FINITE ELEMENTS ANALYSIS AND CRACKING CALCULATION FOR PINION- RACK MECHANISM REGARDING THE DIRECTION MECHANISM

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Abstract: In this paper, a finite elements analysis of the pinion-rack mechanism (part of the direction system) is made. Based on the obtained results, a calculation of the lifetime of this mechanism will be made, considering that a micro crack has initiated at the base of the rack's tooth.

Keywords: loading, cracking, elastic analysis, failure

1. INTRODUCTION

The direction system is one of the main mechanisms of an automobile, having a strong purpose regarding the safety of the traffic, especially in conditions of increase of automobiles parks and their speed. The direction system serves its purpose of guiding the car on the desired track. The change of direction is obtained by changing the plan of the direction wheels reported to the longitudinal plan of the automobile. The direction system must ensure a good handling and stability. For a better experience regarding the handling and stability of direction, a special geometry of wheels is being used. The mechanism of pinion-rack, *Figure 1*, is used frequently at automobiles with independent wheels suspension and transversal direction bar.

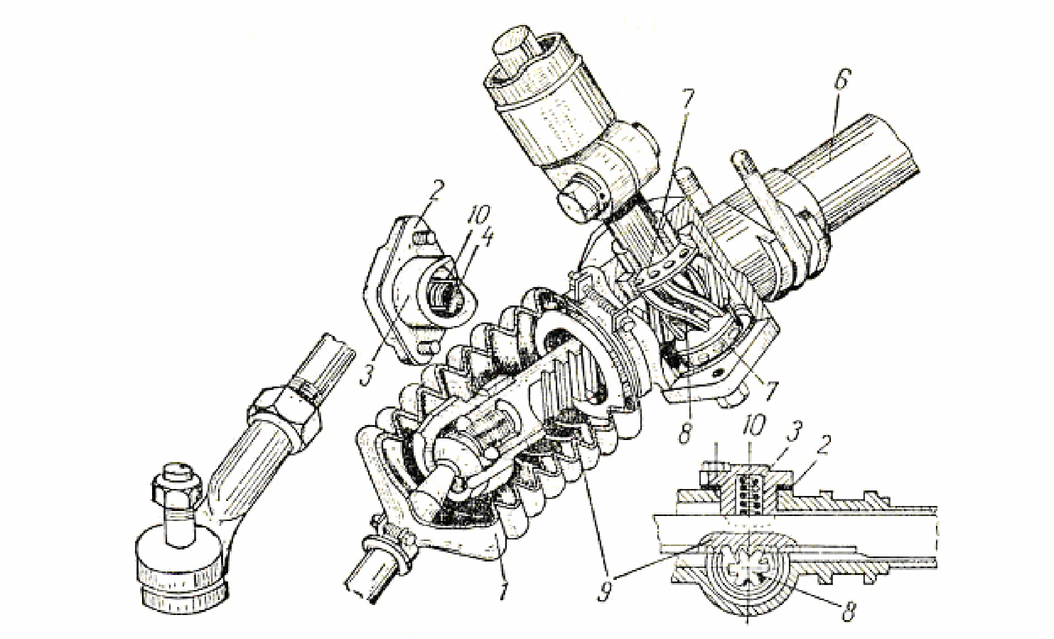

Fig. 1. The mechanism of direction formed of pinion and rack

This way, the number of trapeze articulations is reduced to 4, considering that other solutions require at least six. The utility of the mechanism is calculated 0.65, when the force from the steering wheel to the levier is transmitted and 0.59, in the reversed case. The transmission report is constant. The mechanisms of setting into work the direction with pinion and rack have a high reversibility. To decrease it, some mechanisms of this type have a repositioning arc, which opposes the spinning of the steering wheel and reduces the high reversibility of this mechanism.

