1} &= \hat{x}_{k+1} - x_{k+1} = \tilde{x}_{k+1} + K_{k+1}\cdot H\cdot x_{k+1} + K_{k+1}\cdot v_{k+1} - K_{k+1}\cdot H\cdot \tilde{x}_{k+1} - x_{k+1} = \\ &= (I - K_{k+1}\cdot H)(\tilde{x}_{k+1} - x_{k+1}) + K_{k+1}\cdot v_{k+1} \end{aligned} \tag{8}$$

Using these expressions, after a simple algebraic calculus, the covariance matrices expressions of the prediction errors P_{k+1} and of the estimation ones S_{K+1}, results as:

$$P_{k+1} = E\{(\tilde{e}_{k+1} \cdot \tilde{e}^t_{k+1})\} = f_k E_k f_k^t + W \tag{9}$$

$$S_{k+1} = E\{(\hat{e}_{k+1} \cdot \hat{e}^t_{k+1})\} = (I - K_{k+1}H)P_{k+1}(I - K_{k+1}H)^t + K_{k+1}VK^t_{k+1} \tag{10}$$

The Kalman matrix expression K_{k+1} determines through the variation decreasing of the estimation error S_{K+1}, on the base of the following equation:

$$\frac{\partial S_{k+1}}{\partial K^t_{k+1}} = -H \cdot (I - K_{k+1}H) \cdot P_{k+1}\, H^t + K_{k+1}V = 0 \tag{11}$$

It results the K matrix expression at the t_{k+1} moment:

$$K_{k+1} = P_{k+1}H^t[HP_{k+1}H^t + V]^{-1} \tag{12}$$

Replacing in eqn (10) the matrix K_{k+1} given by the relation (12), results the minimum variation expression of the estimation error:

$$(P_{k+1})_{min} = (I - K_{k+1}H) \cdot P_{k+1} \tag{13}$$

The estimation observer based on the Kalman Filter needs to know the process quantities and a statistic evaluation of the noise in the system, as well as the measurement errors and the initial state.

Synthesizing, the extended Kalman observer algorithm spreads as follows;

① on the base of some stochastic considerations, the initial values of the state vector and of the covariance matrices of the perturbations and estimation error are setting:

$$x(t_0) = x_0, \quad W = W_0, \quad V = V_0, \quad S = S_0;$$

② the state variables prediction for the next sampling time, t_{k+1}, obtains on the base of the entering u_k and state x_k sizes, determined at t_k sampling time, in conformity with the input-state equation:

$$\tilde{x}_{k+1} = F_k x_k + G_k u_k;$$

③ the covariance matrix of the prediction error calculates with the relation :

$$P_{k+1} = f_k \cdot S_k \cdot f_k^t + W;$$

④ the Kalman matrix is determined with the relation:

$$K_{k+1} = P_{k+1}H^t[HP_{k+1}H^t + V]^{-1}$$

⑤ the state vector x_{k+1} is estimated at the t_{k+1} sampling time:

$$\hat{x}_{k+1} = \tilde{x}_{k+1} + K_{k+1}(y_{k+1} - \tilde{y}_{k+1})$$

in which $y_{k+1} = H_{k+1}x_{k+1}$ are the measured output, and $\tilde{y}_{k+1} = H_{k+1} \cdot \tilde{x}_{k+1}$ are the predicted output.

⑥ the covariance matrix of the error is calculated with the relation:

$$S_{k+1} = (I - K_{k+1}H)P_{k+1}$$

⑦ there sets: k = k+1, $x_k = x_{k-1}$, $S_k = S_{k-1}$ and it takes again the algorithm from the stage ②.

3. ALGORITHM OF THE ROTOR SPEED AND FLUX ESTIMATION

The estimation of a machine state or parameters using an extended Kalman filter, performes by taking these sizes as a state variable in the induction machine model. As follows, the Kalman filter appliance supposes the establishment of the time-discrete model with the state-variables equations for the induction motor, one of the states-variables being the estimated parameter. By choosing as the state variables the stator and rotor flux components in a stationary d-q axis and the relative rotor speed *v*, the discrete model of the machine in matrix form is [4]:

