THE INFLUENCE OF THE ASYMMETRY COEFFICIENT WITH NEGATIVE VALUES (R<0) ON THE FATIGUE CRACK GROWTH RATE

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Abstract: The crack propagation speed is an important parameter used to predict the life time of a part or a mechanical structure. Hence, in the last 50 years, many researchers have tried to explain and to create a mathematic model able to describe the phenomenon of fatigue crack initiation and propagation. In our days, the influence of the asymmetry coefficient having negative values, R, on the crack growth rate is considered an important issue. This paper presents theoretical aspects considered as basics for the NASGROS 5.0 software and the obtained results.

Keywords: crack growth, fatigue, fracture, stress ratio.

1. INTRODUCTION

The theoretical research effort carried out to calculate the crack propagation speed for positive values of the asymmetry coefficient R, have offered results similar with the results obtained experimentally.

In case of negative values of the asymmetry coefficient, the phenomenon of crack growth rate becomes more complicated. Hence, for some materials the crack propagation, for negative values of the R coefficient, it happen similar with the case when the material is loaded with a pulsating cycle. However, for mostly all the materials, the difference is obvious, so it must be taken into account the influence of the compression residual stresses which cause a delay effect for the crack initiation. This effect cause variation of the principal parameters of the crack growth rate: the effective values of stress intensity factor, the effective value of the symmetry coefficient.

In this paper are presented the computational relations used by the NASGRO 5.0 software to study the influence of the asymmetry coefficient R with negative values and the results obtained by simulation.

2. THE CRACK GROWTH RATE

To calculate the crack growth rate have been used equations written by authors as: Paris, Head, Donahue, Forman, Walter, Erdogan, so on.

To describe all three regions of the sigmoid curve, $\frac{da}{dN} = f\left(\Delta K\right)$, good results are obtain using the equation resulted from the research work carried by Forman, Newman, Shivakumar, Koning şi Henriksen. This equation is used by the NASGRO 5.0 software for variable loads with constant amplitude:

$$\frac{da}{dN} = C \cdot \left[\frac{1-f}{1-R} \cdot \Delta K\right]^n \cdot \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1 - \frac{K_{\max}}{K_c}\right)^q} \tag{1}$$

in which:

a - the crack length;
c, n, p, q - constants of the material, determined by experimental tests;
f - the crack opening function;
R - the asymmetry coefficient of the loaded cycle;
$\Delta K = K_{max} - K_{min}$ the stress intensity factor;
K_{th} - the threshold value of the stress intensity factor;
K_c - the critical value of the stress intensity factor.

The crack opening function is calculated using the Newman equation:

$$f = \frac{K_{op}}{K_{max}} = \max\left(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3\right) \tag{2}$$

in case when $R \geq 0$
or:

$$f = \frac{K_{op}}{K_{max}} = A_0 + A_1 R \tag{3}$$

in case when $-2 \leq R < 0$ and is taken in consideration the influence of the asymmetry coefficient R. In equation 2 and 3, K_{op} is the value of the stress intensity factor for which the crack starts to open.

The coefficients A_0, A_1, A_2 and A_3 are calculated with the following relations:

$$A_0 = (0,825 - 0,34\alpha + 0,05\alpha^2)\left[\cos\left(\frac{\pi}{2} \cdot \frac{S_{max}}{\sigma_0}\right)\right]^{\frac{1}{\alpha}} \tag{4}$$

$$A_1 = \left(0,415 - 0,071\alpha\right)\frac{S_{max}}{\sigma_0} \tag{5}$$

$$A_2 = 1 - A_0 - A_1 - A_3 \tag{6}$$

$$A_3 = 2A_0 + A_1 - 1 \tag{7}$$

in which:

α - the restrain coefficient for the state of plane strain or plane stress;
S_{max} - the maxim value of the loaded stress;
σ_0 - the actual stress.

There are materials for which the crack propagation rate is not so much affected by the asymmetry coefficient R. In this case, the crack opening function is calculated with the following equations:

- for $0 \leq R < 1$; $f = R$.
- for $R < 0$; $f = 0$.

In these conditions, equation (1) becomes:

$$\frac{da}{dN} = \frac{C \cdot \Delta K^n \cdot \left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1 - \frac{K_{max}}{K_c}\right)^q}; \; 0 \leq R < 1 \tag{8}$$

$$\frac{da}{dN} = C \cdot \left(\frac{\Delta K}{1-R} \right)^n \cdot \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{\left(1 - \frac{K_{\max}}{K_c} \right)^q} ;\ R < 0 \qquad (9)$$

Taking in consideration that $\frac{\Delta K}{1-R} = K_{\max}$, equation (9) becomes:

$$\frac{da}{dN} = \frac{C \cdot K_{\max}^n \cdot \left(1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{\left(1 - \frac{K_{\max}}{K_c} \right)^q} \qquad (10)$$