2. FINITE ELEMENTS ANALYSIS OF THE PINION – RACK SYSTEM IN THE DIRECTION ONE

Finite elements analysis has been made using ALGOR. The purpose is to determine loads and deviations which appear at the contact between pinion and rack. Too high loads may lead to the appearance of cracks on the base of the teeth and too long deviations may lead to blocking the direction mechanism. The 3D geometric model of the pinion-rack system is being created in Superdraw III. First, an analysis about the ways how the geometric 3D model is created should be made. For the pinion-rack system, from the vehicle direction system code we have in view that the 2 components will have to be put into contact. This may be accomplished using classic elements like Gap, introduced at the contact between the 2 components. Also related to this, we have in view, that the coordinates axe system should be the same for both components. The result is represented in *Figure 2.*

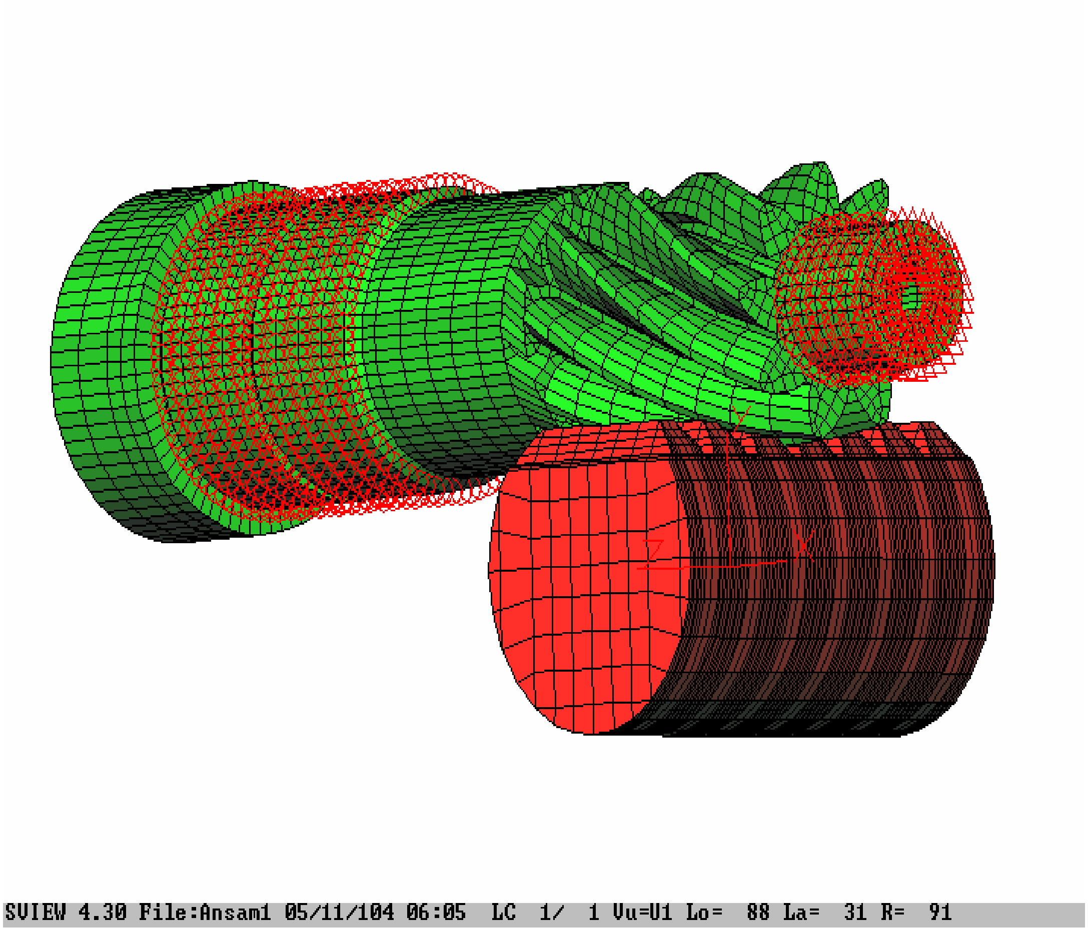

Fig. 2. Finite elements model

A very close contact between the teeth of the pinion and the ones of the rack can be observed. Between 2 teeth, that of the pinion and that of the rack, presented in *Figure 2*, elastic Gap elements were introduced. After the analysis with SSAPO CPU, which affects linear elastic analysis, the map of the loads has resulted, *Figure 3*. As an observation, the maxim loads appear at the contact between the 2 teeth.

