

ANNULAR PLATE WITH SIMPLE LEANED CONTOURS, UNDER THE ACTION OF AN UNIFORM DISTRIBUTED PRESSURE

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Abstract: The paper proposes to establish the expressions of the angular and linear deformations of a circular and annular plate, having simple leaning on the external contours, under the action of a uniform distributed pressure on one face (fig. 1). We propose, too, to determine the calculus relations of the radial and annular stresses, in the domain of the elastic solicitation. To reach this purpose we take into account two variants of study.

Keywords: Plane and annular plates, deformations and stresses states

1. GENERALITIES

This analyze take into consideration the thin plates (those structures whose linear deformations produced of the external loads don't excel an half of thickness), distinguishing the corresponding deformations and stresses states, in the case of a pressure distributed on a face, in the conditions of the elastic domain solicitation, characteristic of the construction material. In this meaning we take into consideration the adequate simplifier hypotheses [1]. We imagine two study variants, taking into account the expressions of the utilized sizes in the papers [2, 3], for establish the corresponding expressions of the linear or angular deformations, respectively the radial and annular stresses:

► **A variant** – fig. 2 - (we suppose that the external leaning exist and we simulate the presence of a second leaning by introducing of a uniform distributed force, which produce a linear deformation having an equal, but inverse value with that created by the uniform pressure).

► **B variant** - fig. 3 - (this time it's take into consideration like basis the external leaning and the distributed uniform force being presented on the internal contour of the plate).

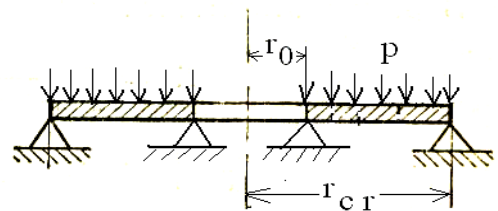


Fig. 1. Plate having simple leaned contours, under the action of a uniform distributed pressure

2. A VARIANT (fig. 2)

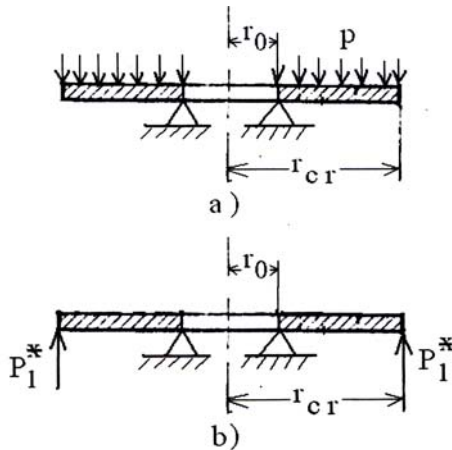
From the equalization of the corresponding expressions for the linear deformation at the level of the external contour, it results:

$$P_1^* = 4 \cdot \pi \cdot p \cdot r_c^2 \cdot \frac{C_{d, x=\alpha}^{(c)}}{C_{d, x=\alpha}^{(a)}}, \quad (1)$$

where:

$$C_{d, x=\alpha}^{(a)} = \frac{4 \cdot (1 + \nu_p) \cdot \ln^2 \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} - 2 \cdot \ln \alpha + \left(\frac{\frac{3 + \nu_p}{1 + \nu_p} + \frac{2 \cdot \alpha^2 \cdot \ln \alpha}{\alpha^2 - 1}}{\alpha^2} \right); \quad (2)$$

$$C_{d, x=\alpha}^{(c)} = \frac{\alpha^2 \cdot (1 - \ln \alpha) - 1}{2 \cdot \alpha^2} + \frac{\alpha^4 - 1}{16 \cdot \alpha^4} - \frac{\alpha^2 - 1}{2} \cdot \left[\frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{1 + \alpha^2}{4 \cdot \alpha^4} + \frac{\ln \alpha}{\alpha^2 - 1} - \frac{1 - \nu_p}{2 \cdot (1 + \nu_p) \cdot \alpha^2} \right] - \frac{\ln \alpha}{1 - \nu_p} \cdot \left[\frac{3 + \nu_p}{4 \cdot \alpha^2} - (1 + \nu_p) \cdot \frac{\ln \alpha}{\alpha^2 - 1} \right]. \quad (3)$$



