STRESS TRIAXIALITY AS FRACTURE TOUGHNESS TRANSFERABILITY PARAMETER FOR NOTCHED SPECIMENS

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Abstract: The problem of fracture toughness transferability is treated by using the stress triaxiality and by introduction of a new transferability parameter called p. This parameter is a combination of effective critical stress triaxiality (mean value of stress triaxiality over effective distance) and multiplies by a geometrical function. Application of this method has been made on three point bending specimens made in XC 38 steel. Comparison of 2D and 3D approaches is made.

Keywords : Transferability parameter, Stress triaxiality, Notch radius, Notch depth, Steel.

1. INTRODUCTION

Transferability of mechanical properties means that these properties measured in some conditions of geometry, loading mode, constraint etc. have to be modified to be apply in other similar conditions. The mechanical properties measured in reference conditions are naturally the reference properties. This means fundamentally, that the mechanical properties are not intrinsic to material, which is a seldom assumption used in structure design. However, this problem is known since a long time and Galileo Galilee have said during the 17th century "I t is not so simple to go from the small to the big". Several theories are used to examine this problem, constrain plasticity, fractal theory, probabilistic approach, dimensional equations etc. Transferability of fracture toughness is an important problem for structural design because change of geometrical or constraint conditions may promote brittle fracture. Fracture toughness transferability (FTT) is made by the way of a transferability function t(p) which depends of transferability parameter p. If R is the fracture resistance in given conditions and R_{ref}, the fracture toughness in reference conditions, transferability function is defined as :

$$R = R_{ref} \, . \, t(p) \tag{1}$$

p is a transferability parameter chosen according to different approaches.

The choice of a transferability parameter remains an open question when we study the transferability with simultaneously two or seve ral parameters. It is well known that ductile fracture is sensitive to stress triaxiality. In the literature, one finds several indicators to quantify the state of the constraints at defect tip . Over the list of these indicators, one can quote the constraint T [1], the Q parameter [2] and the multiaxiality parameter q [3]. In this work, b is used as a measure of stress triaxiality. This parameter is defined as the ratio of the hydrostatic stress σ_h over the equivalent Von Mises stress $\sigma_{eq,VM}$.

$$\beta = \frac{\sigma_h}{\sigma_{eq,VM}} \tag{2}$$

where:

$$\sigma_h = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} \text{ and } \sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{\sigma_{zz}} - \sigma_{xx}\right)^2}$$

The most recent transferability parameters are based on stress distribution at defect tip. Fracture toughness transferability is presented in term of Q parameter fom Rugieri et Al. [4], T stress or for fracture toughness measured on notched specimens, the Q* parameter [5]. This parameter is issue from the stress distribution at notch tip using Notch Fracture Mechanics (NFM) and particularly a local fracture criterion : the Volumetric Method.

In this paper, a tentative to use stress triaxility as a transferability parameter is presented. Application has been made on three point bending specimens made in steel with several notch radii and ligament sizes.

2. MATERIAL AND SPECIMEN

The material is a carbon steel XC38, French standard. Its mechanical properties are listed in table 1.

Table 1 Mechanical properties of XC 38 carbon steel

Yield stress Re	Ultimate strength Rm	Elongation at fracture	Hardness Hv
304 MPa	430 MPa	30 %	137

This strain hardening materials obeys to Ludwik's law according to:

$$\sigma = 830.\varepsilon^{0.257} \tag{3}$$

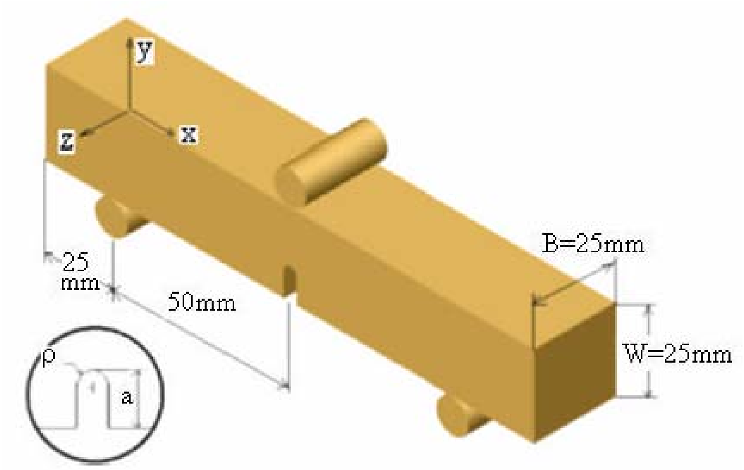


Fig. 1. Specimen geometry.