$$\begin{bmatrix}\psi_{1d}\\ \psi_{1q}\\ \psi_{2d}\\ \psi_{2q}\\ v\end{bmatrix}_{k+1}=\begin{bmatrix}1-\frac{T_e}{\sigma T_1} & 0 & \frac{T_e}{\sigma T_1}\frac{x_m}{x_2} & 0 & 0\\ 0 & 1-\frac{T_e}{\sigma T_1} & 0 & \frac{T_e}{\sigma T_1}\frac{x_m}{x_2} & 0\\ \frac{T_e}{\sigma T_2}\frac{x_m}{x_2} & 0 & 1-\frac{T_e}{\sigma T_2} & -\omega_b T_e \nu & 0\\ 0 & \frac{T_e}{\sigma T_2}\frac{x_m}{x_2} & \omega_b T_e \nu & 1-\frac{T_e}{\sigma T_2} & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}_k \cdot \begin{bmatrix}\psi_{1d}\\ \psi_{1q}\\ \psi_{2d}\\ \psi_{2q}\\ v\end{bmatrix}_k + \begin{bmatrix}\omega_b T_e & 0\\ 0 & \omega_b T_e\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}u_{1d}\\ u_{1q}\end{bmatrix} \tag{14}$$

$$\begin{bmatrix}i_{1d}\\ i_{1q}\end{bmatrix}_k=\begin{bmatrix}\frac{1}{\sigma x_1} & 0 & -\frac{x_m}{\sigma x_1 x_2} & 0 & 0\\ 0 & \frac{1}{\sigma x_1} & 0 & -\frac{x_m}{\sigma x_1 x_2} & 0\end{bmatrix}\cdot\begin{bmatrix}\psi_{1d}\\ \psi_{1q}\\ \psi_{2d}\\ \psi_{2q}\\ v\end{bmatrix}_k$$

The equations (14) have been done in normed quantities and there have been used the notations:

- $T_1=\frac{x_1}{\omega_1 r_1}=\frac{L_1}{R_1}$ – the stator time constant, L_1 and R_1 being the phase self inductance and the phase resistance of the stator winding;
- $T_2=\frac{x_2}{\omega_1 r_2}=\frac{L_2}{R_2}$ – the rotor time constant, L_2 and R_2 being the phase self inductance and the phase resistance of the rotor winding;
- $\sigma=1-\frac{x_m}{x_1 x_2}$ – the total dispersion coefficient, x_1, x_2 and x_m being respectively, the stator, rotor and magnetizing normed reactance.
- T_e – the sampling period.

If there take as the command quantities the components u_{1d} and u_{1q} of the stator voltages and as output (measured) quantities the components i_{1d} and i_{1q} of the stator current, the complete model input-state-output of the asynchronous machine used for the flux estimation, takes the following form:
From the comparison of the model descbribed of eqn. (14) with the discrete model in its general form

$$\begin{aligned}x(k+1)&=F_k x(k)+G\cdot u(k)\\ y(k)&=H\cdot x(k)\end{aligned} \tag{15}$$

the matrices that interfere in the eqn. (14) are obviously.
As the considered state sizes are invariant at the reference frame change, there uses here the machine model in the stationary reference frame. For the model smoothing regarding the appliance of the extended Kalman filter recursive algorithm, there establishes the added matrix $F_k(x_k,u_k)$ for the state equation in conformity with (2):

$$F_k=\begin{bmatrix}\left(1-\frac{T_e}{\sigma T_1}\right)\psi_{1d}+\frac{T_e}{\sigma T_1}\frac{x_m}{x_1}\psi_{2d}+\omega_1 T_e u_{1d}\\ \left(1-\frac{T_e}{\sigma T_1}\right)\psi_{1q}+\frac{T_e}{\sigma T_1}\frac{x_m}{x_1}\psi_{2q}+\omega_1 T_e u_{1q}\\ \frac{x_m T_e}{\sigma x_1 T_2}\psi_{1d}+\left(1-\frac{T_e}{\sigma_1 T_2}\right)\psi_{2d}-\omega_1 T_e \upsilon\psi_{2q}\\ \frac{x_m T_e}{\sigma x_1}d_2\psi_{1q}+\left(1-\frac{T_e}{\sigma_1 T_2}\right)\psi_{2q}+\omega_1 T_e \upsilon\psi_{2d}\\ v\end{bmatrix} \tag{16}$$