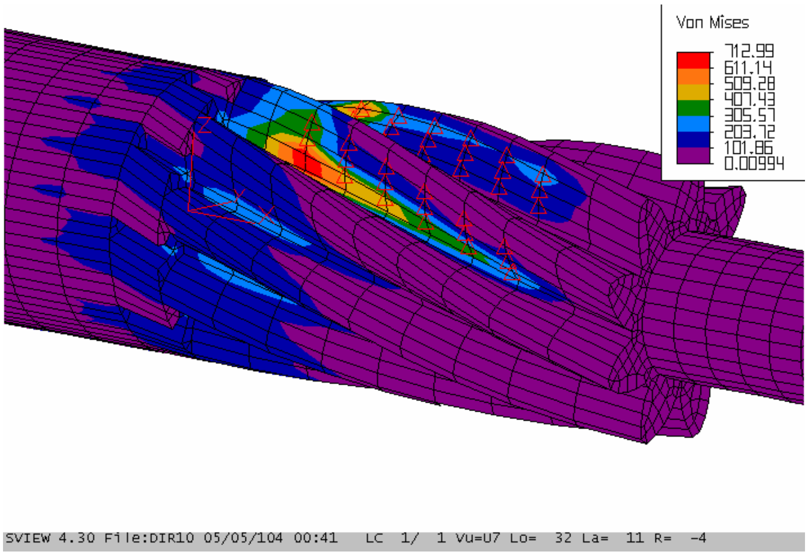

Fig. 3. The map of the pinion loads

3. DETERMINATION OF THE LIFETIME OF PINION-RACK MECHANISM

Considering the info previously, we propose a calculation model, which has the purpose to determine the lifetime or the number of cycles, necessary to obtain critical status in the rack of the direction system. We deduct, that at the base of the rack teeth, a 0.5 mm crack has been detected (an indestructible control being used). The main idea is to determine the number of cycles, necessary to extend this crack until it reaches its critical value. The tooth of the rack is loaded at fatigue after a pulsing cycle between F_{min}=0 and F_{max}=770 N.

The material from which the rack is made is similar to steel $36VS_1 \cdot$WmoCr53, with the next characteristics: $R_{p0,2}$ = 680 N/mm^2; R_m = 1050 N/mm^2; A_5 = 14,5 %; E = $21 \cdot 10^4$ N/mm^2 and hardness 63 HRC. The fracture tenacity of this steel is K_{Ic} = 110 MPa$\sqrt{m}$. The propagation speed of the crack is $\frac{da}{dN} = 1{,}36 \cdot 10^{-10} (\Delta K)^2$, where ΔK represents the variation of the intensity of the load, which is being measured in MPa$\sqrt{m}$, and da/dN, will result in an m/cycle.

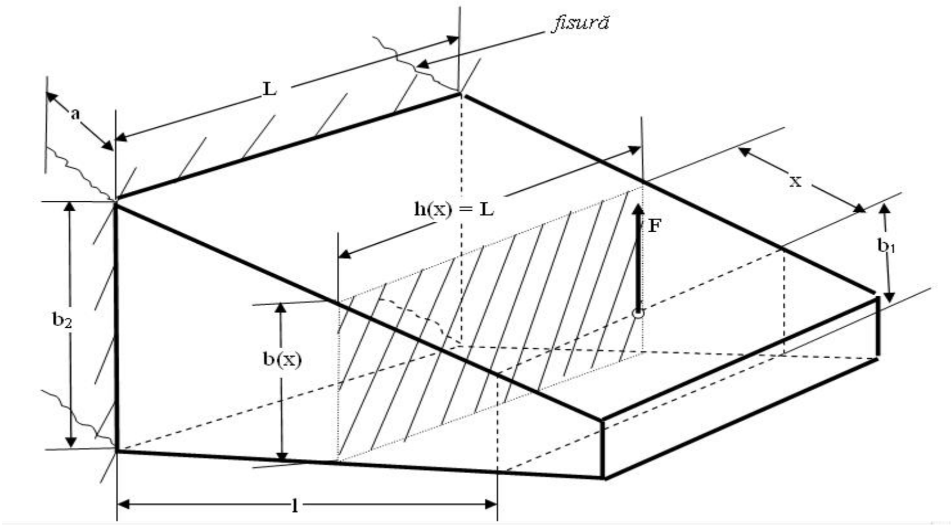

Fig. 5. Model for the tooth of the rack

The calculation relation for intensity factor is, in the case of the model from drawing 5, given by the following:

$$K_{I_c}^2 = \frac{EF^2}{2L}\frac{\partial c}{\partial a} \tag{1}$$

In which:
E- longitudinal elastic module of the material;
F- compression loading on the tooth of the rack;
L- the width of the tooth of the rack;
$\frac{\partial c}{\partial a}$ - the partial differential of the compliance of the tooth of the rack reported to the length of the propagated crack.
The compliance of the tooth of the rack is given by the relation:

$$c = v/F \tag{2}$$

The model from drawing 5 represents a bar, loaded at bending by force F. The displacement **v** on the direction of the F force is given by the Castigliano expression:

$$v = \int_o^l \frac{M(x)}{EI_z} \cdot \frac{\partial M(x)}{\partial F} dx \tag{3}$$

where:
-M(x) represents bending moment in a common section;
-$\frac{\partial M(x)}{\partial F}$ represents the partial differential of the moment reported to the load:

$$\frac{\partial M(x)}{\partial F} = x$$

-I_z represents inertia moment of the section x: $I_z = \frac{L \cdot b(x)^3}{12}$.
The b(x) height of a common x section is given by the relation:

$$b(x) = \frac{b_1 l - b_1 x + b_2 l}{l}$$

As a result, for the arrow, on the direction of the force, there will be:

$$v = \frac{1}{E}\int_o^l \frac{F \cdot x}{\frac{L}{12}\left(\frac{b_1 l - b_1 x + b_2 x}{l}\right)^3} \cdot x \cdot dx \tag{4}$$

After all this calculations, the following is obtained:

$$v = \frac{12Fl^3}{E \cdot L(b_2 - b_1)^3}\left[\ln\frac{b_2}{b_1} + \frac{2b_1}{b_2 - b_1} - \frac{b_1^2}{2(b_2^2 - b_1^2)}\right] \tag{5}$$

In the conditions in which the appearance of a length a crack is observed, relation 4 will be the following:

$$v = \frac{12F(l+a)^3}{E \cdot L(b_2 - b_1)^3}\left[\ln\frac{b_2}{b_1} + \frac{2b_1}{b_2 - b_1} - \frac{b_1^2}{2(b_2^2 - b_1^2)}\right] \tag{6}$$

Introducing the **v** expression for relation 6 into relation 2 and after the partial derivation:

$$\frac{\partial c}{\partial a}=\frac{36(l+a)^2}{E\cdot L(b_2-b_1)^3}\left[\ln\frac{b_2}{b_1}+\frac{2b_1}{b_2-b_1}-\frac{b_1^2}{2\left(b_2^2-b_1^2\right)}\right] \tag{7}$$

As a result, after relation 1, for the intensity factor of the load, there will be:

$$K_1^2=\frac{18F^2(l+a)^2}{L^2(b_2-b_1)^3}\left[\ln\frac{b_2}{b_1}+\frac{2b_1}{b_2-b_1}-\frac{b_1^2}{2\left(b_2^2-b_1^2\right)}\right] \tag{8}$$

Considering the characteristics of the material, there is a critical length of the crack, from which this may propagate. The critical length of the crack is obtained if the equality between the intensity, given by *relation 8*, and the critic intensity of the loads (cracking strength) is obtained:

$$K_I = K_{Ic}$$

There will result:

$$(l+a_{cr})^2=\frac{K_{Ic}^2L^2(b_2-b_1)^3}{18F_{cr}^2\left[\ln\frac{b_2}{b_1}+\frac{2b_1}{b_2-b_1}-\frac{b_1^2}{2\left(b_2^2-b_1^2\right)}\right]} \tag{9}$$

The critical force is obtained from the maxim load equality with the material flowing load:

$$\sigma_{\max}=\frac{M_{\max}}{\omega_z}=\frac{F_{cr}\cdot l}{b_2^2\cdot L/6}=\sigma_c$$

$$F_{cr}=\frac{\sigma_c b_2^2 L}{6l}$$

The size of the rack teeth are: L = 22mm; b_2 = 5mm; b_1 = 2,5mm; l = 2,5mm. With these values, the critic force becomes:

$$F_{cr}=\frac{640\cdot 22{,}5^2}{6\cdot 2{,}5}=23467N$$

By introducing these values in relation 9, there will result: $(l+a_{cr})$= 2,53mm.