2. 1. Deformations states

The expression of the rotation on the internal leaning of the plate is inferred under the form:

$$\varphi_{i, x=1}^{(A)} = \frac{p \cdot r_{cr}^4}{\Re_p \cdot r_0} \cdot \left[\frac{1}{4} \cdot C_{r, x=1}^{(c)} - \frac{C_{d, x=\alpha}^{(c)}}{C_{d, x=\alpha}^{(a)}} \cdot C_{r, x=1}^{(a)} \right], \quad (4)$$

Fig. 2. Analyze diagram – A variant

like difference between the rotations produced by the distributed uniform pressure and that which correspond at

P_1^* distributed force, where:

$$C_{r, x=1}^{(a)} = \frac{(1 + \nu_p) \cdot \ln \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} + \left(\frac{1}{1 + \nu_p} + \frac{\alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \right) \cdot \frac{1}{\alpha^2}; \quad (5)$$

$$C_{r, x=1}^{(c)} = \frac{3}{4 \cdot \alpha^2} - \frac{3 + \nu_p}{4 \cdot (1 + \nu_p)} \cdot \frac{1 + \alpha^2}{\alpha^4} - \frac{\ln \alpha}{\alpha^2 - 1} + \frac{1 - \nu_p}{2 \cdot (1 + \nu_p) \cdot \alpha^2} - \frac{1}{1 - \nu_p} \cdot \left(\frac{3 + \nu_p}{4 \cdot \alpha^2} - (1 + \nu_p) \cdot \frac{\ln \alpha}{\alpha^2 - 1} \right). \quad (6)$$

The angle of the neutral surface of the plate on the external leaning can be determined with the formula:

$$\varphi_{e, x=\alpha}^{(A)} = \frac{p \cdot r_{cr}^4}{\Re_p \cdot r_0} \cdot \left[\frac{C_{d, x=\alpha}^{(c)}}{C_{d, x=\alpha}^{(a)}} \cdot C_{r, x=\alpha}^{(a)} - \frac{1}{4} \cdot C_{r, x=\alpha}^{(c)} \right], \quad (7)$$

inferred through the difference between the angles produced by the $P_1^\bullet$ distributed force and the uniform distributed pressure on the surface of the plate, where the following notations are utilized, too:

$$C_{r, x=\alpha}^{(a)} = \frac{1}{\alpha} \cdot \left[\frac{(1 + \nu_p) \cdot \ln \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} - \ln \alpha + \frac{1}{1 + \nu_p} + \frac{\alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \right]; \quad (8)$$

$$C_{r, x=\alpha}^{(c)} = \frac{1 - 2 \cdot \ln \alpha}{2 \cdot \alpha} + \frac{1}{4 \cdot \alpha^2} - \frac{3 + \nu_p}{4 \cdot (1 + \nu_p)} \cdot \frac{1 + \alpha^2}{\alpha^3} - \frac{\alpha \cdot \ln \alpha}{\alpha^2 - 1} +$$

$$+ \frac{1 - \nu_p}{2 \cdot (1 + \nu_p) \cdot \alpha} - \frac{1}{1 - \nu_p} \cdot \left[\frac{3 + \nu_p}{4 \cdot \alpha^2} - (1 + \nu_p) \cdot \frac{\ln \alpha}{\alpha^2 - 1} \right] \cdot \frac{1}{\alpha}. \quad (9)$$

The expression of the displacements of the points of the neutral surface of the plate is established like difference between the specific relations of the uniform pressure, respectively of the $P_1^\bullet$ distributed force:

$$w_{x, p}^{(A)} = \frac{p \cdot r_{cr}^4}{4 \cdot \Re_p} \cdot \left[C_{d, x, p}^{(c)} - C_{d, x, P_1^\bullet}^{(a)} \cdot \frac{C_{d, x=\alpha}^{(c)}}{C_{d, x=\alpha}^{(a)}} \right], \quad (10)$$

with the notations:

$$C_{d, x, p}^{(c)} = \frac{x^2 \cdot (1 - \ln x) - 1}{2 \cdot \alpha^2} + \frac{x^4 - 1}{16 \cdot \alpha^4} - \frac{x^2 - 1}{2} \cdot \left(\frac{\frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{1 + \alpha^2}{4 \cdot \alpha^4} +}{+ \frac{\ln \alpha}{\alpha^2 - 1} - \frac{1 - \nu_p}{2 \cdot (1 + \nu_p) \cdot \alpha^2}} \right) -$$

$$- \frac{\ln x}{1 - \nu_p} \cdot \left[\frac{\frac{3 + \nu_p}{4 \cdot \alpha^2} -}{- (1 + \nu_p) \cdot \frac{\ln \alpha}{\alpha^2 - 1}} \right]; \quad (11)$$

$$C_{d, x, P_1^\bullet}^{(a)} = \frac{4 \cdot (1 + \nu_p) \cdot \ln \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} \cdot \ln x - \frac{2 \cdot x^2 \cdot \ln x}{\alpha^2} - \left(\frac{\frac{3 + \nu_p}{1 + \nu_p} +}{+ \frac{2 \cdot \alpha^2 \cdot \ln \alpha}{\alpha^2 - 1}} \right) \cdot \frac{1 - x^2}{\alpha^2}. \quad (12)$$