The gradient matrix f_k calculates as in relation (3). Thus it results:

$$f_k = \left(\frac{\partial F_k(x_k, u_k)}{\partial x_k}\right)_{x=\hat{x}_k} = \begin{bmatrix} 1-\frac{T_e}{\sigma T_1} & 0 & \frac{T_e}{\sigma T_1}\frac{x_m}{x_2} & 0 & 0 \\ 0 & 1-\frac{T_e}{\sigma T_1} & 0 & \frac{T_e}{\sigma T_1}\frac{x_m}{x_2} & 0 \\ \frac{T_e}{\sigma T_2}\frac{x_m}{x_2} & 0 & 1-\frac{T_e}{\sigma T_2} & -\omega_b T_e \nu & \frac{T_e}{\sigma T_2}\frac{x_m}{x_1}\psi_{1d} - \frac{T_e}{\sigma T_2}\psi_{2d} \\ 0 & \frac{T_e}{\sigma T_2}\frac{x_m}{x_1} & \omega_b T_e \nu & 1-\frac{T_e}{\sigma T_2} & \frac{T_e}{\sigma}\frac{x_m}{x_1}\psi_{1q} - \frac{T_e}{\sigma}\psi_{2q} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_k \quad (17)$$

With the matrices G_k, F_k and f_k thus established and imposing the initial values of the state variables vector X_0 of the covariance matrices of the perturbations (W_0 and V_0) and of the estimation error S_0, can be reelled off the recursive algorithm of the EKF previously presented that will estimate the machine states, the rotor and stator fluxes and the rotor speed *ν*.

4. SIMULATION RESULTS. CONCLUSIONS

For the numerical simulation of the extended Kalman filter algorithm regarding the rotor speed estimation and magnetic fluxes of the induction machine there have been used a simulation diagram presented in figure 2.

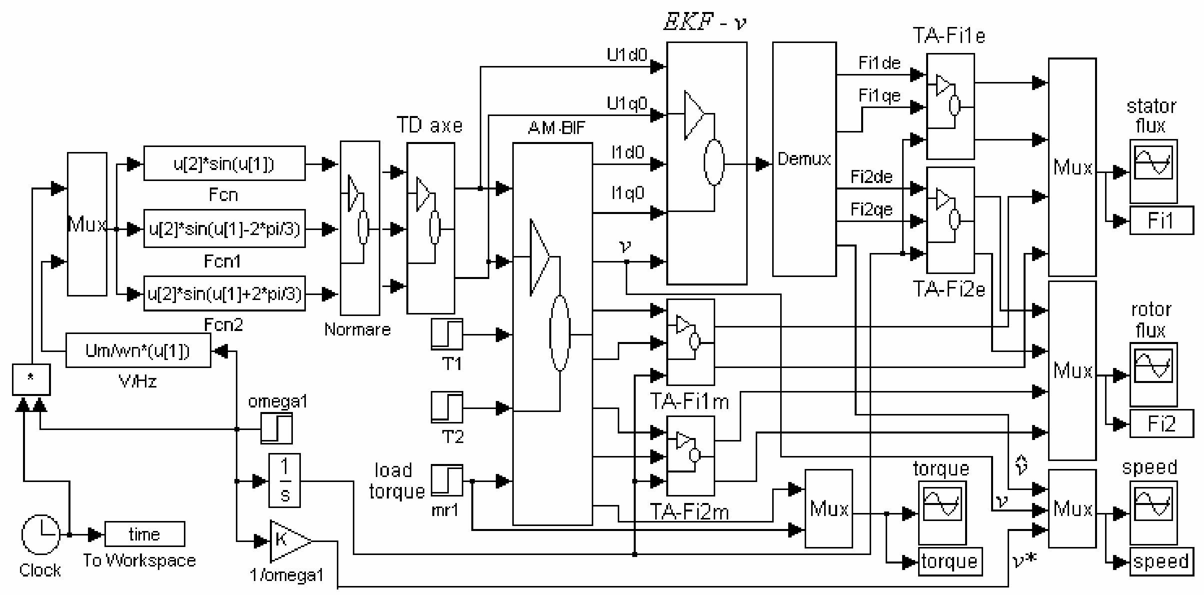

Fig. 2. The SIMULINK block diagram.

The dynamic system of the EKF estimator is performed using a SIMULINK S-function, implemented by using Matlab routine.

The simulation program was developed after the following stages:

1) t = 0 – the machine was started up nominal value speed reference ;
2) t = 0,5s – the load torque mr of nominal value was applied;
3) t = 0,8s – it was suddenly increased the rotor resistance T_2 with 50% in its nominal value;
4) t = 1s – it was suddenly decreased load torque at zero;
5) t = 1,4s – it was suddenly reversed speed reference;
6) t = 2,5s – the load torque was suddenly increased at the nominal value;
7) t = 2,8s – the load torque was suddenly decreased at half (50%);
8) t = 3s – the speed reference was suddenly decreased at 50%;
9) t = 3,5s – the load torque and the speed reference were annulled.