Next, the number of cycles necessary for the crack to extend from 0.5mm to 2.53mm must be calculated. The steps followed are:

1. the length of the crack must increase:

   $$\Delta a_i = 0{,}2\text{mm}$$

2. the medium value of the length of the crack must be calculated:

   $$a_{med(1)} = (0{,}5+0{,}7)/2= 0{,}6\text{mm}$$

3. the variation of the intensity for the medium value of the crack must be calculated $a_{med(1)}$:
4.

$$\Delta K_{I(1)}^2=\frac{18(\Delta F)^2(l+a_{med1})^2}{L^2(b_2-b_1)^3}\left[\ln\frac{b_2}{b_1}+\frac{2b_1}{b_2-b_1}-\frac{b_1^2}{2\left(b_2^2-b_1^2\right)}\right]$$

Force F, which presses on the teeth, has been determined from the finite elements analysis presented in the previous chapter:

$$\Delta F = 9700\text{ N}$$

By introducing the known values, there is obtained:

$$\Delta K_{I(1)}=2331{,}8MPa\sqrt{mm}$$

5. corresponding to $\Delta K_{I(1)}$ previously obtained, the propagation speed of the crack was calculated. For the mentioned steel, the calculation relation is:

$$\frac{da}{dN} = 4{,}36 \cdot 10^{-12} (\Delta K)^2$$

in which Δ K must be introduced in $\sqrt{m}$, and $\left(\frac{da}{dN}\right)$, is obtained in m/cycle:

$$\Delta K = 2331{,}8 \text{ MPa } \sqrt{m} = \frac{2331{,}8}{\sqrt{1000}} \text{ MPa } \sqrt{m} = 73{,}73 \text{ MPa } \sqrt{m}$$

In these conditions, for the propagation speed of the crack:

$$\left(\frac{da}{dN}\right) = 7{,}36 \cdot 10^{-12} (73{,}73)^2 = 4 \cdot 10^{-8} \text{ m/cycle.}$$

6. the number of cycles necessary for the crack to extend with 0.2mm is calculated:

$$\Delta N_1 = \frac{\Delta a_i}{\left(\frac{da}{dN}\right)_1} = \frac{0{,}2 \cdot 10^{-3}}{4 \cdot 10^{-8}} = 5000 \text{ cycles}$$

The next step must be followed with another increase: Δa_2 = 0.2 mm. The algorithm is repeated until the dimension of the critical crack is reached: a_{cr} = 2.12 mm. The data obtained from the calculations are in table 1.

Step	a_i [mm]	l_i+a_{med} [mm]	$\Delta K_{Ii} \left[MPa\sqrt{m}\right]$	$\left(\frac{da}{dN}\right)_i [m/cycle]$	ΔN_i [cycles]
1	0,7	3,1	73,73	$4 \cdot 10^{-8}$	5000
2	0,9	3,3	78,49	$4{,}53 \cdot 10^{-8}$	4415
3	1,1	3,5	83,25	$5{,}10 \cdot 10^{-8}$	3921
4	1,3	3,7	88,01	$5{,}70 \cdot 10^{-8}$	3508
5	1,5	3,9	92,76	$6{,}33 \cdot 10^{-8}$	3159
6	1,7	4,1	97,52	$6{,}99 \cdot 10^{-8}$	2861
7	1,9	4,3	102,28	$7{,}69 \cdot 10^{-8}$	2600
8	2,1	4,5	107,03	$8{,}43 \cdot 10^{-8}$	2372
9	2,3	4,7	111,79	$9{,}19 \cdot 10^{-8}$	2176
10	2,5	4,9	116,55	$9{,}99 \cdot 10^{-8}$	2001
11	2,53	5,015	119,28	$10{,}45 \cdot 10^{-8}$	1913
				$\sum \Delta i$	33.927

There can be observed, that at the base of the tooth a crack of only 0.5 mm appears and the rack can work another 34.000 loading cycles until a critical crack is reached. This will lead ultimately to the breaking of the tooth.

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