2. 2. Stresses state

For establish the expressions of the radial and annular stresses we proceed, like in the previous case, too, at the difference of the characteristic magnitudes of the action of the distributed pressure, respectively of the $P_1^\bullet$ load, and in finale it result:

► **for radial stresses:**

$$\sigma_{r,x}^{(A)} = \frac{3 \cdot p \cdot r_{c,r}^2}{\delta_p^2} \cdot \left(\frac{1}{2} \cdot C_{r,\sigma,x}^{(c)} - 2 \cdot C_{r,\sigma,x}^{(a)} \cdot \frac{C_{d,x=\alpha}^{(c)}}{C_{d,x=\alpha}^{(a)}} \right), \quad (13)$$

with the notations:

$$C_{r,\sigma,x}^{(a)} = (1 + \nu_p) \cdot \left[\ln x - \frac{x^2 - 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2 \cdot \ln x \right]; \quad (14)$$

$$C_{r,\sigma,x}^{(c)} = \frac{3 + \nu_p}{4} \cdot (x^2 - 1) \cdot \frac{\alpha^2 - x^2}{\alpha^2 \cdot x^2} + (1 + \nu_p) \cdot \left(\ln x - \frac{\alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \cdot \frac{x^2 - 1}{x^2} \right); \quad (15)$$

► **for annular stresses:**

$$\sigma_{\theta,x}^{(A)} = \frac{3 \cdot p \cdot r_{c,r}^2}{\delta_p^2} \cdot \left(\frac{1}{2} \cdot C_{\theta,\sigma,x}^{(c)} - 2 \cdot C_{\theta,\sigma,x}^{(a)} \cdot \frac{C_{d,x=\alpha}^{(c)}}{C_{d,x=\alpha}^{(a)}} \right), \quad (16)$$

with the notations:

$$C_{\theta,\sigma,x}^{(a)} = (1 + \nu_p) \cdot \left[\ln x - \frac{x^2 + 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2 \cdot \ln \alpha - \frac{1 - \nu_p}{1 + \nu_p} \right]; \quad (17)$$

$$C_{\theta,\sigma,x}^{(c)} = \frac{3 + \nu_p}{4} \cdot \left(1 + \frac{1}{\alpha^2} + \frac{1}{x^2} \right) + (1 + \nu_p) \cdot \ln x \cdot \frac{1 + 3 \cdot \nu_p}{4} \cdot \left(\frac{x}{\alpha} \right)^2 - \frac{(1 + \nu_p) \cdot \alpha^2 \cdot (x^2 + 1) \cdot \ln \alpha}{(\alpha^2 - 1) \cdot x^2} - 1 + \nu_p. \quad (18)$$

3. B VARIANT (fig. 3.)

To establish the corresponding relation of the rotation of the plate on the external leaning, we ask for the previous enunciated methodology. In this sense, an annular plate is considerate, having on its internal contour a uniform distributed force, with inverted action of the pressure, having the relation under the form:

$$P_2^\bullet = \frac{1}{4} \cdot \pi \cdot p \cdot r_{c,r}^2 \cdot \frac{C_{d,x=1}^{(d)}}{C_{d,x=\alpha}^{(a)}}, \quad (19)$$

where:

$$\begin{aligned}
 C_{d, x=1}^{(d)} = & \frac{9}{\alpha^4} - 1 - \frac{2 \cdot (3 + \nu_p) \cdot (1 - \alpha^4)}{\alpha^4} + \\
 & + 4 \cdot \frac{3 + \nu_p}{(1 - \nu_p) \cdot \alpha^2} \cdot \ln \alpha + 4 \cdot \frac{1 - \nu_p}{1 + \nu_p} \cdot \frac{1 - \alpha^2}{\alpha^4} + \\
 & + 8 \cdot \frac{\ln \alpha - 1}{\alpha^2} + 8 \cdot \frac{\ln \alpha}{(\alpha^2 - 1) \cdot \alpha^2} \cdot \left(1 - \alpha^2 - 2 \cdot \frac{1 + \nu_p}{1 - \nu_p} \cdot \ln \alpha \right).
 \end{aligned} \quad (20)$$

