SOLICITATION STATES IN THE ANNULAR AND SIMPLE LEANED PLATES ON THEIR CONTOURS, UNDER THE ACTION OF SOME RADIAL BENDING MOMENTS (I)

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Abstract. In the composition of the mechanical equipments for the industrial processing of the substances, some annular plates, having the role of separation of the working spaces, or fixing of some specific constructive elements can be met. For dimension or verify the respective geometries, one of the analyze variants is that which hypothetical devises the adjacent elements and establish the equilibrium connection loads. For an annular plate, having the simple leaned contours, the effects about the deformation and stresses state of some connection radial bending moments are presented in one first variant in this paper. In this sense, in line, the simple leanings are assimilated with the effects of some normal and uniform distributed forces at the medium surface. It is mentioned, too, that the solicitation of the plate is considered in the elastic domain, which is specific to the construction material.

Keywords: Annular plate, simple leaned contours, connection loads

1. GENERALITIES

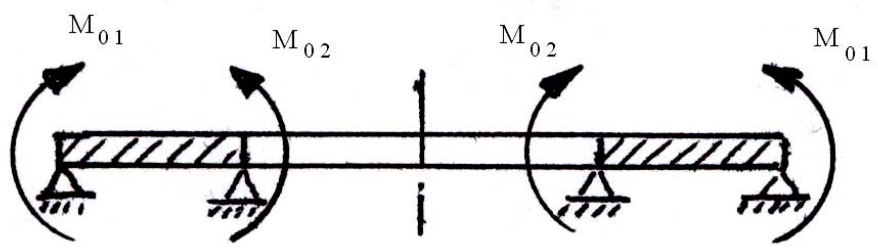

Fig. 1. Annular plate charged with bending moments which are uniform distributed on the contour

Especially in the mechanical structures like under pressure recipients, are often met annular plates which divides two working spaces, or attach two cylindrical covers, for example, having technological or mechanical resistance role, static [1 – 9, 11, 13] or dynamic [9, 10, 12, 13]. In this case, the analyze of the solicitation states is enforced for validate the respective geometries. The annular plates can have one alone or the both simple leaned or embedded contours. On these circumferences, connection radial bending moments are developed. The presented paper proposes to establish the expressions of the rotations, linear deformations, radial and annular stresses, and we can to evaluate the equivalent stresses. In this sense, two analyze variants are taken into consideration where one or other of the contours is assimilated, in line, being charged with normal loads which are uniform distributed on the logway of the respective circumference at the median surface of the plate. Their effect is identically with the existed bending moments on the other leaning (the same linear deformation, having inverse sense is developed).

In this presented case, the deformations and stresses states have a charging like in figure 1 and we adopt for study the calculus adequate relations from the papers [1, 2].

The method of cancellation of the linear deformation on one or other leaning is used, through the introduction of a uniform distributed force on the considered base and two study variants result, too:

► **A variant–** fig. 2 – (we suppose that the external leaning exist and we simulate the presence of the second leaning (the internal leaning), by the introduction of a uniform distributed force, which produces a linear deformation, having an equal value but with inverse sense with the value of the active bending moment on the peripheral leaning;

► **B variant** - fig. 3 – (the internal leading is considered like basis, this time, and the uniform distributed force is presented on the external contour of the plate.