The threshold value of the stress intensity factor it is calculated with the following equations:

$$\Delta K_{th} = \Delta K_1 \left(\frac{a}{a+a_0} \right)^{\frac{1}{2}} \cdot \left(\frac{1-R}{1-f(R)} \right)^{\left(1+R\cdot C_{th}^p\right)} \cdot \frac{1}{\left(1-A_0\right)^{(1-R)C_{th}^p}} \quad \text{for } R \geq 0 \qquad (11)$$

$$\Delta K_{th} = \Delta K_1 \left(\frac{a}{a+a_0} \right)^{\frac{1}{2}} \cdot \left(\frac{1-R}{1-f(R)} \right)^{\left(1+R\cdot C_{th}^m\right)} \cdot \frac{1}{\left(1-A_0\right)^{\left(C_{th}^p - R\cdot C_{th}^m\right)}} \quad \text{for } R < 0 \qquad (12)$$

in which:

a - the crack length;

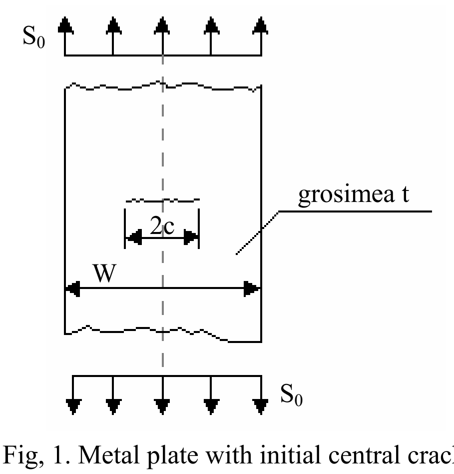
a_0 - parameter of the crack length (approximately 0,0381 *mm*);

C_{th} - constant parameter with different values for $R \geq 0$; $\left(C_{th}^p\right)$ and $R < 0$; $\left(C_{th}^m\right)$.

3. NUMERICAL SIMULATION

For the numerical simulation have been considered a metal plate with a central crack (Fig. 1), having the following parameters:

- plate wideness: $W = 50mm$
- plate thickness: $t = 2mm$
- initial crack length: $a = 0,5mm$



Fig, 1. Metal plate with initial central crack

The material used for the simulation is a hot rolled steel having the following mechanical properties:

- braking stress: $\sigma_r = 310{,}3\,N/mm^2$;
- upper yield stress: $\sigma_c = 172{,}4\,N/mm^2$
- braking tenacity: $K_c = 2432$
- the material coefficients from equation (1): $C = 0{,}5762$; $n = 3{,}6$; $p = 0{,}5$; $q = 0{,}5$.

The values of the σ tension, with constant amplitude, and the values of the asymmetry coefficient R, used for the numerical simulation, are presented in table 1.

Table 1. Numerical simulation inputs

No.	$\sigma_{min}\left[N/mm^2\right]$	$\sigma_{max}\left[N/mm^2\right]$	**R**
1	-170	230	-0,74
2	-180	220	-0,81
3	-190	210	-0,90
4	-200	200	-1,0
5	-210	190	-1,1

The results obtained with NASGRO 5.0 software are presented in the following figures:

- Fig. 2 illustrates the variation of the crack length function to the number of the cycles, $a - N$;
- Fig. 3 presents the variation of the crack growth rate function to the number of the cycles, $da/dN - N$;
- Fig. 4 presents the variation of the crack growth rate function to the crack length, $da/dN - a$.