Specimen geometry is given in figure 1. Tests have been performed on three point bending notched specimens with five a/W ratios and nine notches radii according to table 2. The notch or crack length is 'a' and 'W' is the specimen width.

Table 2. Geometrical characteristics of 3.P.B notched specimens.

Relative notch depth : a/W	Notch radius (mm)								
0.2	0	0.15	0.48	0.75	1.02	1.25	1.54	1.75	2
0.3	0	0.15	0.48	0.75	1.02	1.25	1.54	1.75	2
0.4	0	0.15	0.48	0.75	1.02	1.25	1.54	1.75	2
0.5	0	0.15	0.48	0.75	1.02	1.25	1.54	1.75	2

Specimens are loaded until fracture. Typical experimental examples of load displacement diagrams for a 3PB notched specimen with a 0.48 mm notch radius are given in figure 2. We can note that the material exhibits a fully ductile failure with a critical load close to the maximum load.

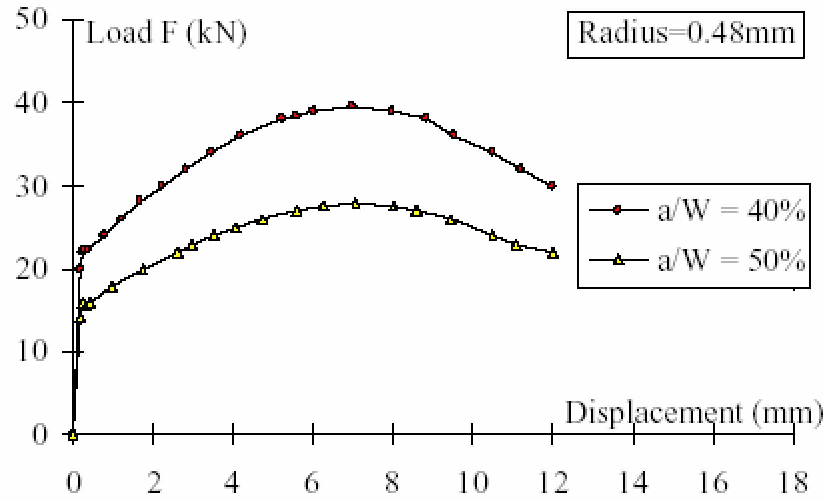


Figure 2. Examples of experimental load-displacement diagrams of T.P.B notched (r=0.48mm).

3. FRACTURE TOUGHNESS OF NOTCHED SPECIMEN

Fracture loads extracted from load displacement diagrams have been converted into fracture toughness using the Notch Fracture mechanics and particularly the Volumetric Method. The Volumetric method [6] is a local fracture criterion, which assumed that the fracture process requires a certain volume. This volume is assumed as cylindrical with effective distance as its diameter. Physical meaning of this fracture process volume is "the high stressed region" where the necessary fracture energy release rate is stored. The difficulty is to find the limit of this "high stressed region". This limit is a priori not a material constant but depends on loading mode, structure geometry and load level. Th e size of the fracture process reduced to the effective distance according to the above mentioned assumptions is obtained by analysis of the stress distribution.

The bi-logarithmic elastic -plastic stress distribution, figure 3, along the ligament exhibits three distinct zones which can be easily distinguished. The elastic-plastic opening stress primarily increases and it attains a peak value (zone I) then it gradually drops to the elastic plastic regime (zone II). Zone III represents linear behaviour in the bi-logarithmic diagram governs by the so called notch stress intensity factor. It has been proof [6] by examination of fracture initiation sites that the effective distance correspond to the beginning of zone III which is in fact an inflexion point on this bi logarithmic stress distribution. A graphical method based on the relative stress gradient χ, associates the effective distance with minimum of χ. The relative stress gradient is given by:

$$?(r) = \frac{1}{s_{xx}(r)} \frac{\partial s_{yy}(r)}{\partial r} \tag{4}$$

where ?(r) and s_{yy} (r) are the relative stress gradient and maximum principal stress or crack opening stress, respectively, r is distance.

The effective stress for fracture is then considered as the average volume of the stress distribution over the effective distance. However stresses are multiply by a weight function in order to take into account the influence of the stress gradient due to geometry and loading mode.