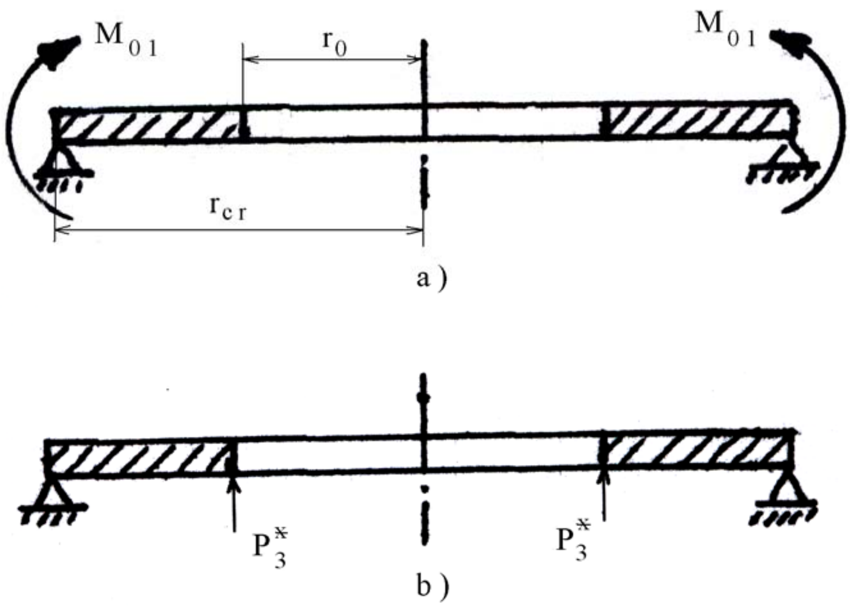

Fig. 2. The analyze diagram – A variant

2. THE DEFORMATIONS AND STRESSES STATES

2. 1. A Variant (fig. 2)

2. 1. 1.The expression of the equivalent force

After the equality between the linear deformations produced of the M_{01} bending moment and the $P_3^{\bullet}$ uniform distributed force on the internal contour of the plate:

$$w_{M,\,x=1}^{(f)} = w_{P^{\bullet},\,x=1}^{(b)}, \tag{1}$$

we reach at the expression:

$$P_3^{\bullet} = 16 \cdot \pi \cdot \frac{C_{d,M,x=1}^{(f)}}{C_{d,P^{\bullet},x=\alpha}^{(a)}} \cdot M_{01}, \tag{2}$$

where we note:

$$C_{d,M,x=1}^{(f)} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{1-\alpha^2}{2 \cdot \left(1+\nu_p\right)} - \frac{\ln \alpha}{1-\nu_p} \right]; \tag{3}$$

$$C_{d,P^{\bullet},x=\alpha}^{(a)} = \frac{4 \cdot \left(1+\nu_p\right) \cdot \ln^2 \alpha}{\left(1-\nu_p\right) \cdot \left(\alpha^2 - 1\right)} - 2 \cdot \ln \alpha + \left(\begin{array}{l} \dfrac{3+\nu_p}{1+\nu_p} + \\ + \dfrac{2 \cdot \alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \end{array} \right) \cdot \frac{\alpha^2 - 1}{\alpha^2}. \tag{4}$$

2. 1. 2. The deformations state

The rotation of the median surface of the plate on the internal leaning has the following formula:

$$\varphi_{M,x=1}^{(A)} = \frac{M_{01} \cdot r_{cr}^2}{\Re_p \cdot r_0} \cdot \left[C_{r,M,x=1}^{(f)} - 4 \cdot \frac{C_{d,M,x=1}^{(e)}}{C_{d,P^{\bullet},x=\alpha}^{(a)}} \cdot C_{r,P^{\bullet},x=\alpha}^{(a)} \right], \tag{5}$$

with the notations:

$$C_{r,M,x=1}^{(f)} = \frac{2}{\left(\alpha^2 - 1\right) \cdot \left(1 - \nu_p^2\right)}; \tag{6}$$

$$C_{r,P^{\bullet},x=\alpha}^{(a)} = \frac{1}{\alpha} \cdot \left[\frac{\left(1+\nu_p\right) \cdot \ln \alpha}{\left(1-\nu_p\right) \cdot \left(\alpha^2 - 1\right)} - \ln \alpha + \frac{1}{1+\nu_p} + \frac{\alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \right], \tag{7}$$

while the rotation on the external leaning has the expression:

$$\varphi_{M,x=\alpha}^{(A)} = \frac{M_{01} \cdot r_{cr}^2}{\Re_p \cdot r_0} \cdot \left[C_{r,M,x=\alpha}^{(f)} - 4 \cdot \frac{C_{d,M,x=1}^{(f)}}{C_{d,P^{\bullet},x=\alpha}^{(a)}} \cdot C_{r,P^{\bullet},x=\alpha}^{(a)} \right], \tag{8}$$

where:

$$C_{r,M,x=\alpha}^{(f)} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{\alpha}{1+\nu_p} + \frac{1}{\left(1-\nu_p\right) \cdot \alpha} \right]. \tag{9}$$

