THE RTR ROBOT PATH OF MOTION, WHEN THE TRANSLATION COUPLING LAW IS SINUSOIDAL

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Abstract: This paper studies the path of motion of the characteristic point of the RTR robot, when at the coupling level the working laws are replaced by the sinusoidal laws, used mathematical model of the quaternion.

Keywords: RTR robot, translations coupling, quaternion, end-effectors

1. INTRODUCTION

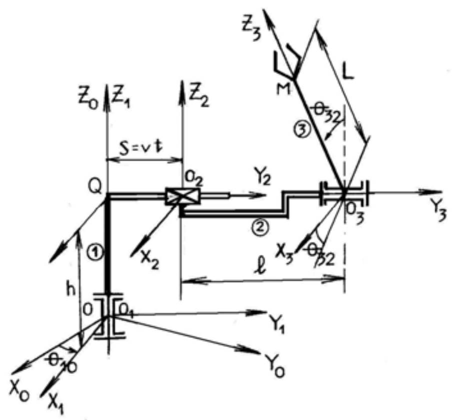


Fig. 1. The constructive design of the RTR robot

In figure 1 it is considerate the kinematics designs of a stationary RTR robot with base solid space with three degree of freedom. The robot it is composed from three tie-bar joint with cylindrically coupling so that the frame of robot represented the rod adapter for the rigid bars. At the final of the bars is fixed one end-effector's used for the manipulations of the objects. The end-effectors are marks with M [6], [10], [11].

The parametric equations of the M path of motion in $O_0X_0Y_0Z_0$ fix frame were calculated used the mathematical models of the quaternion:

$$\begin{aligned} r_{0M} &= [-(\ell + v\cdot t)\sin\omega_1 t + L\cos\omega_1 t\cdot\sin\omega_3]\cdot e_1 + \\ &+[(\ell + v\cdot t)\cos\omega_1 t + L\sin\omega_1 t\cdot\sin\omega_3 t]\cdot e_2 + \\ &+[h + L\cos\omega_3 t]\cdot e_3 \end{aligned}$$

A translation coupling may be manipulated in diverse ways; usually the most used coupling is the nut bolt mechanism and şeping mechanism.

Using the şeping mechanism [9] it is possible to obtain a finite working operation of the translation coupling. Figure 2 shows the more simple mechanism like that, thus de moving laws is sinusoidal: $s(t) = r\cdot\sin\omega\cdot t$, where the length of the stroke is equal with 2r. Results:

$$OA = s(t) = r\sin\omega t; \quad v_A = \dot{s}(t) = \omega r\cos\omega t;,$$

The bar stroke is equal with 2r, between $x_A = r$ and $x_A = -r$. Discussions:

$$v_{A\,t=0} = \omega\cdot r\ (\text{max imă}); \quad v_{A\,t=\pi/2\omega} = 0(\text{nulă})$$

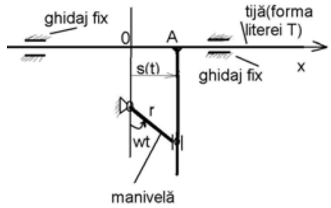


Fig. 2. Şeping mechanism

In technical practice for one translation coupling a moving law like $s(t) = v\cdot t + s_0$ isn't recommended because s(t) it is increasing (when $v > 0$), and the work space will be too much; in present day the working space must be smaller.

2. THE REPLACEMENT OF THE TRANSLATION COUPLING WITH THE SINUSOIDAL LAW

It is used the mathematical method of the quaternion by modifying the translation quaternion r_{21}.

$$\begin{array}{l} r_{10} = 0, \\ r_{21} = h \cdot e_3 + R \cdot \sin\Omega \cdot t \cdot e_2, \\ r_{32} = \ell \cdot e_2, \\ O_3M = r_{43} = L \cdot e_3 \end{array} \qquad \begin{array}{l} q_{10} = C_1 e_0 - S_1 e_3 \\ q_{21} = e_0 \\ q_{32} = C_3 e_0 - S_3 e_2 \end{array} ; \qquad \begin{array}{l} q_{20} = q_{10} = C_1 e_0 - S_1 e_3 \\ q_{30} = C_1 C_3 e_3 + S_1 S_3 e_1 - C_1 S_3 e_2 - S_1 C_3 e_3 \end{array} ;$$

Where: $n_0 = C_1 C_3,\ n_1 = S_1 S_3,\ n_2 = C_1 S_3, n_3 = S_1 C_3$. q_{30} became:

$$q_{30} = n_0 e_0 + n_1 e_1 - n_2 e_2 - n_3 e_3$$

The expression of the path of motion of the M point will be [3], [4]:

$$\begin{array}{l} r_{OM} = q_{10} + q_{10}^{-1} * r_{21} * q_{10} + q_{20}^{-1} * r_{32} * q_{20} + q_{30}^{-1} * r_{43} * q_{30} \Rightarrow \\ \Rightarrow r_{OM} = q_{10}^{-1} * (r_{21} + r_{32}) * q_{10} + q_{30}^{-1} * r_{43} * q_{30} \end{array} \qquad (1)$$

Using (23), (24), (25), (26), (27) for the double producing after that it will be identify *p* with q_{10} results: $p_0 = C_1,\ p_1 = p_2 = 0,\ p_3 = -S_1$.