The stress distribution is then given by:

$$s_{ef} = \frac{1}{X_{ef}} \int_0^{X_{ef}} s_{xx}(r) \times (1 - r \times ?(r))\, dr \tag{5}$$

Therefore, the notch stress intensity factor (N.S.I.F) is defined as a function of effective distance and effective stress:

$$K_{?} = s_{ef}\sqrt{2pX_{ef}} \tag{6}$$

where K_r , s_{ef} and X_{ef} are notch stress intensity factor, effective stress and effective distance, respectively. Description of this kind of stress distribution at notch tip and the procedure with the help of the relative stress gradient is given in figure 3.

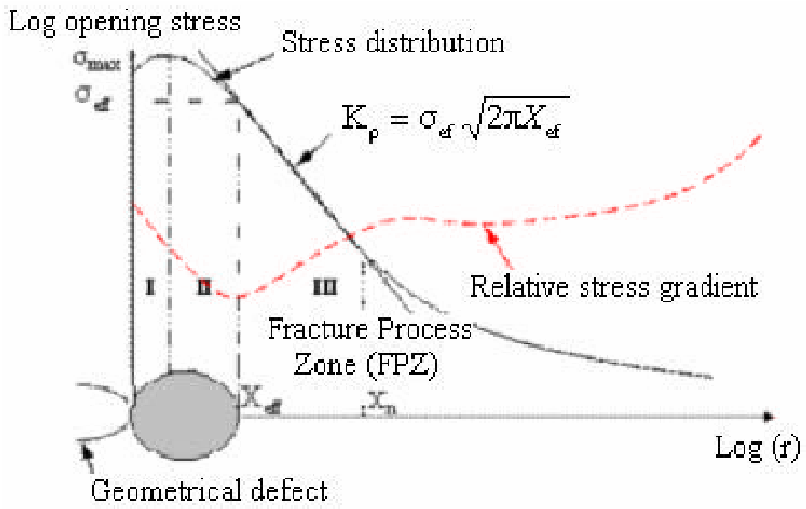


Fig. 3. Schematic elastic-plastic stress distribution along notch ligament and notch stress intensity concept.

4. FINITE ELEMENT ANALYSIS

4.1. 2D analysis

By exploiting the symmetry of the geometry and the mechanical loading, half of the specimen was used with the boundary conditions shown on figure 4.

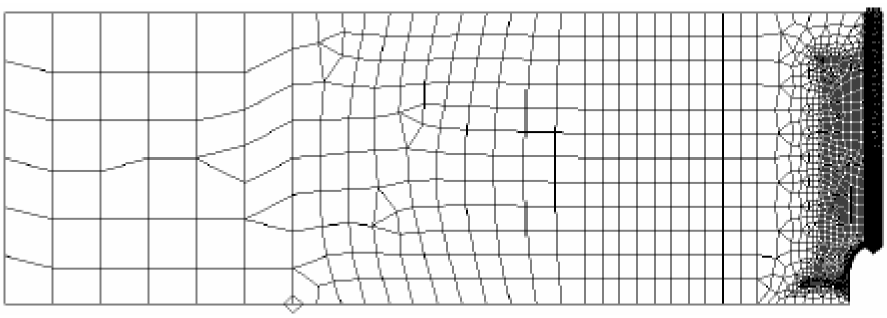


Fig. 4- Geometrical model and boundary conditions (a/W=20%, radius r=2mm) .

The part modelled in plane stain is meshed by quadrangular elements with eight nodes and isoparametric triangles with six nodes. Computing was carried out on Castem software 2000 [7], using the true stress versus true train function described above.

Using the above mentioned procedure, it has been seen that the effective distance depends of notch radius and ligament size according to the following experimental relationships:

$$X_{ef}^{c} = A.\rho + B \tag{7}$$

$$\text{for a/W} = 0.2: \; X_{ef}^{c} = 2.\rho + 1.43 \tag{8}$$

$$\text{for a/W} \geq 0.3: \; X_{ef}^{c} = 1.1.\rho + 1.8\frac{W-a}{W} \tag{9}$$

The size of the effective distance for current notch radius is of the order of millimetre and close to values found in literatures (see table 3).

Table 3. Effective distance and yield stress for three different materials

Material	Yield stress (MPa)	Notch radius (mm)	Effective distance (mm)
XC 38 [our results]	304	0.15 to 2.0	1 to 5
CrMoV Rotor steel [8]	771	0.25	0.38
Aluminium alloy	228	0.15 to 12.0	0.25 to 1

The effective stress decreases with effective distance according to a linear relationship:

$$\sigma_{ef}^{c} = \text{-C}.\, X_{ef}^{c} + \text{D} \tag{10}$$

where C and D depends of notch radius and ligament size as can be seen in table 4.