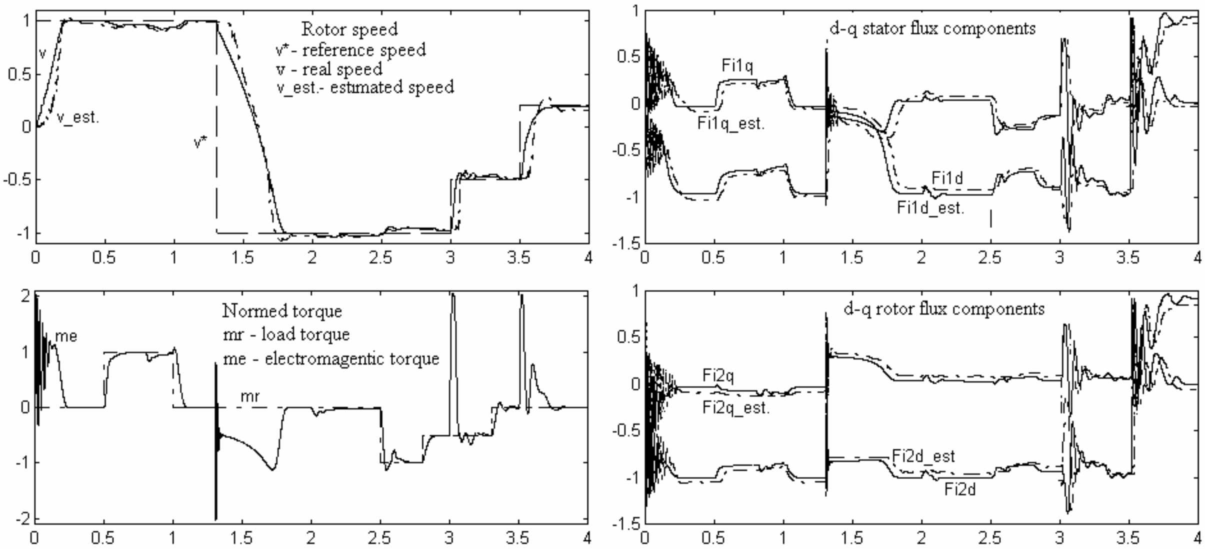

Fig. 3. The simulation results of the Kalman observer.

The simulation results, the variation curves for speed, torque (load and electromagnetic torque) and for the rotor and stator fluxes components in normalized sizes are presented in figure 3. To show better the estimation quality, the magnetic fluxes have been presented in an orthogonal axis system, synchronously with the rotation magnetic field. So, in the conditions of the steady state, these are constant sizes. To perform the simulation program there followed the estimation precision, the estimator dynamic performances at the speed and torque variations and the rotor resistance emphasizing. For the simulation it was used a sampling period Te = 0,0005 seconds.
It was adopted the nule initial state. The covariance matrices values of the system noises (w) and of measure (v), the diagonal matrices were established through repeated attempts, up to the getting the optimal performances in the point of view of the precision estimation states, of the system stability and of the estimator convergence.
The obtained results by simulation prove a very good dynamic behavior of the estimator, the system stability and the estimation precision of the states, maintaining at sudden variations of the speed and of the and of the applied torque at machine ax. As well as the precision, as the estimated state convergence at real values are dependent on the covariance matrices values of the noises. Generally, the choosing of these matrices optimal values in the convergence point of view in the transient conditions, as well as the estimation precision, is dependent on the state variable for which it wants a very correct estimation. If there is any interest to get a better estimation of the entire state vector, regarding the covariance matrices values settlement, it must be accepted a compromise regarding the estimation precision in the favour of a sure and stable, dynamic behaviour assurance of the system and of an acceptable convergence for the all estimated state sizes.

Appendix. *Parameters of the asynchronous machine used for EKF simulation*:
Rated power = 2,2 KW; Rated speed = 2800r/min; Number of poles pair = 1; Stator resistance R_1 = 8,5Ω; Rotor resistance related to stator R_2 = 7,8 Ω; Total stator inductance L_1 = 852 mH; Total rotor inductance related to stator L_2 = = 852 mH; Magnetizing inductance L_m = 815 mH.

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