The current linear deformation of the points of the median surface of the plate is:

$$w_{M,x}^{(A)} = \frac{M_{01} \cdot r_{cr}^{2}}{\Re_p} \cdot \left[\begin{array}{c} C_{d,M,x=1}^{(f)} - C_{d,M,x}^{(f)} - \\ - \dfrac{C_{d,M,x=1}^{(f)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \cdot \left(C_{d,P^\bullet,x=\alpha}^{(a)} - C_{d,P^\bullet,x}^{(a)} \right) \end{array} \right], \tag{10}$$

where the notations are remarked:

$$C_{d,M,x}^{(f)} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{1 - x^2}{2 \cdot \left(1 + \nu_p \right)} - \frac{\ln x}{1 - \nu_p} \right]; \tag{11}$$

$$C_{d,P^\bullet,x}^{(a)} = \frac{4 \cdot \left(1 + \nu_p \right) \cdot \ln \alpha}{\left(1 - \nu_p \right) \cdot \left(\alpha^2 - 1 \right)} \cdot \ln x - \frac{2 \cdot x^2 \cdot \ln x}{\alpha^2} - \left(\begin{array}{c} \dfrac{3 + \nu_p}{1 + \nu_p} + \\ + \dfrac{2 \cdot \alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \end{array} \right) \cdot \frac{1 - x^2}{\alpha^2}. \tag{12}$$

2. 1. 3. The stresses state

The final radial stresses are inferred as difference between the values given of the bending moment which is uniform distributed on the external contour and the equivalent force which is assigned on the internal contour, having the expression:

$$\sigma_{r,M,x}^{(A)} = \frac{6 \cdot M_{01}}{\delta_p^2} \cdot \left[C_{r,\sigma,M}^{(f)} - 4 \cdot \frac{C_{d,M,x=1}^{(f)}}{C_{d,P^\bullet,x=\alpha}} \cdot C_{r,\sigma,P^\bullet}^{(a)} \right], \tag{13}$$

with the notations:

$$C_{r,\sigma,M}^{(f)} = \frac{x^2 - 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2; \tag{14}$$

$$C_{r,\sigma,P^\bullet}^{(a)} = \left(1 + \nu_p \right) \cdot \left[\ln x - \frac{x^2 - 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2 \cdot \ln \alpha \right]. \tag{15}$$

Accepting the same procedure for the annular stresses, too, we reach at the formula:

$$\sigma_{\theta,x}^{\left(M_{0,1} \right)} = \frac{6 \cdot M_{01}}{\delta_p^2} \cdot \left[C_{\theta,\sigma,M}^{(f)} - 4 \cdot \frac{C_{d,M,x=1}^{(f)}}{C_{d,P^\bullet,x=\alpha}} \cdot C_{\theta,\sigma,P^\bullet}^{(a)} \right], \tag{16}$$

with the notations:

$$C_{\theta,\sigma,M}^{(f)} = \frac{x^2 + 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2; \tag{17}$$

$$C^{(a)}_{\theta,\sigma,P^{\bullet}} = \left(1+\nu_p\right)\cdot\left[\ln x - \frac{x^2+1}{\alpha^2-1}\cdot\left(\frac{\alpha}{x}\right)^2\cdot\ln\alpha - \frac{1-\nu_p}{1+\nu_p}\right]. \tag{18}$$

2. 2. B variant (fig. 3)