Table 4. Values of parameters C and D with notch radius (Eq. 10)

r (mm)	0.00	0.15	0.48	0.75	1.02	1.25	1.54	1.75	2.00
C (MPa/mm)	251	302	288	363	299	282	236	161	167
D (MPa)	1090	1239	1392	1638	1588	1613	1543	1335	1388

Critical values of effective stress and effective distance are combined according to equation (6) to get the critical notch stress intensity factor which is also sensitive to notch radius and ligament size as we can see on figure 5. In a first step, the critical N.S.I.F increases with notch depth as long as the latter does not reach half of width ; beyond this lengt h, it is stabilized or decrease. It is clear that this is probably which the change of constraint induced by the relative depth.

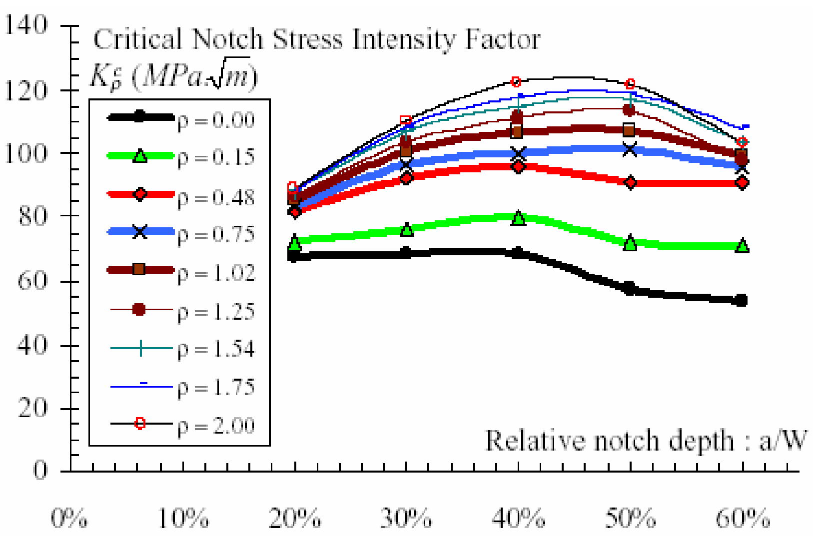


Fig. 5. Influence of relative depth and notch radius on critical notch stress intensity factor.

A linear relationship of the critical notch stress intensity factor with the square root of the notch radius (figure 6) has been found.

$$K_{\rho}^{c} = \alpha \sqrt{\rho} + K_{\rho=0}^{c} \tag{11}$$

where α is called the notch sensitivity. Similar relationship has been found for other materials but with a plateau for a notch radius below a critical value ρ_c [6].

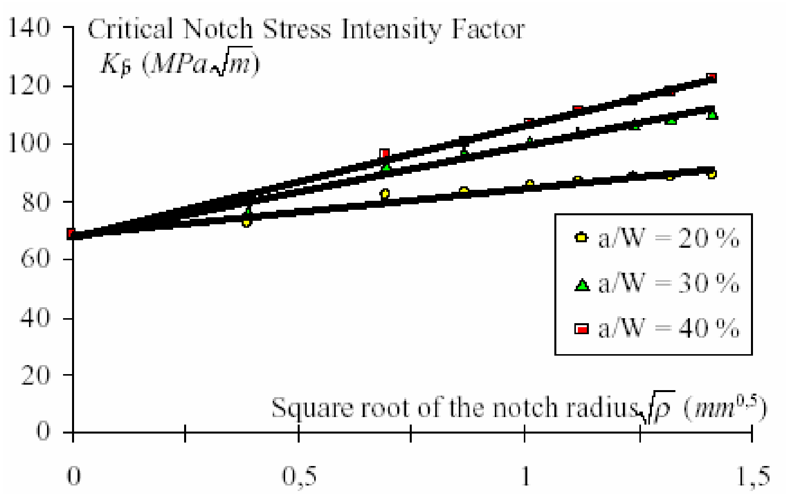


Fig. 6. Evolution of fracture toughness with the square root of the notch radius and non dimensional notch depth.

We can note that fracture toughness and more precisely the critical notch stress intensity factor is sensitive to notch radius and ligament size.