3. 1. Deformations state

The rotation on the external leaning can be calculated with the expression:

$$\varphi_{e, x=\alpha}^{(B)} = \frac{p \cdot r_{cr}^4}{16 \cdot \Re_p \cdot r_0} \cdot \left[\begin{array}{c} C_{r, x=\alpha}^{(d)} - \\ - \frac{C_{d, x=1}^{(d)}}{C_{d, x=\alpha}^{(a)}} \cdot C_{r, x=1}^{(a)} \end{array} \right], \quad (21)$$

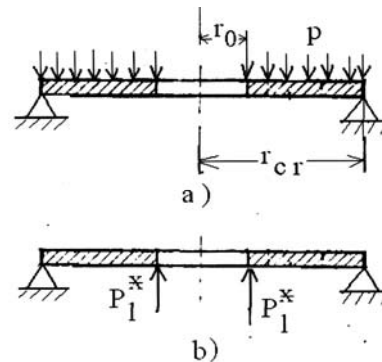


Fig. 3. Analyze diagram – B variant

where the following relation is introduced:

$$C_{r, x=\alpha}^{(d)} = \frac{1}{\alpha^2} \cdot \left[\begin{array}{c} \frac{\alpha + 2 \cdot (1 - 2 \cdot \ln \alpha)}{\alpha} - \frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{\alpha^2 + 1}{\alpha} - \frac{3 + \nu_p}{(1 - \nu_p) \cdot \alpha} + \\ + 2 \cdot \frac{1 - \nu_p}{(1 + \nu_p) \cdot \alpha} + \frac{4 \cdot \ln \alpha}{\alpha^2 - 1} \cdot \left(\alpha - \frac{1 + \nu_p}{1 - \nu_p} \cdot \frac{1}{\alpha} \right) \end{array} \right], \quad (22)$$

while the rotation on the internal leaning has the form:

$$\varphi_{i, x=1}^{(B)} = \frac{p \cdot r_{c, r}^4}{16 \cdot \Re_p \cdot r_0} \cdot \left[\frac{C_{d, x=1}^{(d)}}{C_{d, x=\alpha}^{(a)}} \cdot C_{r, x=1}^{(a)} - C_{r, x=1}^{(d)} \right], \quad (23)$$

with the notation:

$$C_{r, x=1}^{(d)} = \frac{1}{\alpha^2} \cdot \left[\begin{array}{c} \frac{3}{\alpha^2} - \frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{\alpha^2 + 1}{\alpha^2} - \frac{3 + \nu_p}{1 - \nu_p} + \\ + 2 \cdot \frac{1 - \nu_p}{(1 + \nu_p) \cdot \alpha^2} - \frac{8 \cdot \nu_p \cdot \ln \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} \end{array} \right]. \quad (24)$$

The linear deformation of the points of the median surface between the leaning contours can be calculated using the expression:

$$w_{x,p}^{(B)} = \frac{p \cdot r_{c,r}^4}{64 \cdot \Re_p} \cdot \left[C_{d,x,p}^{(d)} - \frac{C_{d,x=1}^{(d)}}{C_{d,x=\alpha}^{(a)}} \cdot \left(C_{d,x=\alpha,P_2}^{(a)} - C_{d,x,P_1}^{(a)} \right) \right], \quad (25)$$

with the notations:

$$\begin{aligned} C_{d,x,p}^{(d)} = & \left(\frac{x}{\alpha} \right)^4 - 1 + \frac{8 \cdot x^2 \cdot (1 - \ln x)}{\alpha^4} - \frac{2 \cdot (3 + \nu_p)}{1 + \nu_p} \cdot \frac{(x^2 - \alpha^2) \cdot (\alpha^2 + 1)}{\alpha^4} + \\ & + 4 \cdot \frac{3 + \nu_p}{(1 - \nu_p) \cdot \alpha^2} \cdot \ln \frac{\alpha}{x} + 4 \cdot \frac{1 - \nu_p}{1 + \nu_p} \cdot \frac{x^2 - \alpha^2}{\alpha^4} + 8 \cdot \frac{\ln \alpha - 1}{\alpha^2} + \\ & + 8 \cdot \frac{\ln \alpha}{(\alpha^2 - 1) \cdot \alpha^2} \cdot \left(x^2 - \alpha^2 + 2 \cdot \frac{1 + \nu_p}{1 - \nu_p} \cdot \ln \frac{x}{\alpha} \right); \end{aligned} \quad (26)$$