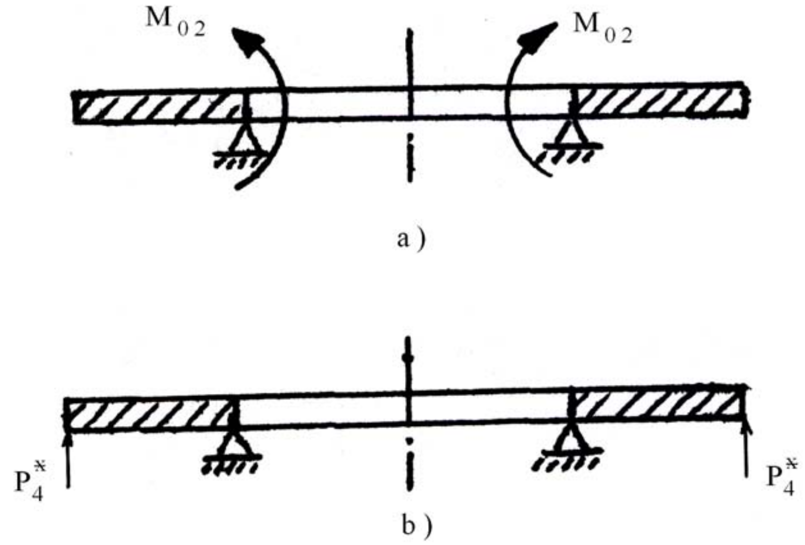

Fig. 3. The analyze diagram – B variant

2. 2. 1. The expression of the equivalent force

The simulation of the external leaning of the annular plate is obtained with the presence of a uniform distributed force on the corresponding circumference. From the equality of the linear deformation with the linear deformation produced of the $M_{0\,2}$ unitary radial bending moment:

$$w^{(h)}_{M,\,x=\alpha} = w^{(a)}_{P^{\bullet},\,x=\alpha}, \tag{19}$$

it is inferred:

$$P^{\bullet}_4 = 16\cdot\pi\cdot M_{0\,2}\cdot\frac{C^{(h)}_{d,M,\,x=\alpha}}{C^{(a)}_{d,P^{\bullet},\,x=\alpha}}, \tag{20}$$

where we used the relation:

$$C^{(h)}_{d,M,\,x=\alpha} = \frac{1}{\alpha^2-1}\cdot\left[\frac{\alpha^2-1}{2\cdot\left(1+\nu_p\right)\cdot\alpha^2} + \frac{1}{1-\nu_p}\cdot\ln\alpha\right]. \tag{21}$$

2. 2. 2. The deformation state

The rotation of the median surface of the plate on the internal leaning has the expression:

$$\varphi^{(B)}_{M,x=1} = \frac{M_{02} \cdot r^2_{cr}}{\Re_p \cdot r_0} \cdot \left[C^{(h)}_{r,M,x=1} - 4 \cdot \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,P^\bullet,x=1} \right], \tag{22}$$

where the following notations are remarked:

$$C^{(a)}_{r,P^\bullet,x=1} = \frac{\left(1+\nu_p\right) \cdot \ln \alpha}{\left(1-\nu_p\right) \cdot \left(\alpha^2 - 1\right)} + \left(\frac{1}{1+\nu_p} + \frac{\alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \right) \cdot \frac{1}{\alpha^2}; \tag{23}$$

$$C^{(h)}_{r,M,x=1} = \frac{1}{\alpha \cdot \left(\alpha^2 - 1\right)} \cdot \left[\frac{1}{\left(1+\nu_p\right) \cdot \alpha} + \frac{1}{1-\nu_p} \right], \tag{24}$$

while the rotation on the external leaning is:

$$\varphi^{(B)}_{M,x=\alpha} = \frac{M_{02} \cdot r^2_{cr}}{\Re_p \cdot r_0} \cdot \left[C^{(h)}_{r,M,x=\alpha} - 4 \cdot \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,P^\bullet,x=\alpha} \right], \tag{25}$$

with the notation:

$$C^{(h)}_{r,M,x=\alpha} = \frac{2}{\alpha \cdot \left(\alpha^2 - 1\right) \cdot \left(1 - \nu_p^2\right)}. \tag{26}$$