4.2. 3D Analysis

A 3-dimensional analysis of the stress distribution at notch tip on five parallel and distant lines of z = 2.5mm. To reduce computer time, only one fourth of the TPB geometry was simulated. Symmetry conditions were imposed to the two planes defined by the equations x=75mm and z =0. Two relative notch depths were considered ; 20% and 50%. Notch radius was respectively 0.15, 1 or 2mm.

Critical N.S.I.F increases linearly with z as we can see on figure 7 due to the lost of constraint when distribution is close to specimen surface.

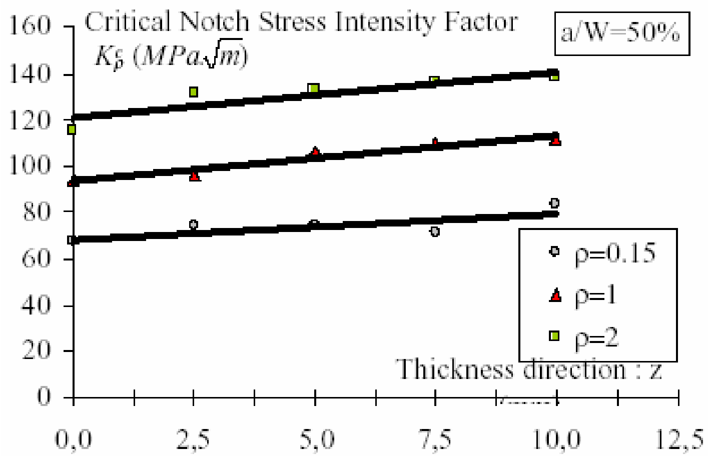


Fig. 7. Critical N.S.I.F versus the thickness direction z (a/W=50%).

4.3. Stress triaxiality

Stress triaxiality has been chosen as a transferability parameter because ductile fracture is there sensitive [9- 10]. Stress triaxiality distribution at notch tip increases until a maximum which for the critical event is called βmax, c and corresponds to the distance Xbmax,c.. After, it decreases and then it increases again and finally then falls until zero when, the distance from notch tip is equal to half the ligament . For small ligament size, fracture by shear is prevalent and we get negative stress triaxiality values.

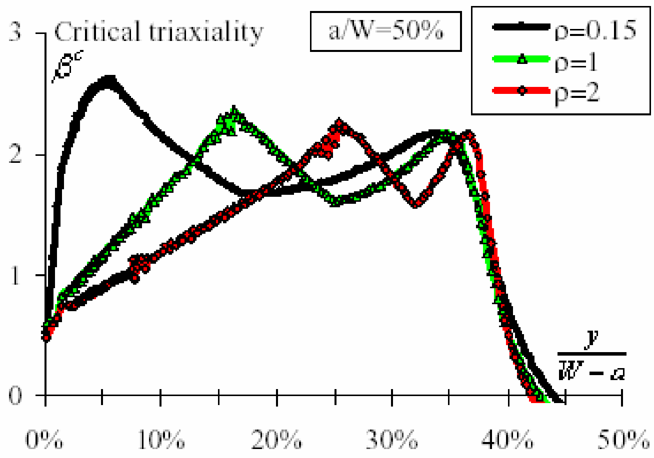


Fig. 8. Examples of the evolutions of the critical triaxiality β^c in the plan of the defect "y/(W-a)"

The maximum triaxiality is sensitive to notch radius and ligament size. It decreases practically linearly with the notch radius and increases with relative notch depth. This evolution can be seen on figure 9.

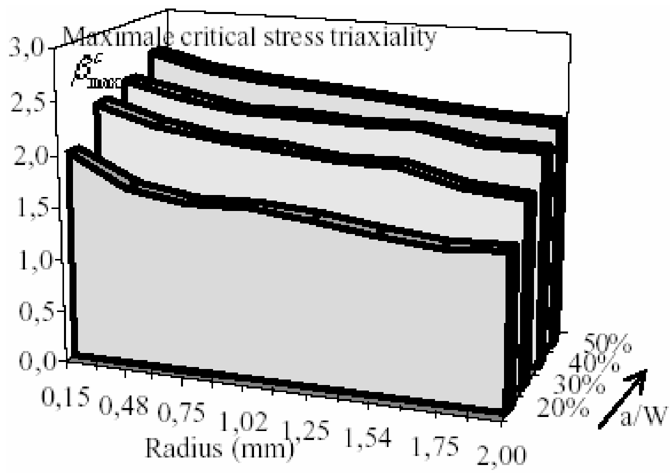


Fig. 9. Evolution of maximal critical stress triaxiality with notch radius and relative notch size

For the whole radii and depths studied, by comparing the effective distance with the distance where the triaxiality reaches its maximum value we noted that:

- the maximum triaxiality always takes place inside the volume of the fracture process zone since $X_{\beta^c_{max}}$ remains lower or equal to X^c_{ef} ,
- with increase of the relative notch depth, the position of maximum triaxiality approaches or reaches the limit of the fracture process zone.