$$C_{d,x=\alpha,P_2}^{(a)} = \frac{4 \cdot (1 + \nu_p) \cdot \ln^2 \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} - 2 \cdot \ln \alpha - \left(\frac{3 + \nu_p}{1 + \nu_p} + \frac{2 \cdot \alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \right) \cdot \frac{1 - \alpha^2}{\alpha^2}. \quad (27)$$

3. 2. Stresses state

The expressions of the radial and annular stresses are inferred, in this case, too, through the difference between the corresponding stresses to the actions given by the distributed uniform pressure and $P_2^\bullet$ force, resulting:

► *for radial stresses:*

$$\sigma_{r,x}^{(B)} = \frac{3 \cdot p \cdot r_{c,r}^2}{2 \cdot \delta_p^2} \cdot \left[C_{r,\sigma,x}^{(d)} - \frac{1}{4} \cdot \frac{C_{d,x=1}^{(d)}}{C_{d,x=\alpha}^{(a)}} \cdot C_{r,\sigma,x}^{(a)} \right], \quad (28)$$

with the notation:

$$\begin{aligned} C_{r,\sigma,x}^{(d)} = & \frac{1 + \nu_p}{\alpha^2} \cdot \ln \alpha - \frac{3 + \nu_p}{4} \cdot \frac{(x^2 - 1) \cdot (x^2 - \alpha^2)}{\alpha^2 \cdot x^2} - \\ & - \frac{(1 + \nu_p) \cdot \ln \alpha}{\alpha^2 - 1} \cdot \frac{x^2 - 1}{x^2}; \end{aligned} \quad (29)$$

► *for annular stresses:*

$$\sigma_{\theta,x}^{(B)} = \frac{3 \cdot p \cdot r_{c,r}^2}{2 \cdot \delta_p^2} \cdot \left[C_{\theta,\sigma,x}^{(d)} - \frac{1}{2} \cdot \frac{C_{d,x=1}^{(d)}}{C_{d,x=\alpha}^{(a)}} \cdot C_{\theta,\sigma,x}^{(a)} \right], \quad (30)$$

with the notation:

$$C_{\theta, \sigma, x}^{(d)} = \frac{1 + \nu_p}{\alpha^2} \cdot \ln x + \frac{3 + \nu_p}{4} \cdot \left(1 + \frac{1}{\alpha^2} + \frac{1}{x^2} \right) - \frac{(1 + \nu_p) \cdot \ln \alpha}{\alpha^2 - 1} \cdot \frac{x^2 + 1}{x^2} - \frac{1 + 3 \cdot \nu_p}{4 \cdot \alpha^2} \cdot x^2 - \frac{1 - \nu_p}{\alpha^2} \quad (31)$$

4. NOTATIONS

E_p – the longitudinal modulus of the material of the plate; P_1^*, P_2^* – the normal forces at the median surface of the plate, which are distributed on the contours of the annular plate; p – the distributed uniform pressure on one face of the plate; r – the current radius of the plate ($r \in [r_0; r_{c,r}]$); $r_{c,r}$ – the radius of the circumference of the external leaning of the plate; r_0 – the radius of the circumference of the internal leaning of the plate; $x = r / r_0$ – current geometrical simplex; w – the linear deformation of the plate (normal at the median surface of the plate); $\Re_p = (E_p \cdot \delta_p^3) / [12 \cdot (1 - \nu_p^2)]$ – the bending cylindrical rigidity of the plate; δ_p – the thickness of the annular plate; $\alpha = r_{c,r} / r_0$ – geometrical simplex; ν_p – the coefficient of the transversal contraction of the material of the plate; φ_i, φ_e – the rotations of the median surfaces of the plate, corresponding to the internal, respectively external leaning, of the plate; σ_r, σ_θ – radial and annular stresses.

5. CONCLUSIONS

Taking into consideration the simplifier hypotheses for the solicitation in elastic domain of the material of the annular and plane plates, the paper establishes, using two study variants, the expressions of the linear and angular deformations, respectively the annular and radial stresses (whose basis we can establish, in given cases, the equivalent stresses and the portent capacity of some structures with known geometry or its determination).

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