The values of the points of the neutral surface of the annular plate can be calculated using the relation:

$$w^{(B)}_{M,x} = \frac{M_{02} \cdot r^2_{cr}}{\Re_p} \cdot \left[C^{(h)}_{d,M,x=\alpha} - C^{(h)}_{d,M,x} - \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{d,P^\bullet,x} \right], \tag{27}$$

where the following notation had been utilized:

$$C^{(h)}_{d,M,x} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{\alpha^2 - x^2}{2 \cdot \left(1+\nu_p\right) \cdot \alpha^2} + \frac{1}{1-\nu_p} \cdot \ln \frac{\alpha}{x} \right]. \tag{28}$$

2. 2. 3. The stresses state

The values of the radial stresses developed in the annular plate under the action of the M_{02} radial bending moment can be established taking into consideration the expression (taking into account the leaning type of the annular plate and the so-called charge):

$$\sigma^{(B)}_{r,M,x} = \frac{6 \cdot M_{02}}{\delta^2_p} \cdot \left[C^{(h)}_{r,\sigma,M} - 4 \cdot \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,\sigma,P^\bullet} \right], \tag{29}$$

with the notation:

$$C^{(h)}_{r,\sigma,M} = \frac{1}{\alpha^2 - 1} \cdot \left[\left(\frac{\alpha}{x} \right)^2 - 1 \right]. \tag{30}$$

The same logic for establish the expression of the annular stress lead to:

$$\sigma^{(B)}_{\theta,M,x} = \frac{6 \cdot M_{02}}{\delta_p^2} \cdot \left[C^{(h)}_{\theta,\sigma,M} - 4 \cdot \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{\theta,\sigma,P^\bullet} \right], \tag{31}$$

where the following notation is found:

$$C^{(h)}_{\theta,\sigma,M} = \frac{1}{\alpha^2 - 1} \cdot \left[\left(\frac{\alpha}{x} \right)^2 + 1 \right]. \tag{32}$$

Note: Where the analyzed annular plate is under the simultaneous action of the radial bending moments (fig. 1), the values of the rotations, linear deformations and the annular and radial stresses are evaluated using the superposition of the effects. The senses of action and the solicitation in elastic domain which characterize the construction materials are taken into consideration.

3. NOTATIONS

p – the uniform distributed pressure on the surface of the annular plate; r – the current radius of the plate; r_{cr} - the external radius of the plate; r_0 - the internal radius of the plate; x - geometric simplex ($x = r / r_0$; $x = 1$, for $r = r_0$; $x = \alpha$, for $r = r_{cr}$); C - influence factors (the inferior indexes show the loads which action, the place which is on the plate; E_p - the elasticity longitudinal modulus of the material of the plate; M - the unitary radial bending moment (M_{01}, M_{02} the moments which action on the external leaning, respectively, on the internal leaning of the plate– figure 1); $P^\bullet$ - the equivalent force, which is uniform distributed, on the external or the internal circumference of the plate; $\Re_p$ - the cylindrical rigidity of the plate ($\Re_p = E_p \cdot \delta_p^3 / \left[12 \cdot \left(1 - \nu_p^2 \right) \right]$; w - the linear deformation of the points of the median surface of the plate; δ_p - the thickness of the plate; φ - the rotation of the median surface of the plate; ν_p - the coefficient of the transversal contraction of the material of the plate; σ_r, σ_θ - the radial and annular stresses (the "r"inferior index represents the radial direction, and "θ" the annular direction (circumferential)).

4. CONCLUSIONS

The paper presents the expressions of the rotations and linear deformations of the points of the median surface of an annular plate, charged with unitary radial bending moments on the both their simple leanings, which are situated at the level of the internal, respectively external circumference. The plate has the same thickness on its entire surface. The solicitation is considered in the elastic domain. The calculus relations of the radial and annular stresses are determined, too. Helping these relations, the equivalent maximal stress is evaluated and this construction is validated, on the basis of an accepted resistance theory. The expressions of the indicated magnitudes are realized using distinct loads on the leanings of the plate and the superposition of the effects, given of the simultaneous action of the bending moments on the both bases of the plate is realized, by case. The passive leanings were had been simulated using the introduction of a fictive load which action lead to at the realization of a similar linear deformation with the linear deformation given by the active bending moment.

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