Non coincidence of fracture process zone and high triaxiality zone with short ligament leads to the conclusion that there is a superimposed effect of triaxiality and ligament size.

In 3D analysis, we note that the critical maximum triaxiality varies in the z direction as can be see in figure 10. It is less in the centre of specimen (z = 0) than on surface and the absolute maximum is not on surface but a little distance behind surface. This needs to study the influence of the transferability parameter on average fracture toughness over the thickness direction $\overline{K}_{\rho}^{c}$.

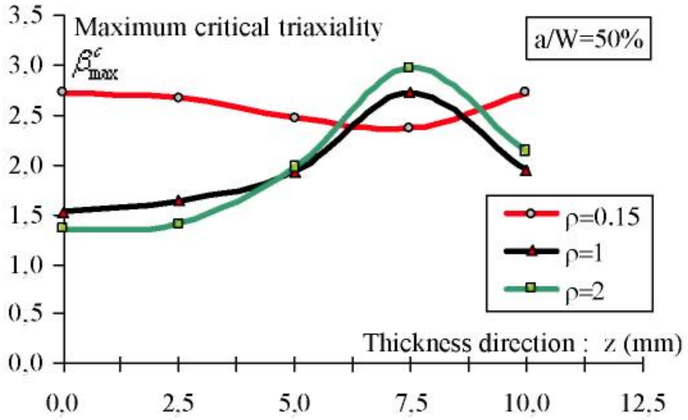


Fig. 10. The maximum critical triaxiality along to the thickness direction (a/W=50%).

4.4. A transferability parameter

Evolution of critical notch stress intensity factor is plotted versus maximal critical stress triaxiality and given in figure 11. We note that fracture toughness decreases with this parameter but this evolution remains sensitive to the relative crack depth. One concludes that maximal critical stress triaxiality is not adequate as transferability parameter.

An improvement has been made using a new transferability parameter : the effective critical stress triaxiality $\beta_{ef,c}$. This parameter is defined as the average value of the critical stress triaxiality over the effective distance at critical event $X_{ef,c}$.

$$\beta_{ef,c} = \frac{1}{X_{ef,c}} \int_0^{X_{ef,c}} \beta(x)\,dx \tag{12}$$

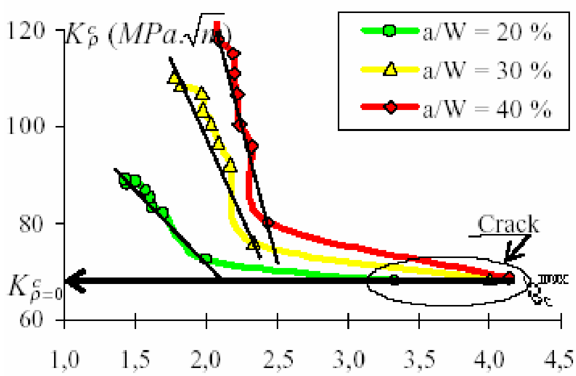


Fig. 11. Evolution of critical notch stress intensity factor versus maximal critical stress triaxiality. T.P.B- Plane Strain.

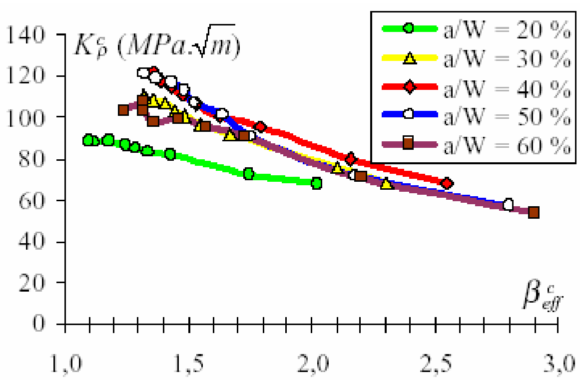


Fig. 12. Evolution of critical notch stress intensity factor versus critical effective stress triaxiality. T.P.B- Plane Strain

We note on figure 12 that the influence of relative notch depth is then reduced but remains mainly for small value of a/W. We consider again that the transferability is not assured in a satisfactory manner.