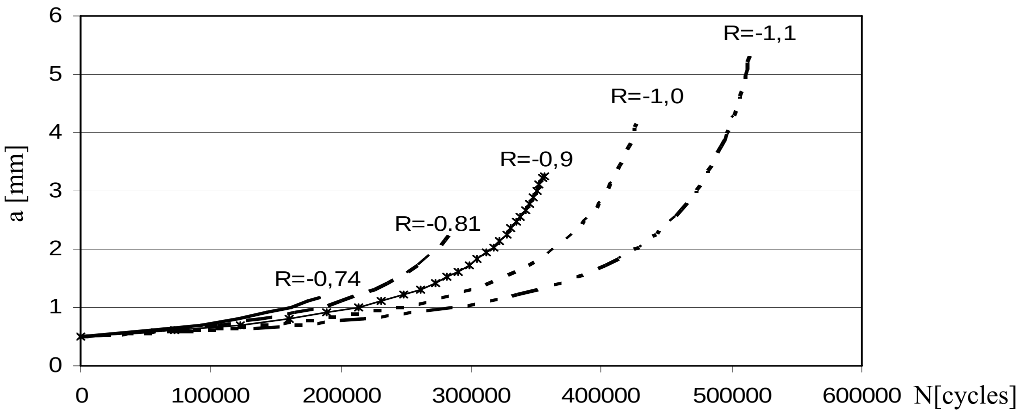


Fig. 2. Variation of the crack length function to the number of the cycles

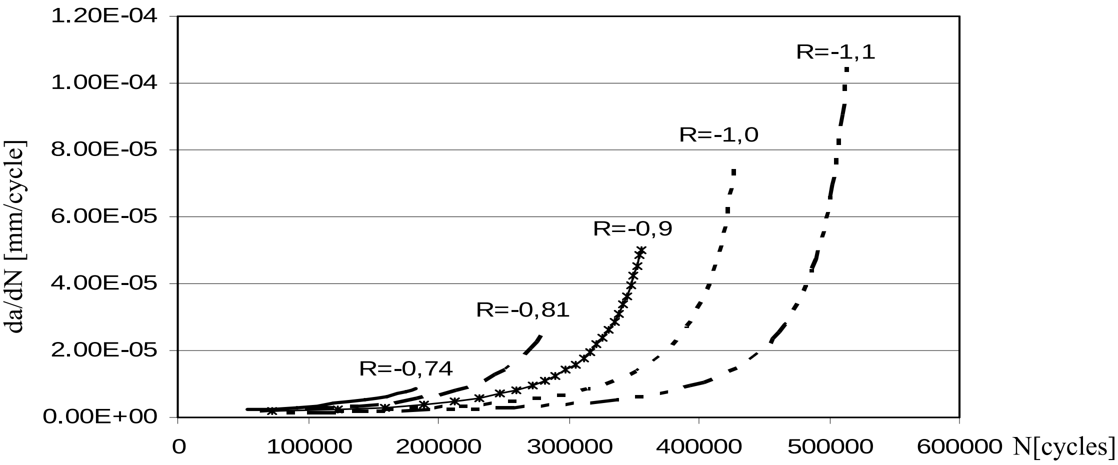


Fig. 3. Variation of the crack growth rate function to the number of the cycles

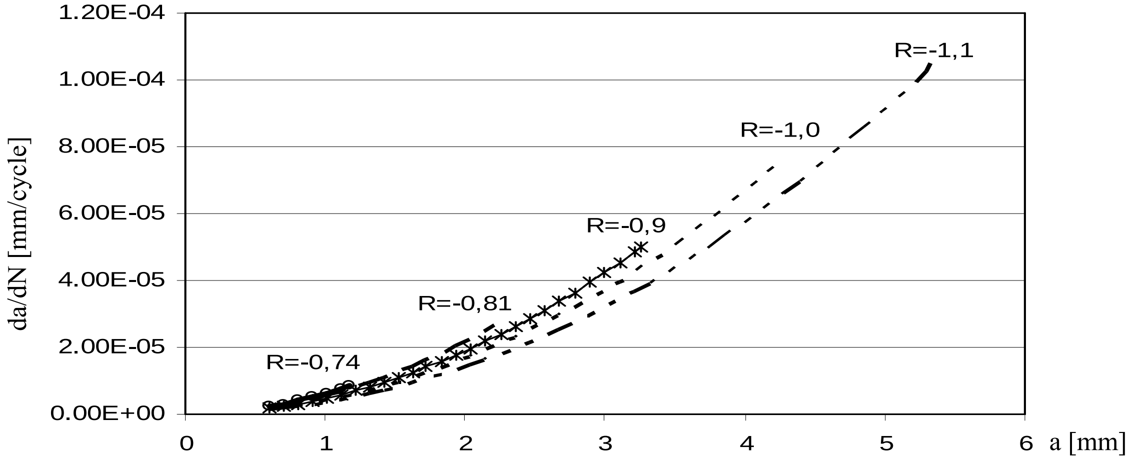


Fig. 4. Variation of the crack growth rate function to the crack length

4. CONCLUSIONS

Based on the results presented at the previous section, the following conclusions can be considered:

- for increasing values of the asymmetry coefficient a larger number of cycles is required to obtain a crack with the same length. Hence, ascending of the negative value of the applied stress determine an decreasing value of the crack growth rate.
- for the same number of cycles, the crack length grows proportionally with reduction of the asymmetry coefficient, therefore simultaneously with reduction of the stress negative values and increasing of the stress positive values.

It must be mentioned that for all simulation tests, the variation of the applied stress it was considered a constant value: $\Delta\sigma = \sigma_{max} - \sigma_{min} = ct.$

For two different simulation described by $\sigma_{max} = 230 N/mm^2$; $\sigma_{min} = -170 N/mm^2$ and $\sigma_{max} = 230 N/mm^2$; $\sigma_{min} = 0 N/mm^2$, having the asymmetry coefficient $R = -0{,}74$, respectively $R = 0$ (the maximal stress value is constant), the required number of cycles needed to obtain the same crack length $a = 1{,}17612 mm$, it is $N = 173.722$ respectively $N = 269.476$ cycles. From this observation results the stress influence in case of stresses with negative value.

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