One concludes that the use of stress triaxiality as a transferability parameter needs to introduce a geometrical correction. This has been made using the following transferability parameter:

$$p = \beta^c_{eff} \cdot \frac{X^c_{\beta_{max}}}{W-a} \tag{13}$$

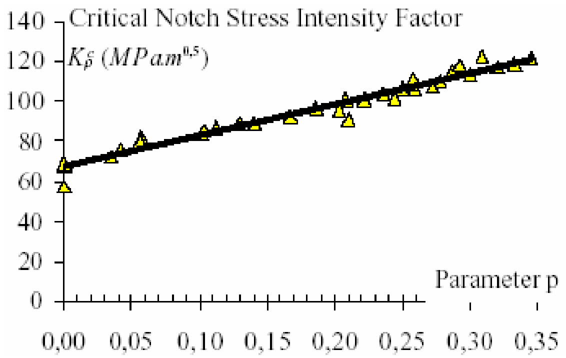
Evolution of critical notch stress intensity factor versus p transferability parameter is shown in figure 13. We note that all data for every notch radius and relative crack size merge into a unique curve which in addition, is simply a linear curve.

$$K^c_\rho(MPa) = 157p + K^c_{\rho=0} \tag{14}$$

The three parameters introduced into the transferability parameter p depend on relative notch depth and radius. Then p can be write, after results analysis in the following form :

$$p\left(\frac{a}{W},\rho\right) = A\left(\frac{a}{W}\right)\rho + B\left(\frac{a}{W}\right) \tag{15}$$

where: $A\left(\frac{a}{W}\right) = -0.955\left(\frac{a}{W}\right)^2 + 0.836\frac{a}{W} - 0.068$ and : $B\left(\frac{a}{W}\right) = 0.351\frac{a}{W} - 0.027$



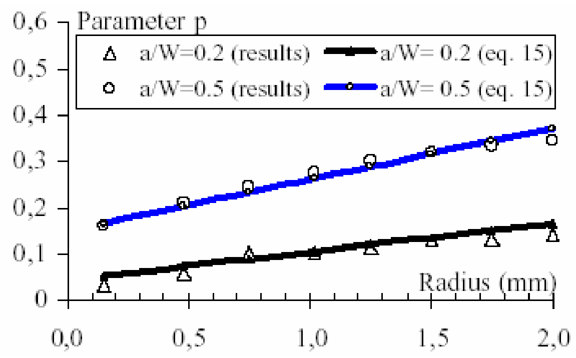


Fig. 13. Evolution of critical notch stress intensity factor versus p parameter, 3PB specimens (XC 38 Steel).

Fig. 14. Evolution of transferability parameter p versus notch radius, 3PB specimens (XC 38 Steel).

Two examples of the linear evolution of the transferability parameter with relative notch depth are given on figure 14.

The interest of equations 14 and 15 is that they make it possible to estimate tenacity in a given geometrical configuration of the notch starting from a tenacity of reference.

As transferability is associated with constraint, it is probably necessary to come back to simple and physical evidence the lateral contraction at crack or notch tip and the associated σ_{zz} stress. Figure 15 indicates a similar correlation K^c_ρ = $f(p)$ and $\overline{K}^c_\rho$ = $f(p)$.

The simple 2-dimentional analysis of the T.P.B specimen notched, assuming a plane strain behaviour, provides an adequate description the 3-dimensional fracture toughness transferability and makes possible to reduce the computing times.

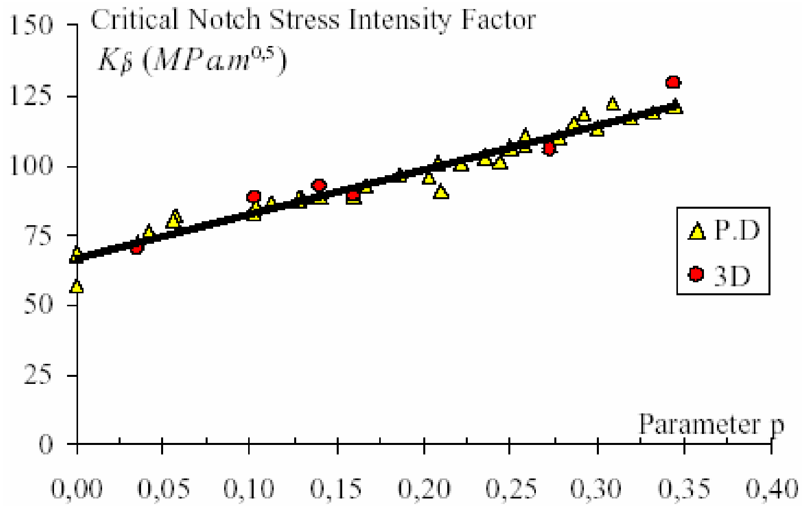


Fig. 15. Evolution of critical notch stress intensity factor versus p parameter (plane- deformations PD and 3D).

5. DISCUSSION

Actually, the most used transferability is Q parameter of Dodds. This parameter is defined as the difference of the relative opening stress value at non dimensional distance r.sy/J = 2 of two distributions a reference distribution generally for small scale yielding (ssy) and the current one.

$$Q = \frac{\sigma_{yy} - \left(\sigma_{yy}\right)_{ssy}}{\sigma_y} \tag{16}$$

The validity of Q is limited to homothetic distribution given by the following rule : the stress gradient between the two non dimensional distance (5) and (1) is less than 10%.

$$gradQ = \frac{Q_{(1)} - Q_{(5)}}{4} \leq 0{,}1 \tag{17}$$

It has been seen by finite element that in this case the distance of maximum opening stress for the two constraint situation coincide. This distance is considered as follows as the characteristic distance in a local fracture criterion.

If we multiply relationship (16) by $\sqrt{\rho \cdot X_c}$ and considers that we have pure brittle fracture:

$$Q = \frac{\sigma_{yy} \cdot \sqrt{\rho . X_c} - \left(\sigma_{yy}\right)_{SSY} \cdot \sqrt{\rho . X_c}}{\sigma_y \cdot \sqrt{\rho . X_c}} \tag{18}$$

$$Q = \frac{K_c - K_{Ic}}{\sigma_y \cdot \sqrt{\rho . X_c}} \tag{19}$$

We see than in this case Q is simple a relative difference between critical stress intensity factor. This means that Q parameter gives only another description of the relative fracture toughness which gives a strong limitation to its interest and justify the use of the parameter p.

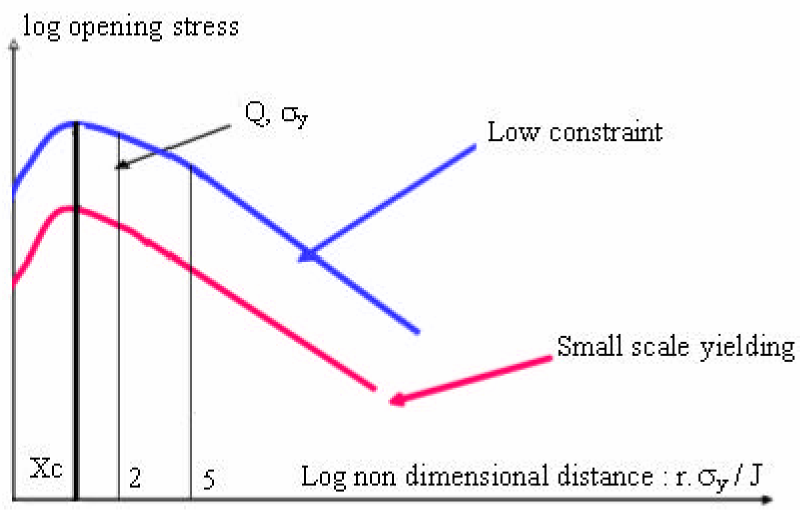


Fig. 16. Definition of Q parameter and characteristic distance used in a local fracture criterion

Another argument to use the stress triaxiality as transferability parameter is the fact that it has been recognised that the stress strain behaviour of the material is sensitive to stress triaxiality [12] according to the following relationship:

$$\sigma = \left(A + B \cdot \beta\right)\varepsilon^{n} \tag{20}$$

The stress distribution is then sensitive to the stress triaxiality. This approach is at present under progress but can be an interesting tool for solving the problem of transferability.

6. CONCLUSION

A new transferability parameter for fracture toughness has been proposed. It is based on effective maximal stress triaxiality and a ligament size correction. This approach is consider by us as a step because its involve several parameters. An alternative approach can be proposed using strain or stress in the thickness direction z.

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