Using (26) and (27) results:

$$q_{10}^{-1} * e_2 * q_{10} = -2S_1C_1e_1 + (C_1^2 - S_1^2)e_2 = -\sin\omega_1 t \cdot e_1 + \cos\omega_1 t \cdot e_2 \qquad (2)$$

$$q_{10}^{-1} * e_3 * q_{10} = (C_1^2 + S_1^2)e_3 = e_3 \qquad (3)$$

Because $r_{21} + r_{32} = (\ell + R\sin\Omega t)e_2 + h \cdot e_3$ (4)

$$q_{10}^{-1} * (r_{21} + r_{32}) * q_{10} = (\ell + R\sin\Omega \cdot t)(-\sin\omega_1 t \cdot e_1 + \cos\omega_1 t \cdot e_2) + h \cdot e_3 \qquad (5)$$

For calculated the second term of the (1) equation identify:

$$p_0 = n_0,\ p_2 = -n_2, p_1 = n_1,\ p_3 = -n_3. \qquad (6)$$

Because $r_{43} = L \cdot e_3$ (7)

results: $p^{-1} * e_3 * p$:

$$2(-p_0p_2 + p_1p_3) = 2(n_0n_2 - n_1n_3) = 2(C_1^2 \cdot C_3 \cdot S_3 - S_1^2 \cdot S_3 \cdot C_3) = (C_1^2 \cdot S_1^2) \cdot 2S_3 \cdot C_3 = \cos\omega_1 t \cdot \sin\omega_3 t \qquad (8)$$

$$2(p_0p_1 + p_2p_3) = 2(n_0n_2 + n_2n_3) = 2(S_1C_1 \cdot S_3C_3 + S_1C_1 \cdot S_3C_3) = 2S_1 \cdot C_3 = \sin\omega_1 t \cdot \sin\omega_3 t \qquad (9)$$

$$p_0^2 - p_1^2 - p_2^2 + p_3^2 = n_0^2 - n_1^2 - n_2^2 + n_3^2 = C_3^2(C_1^2 + S_1^2) - S_3^2(S_1^2 + C_1^2) = C_3^2 - S_3^2 = \cos\omega_3 t \qquad (10)$$

Using the equations (8), (9), (10), [8], results:

$$\begin{array}{l} q_{30}^{-1} * r_{43} * q_{30} = q_{30}^{-1} * (Le_3) * q_{30} = Lp^{-1} * e_3 * p = L\cos\omega_1 t \cdot \sin\omega_3 t \cdot e_1 + \\ \quad + L\sin\omega_1 t \cdot \sin\omega_3 t \cdot e_2 + L\cos\omega_3 t \cdot e_3 \end{array} \qquad (11)$$

$$\begin{array}{l} r_{OM} = [L\cos\omega_1 t \cdot \sin\omega_3 t - (\ell + R\sin\Omega t)\sin\omega_1 t]e_1 + \\ \quad + [L\sin\omega_1 t \cdot \sin\omega_3 t + (\ell + R\sin\Omega t)\cos\omega_1 t]e_2 + (L\cos\omega_3 t + h)e_3 \end{array} \qquad (12)$$

Equation number (12) gives us the path of motion of the M point which is:

$$\begin{array}{l} x_{0M} = L\cos\omega_1 t \cdot \sin\omega_3 t - (\ell + R\sin\Omega t)\sin\omega_1 t \\ y_{0M} = L\sin\omega_1 t \cdot \sin\omega_3 t + (\ell + R\sin\Omega t)\cos\omega_1 t \\ z_{0M} = L\cos\omega_3 t + h \end{array} \qquad (13), (14), (15)$$

In equations (13), (14), (15) it is neglected the coordinate indicators, it is squaring and it is adding the equation (13) with (14). In equation (15) *h*, will be moving in left member, it is squaring and results:

$$\begin{array}{l} x^2 + y^2 = (L\sin\omega_3 t)^2 + (\ell + R\sin\Omega t)^2 \\ (z - h)^2 = (L\cos\omega_3 t)^2 \end{array} \qquad (16), (17)$$

From (16) and (17) result: $x^2 + y^2 + (z - h)^2 = L^2 + (\ell + R\sin\Omega t)^2$ (18)

The geometrical interpretation for the equation number (18) it is that working space of the RTR robot it is functions in “spherical upper crust” [5], folding between two concentring sphere with semi diameters:

$$r_{min} = \sqrt{L^2 + (\ell - R)^2} \tag{19}$$

$$R_{max} = \sqrt{L^2 + (\ell + R)^2} \tag{20}$$

The inner space of the two spheres represented the spherical field where will be marks the characteristic point path of motion. That will be realized with helper of the 3D graphics. For the geometrical interpretation it will be used the MathCAD program.

$$h := 2 \qquad R := 5 \qquad b := \frac{\pi}{2\cdot\Omega} \qquad c := \frac{3\cdot\pi}{2\cdot\Omega} \qquad \omega 3 := \frac{\pi}{4} \qquad \omega 1 := \frac{\pi}{2} \qquad t := 0, 1 .. 5$$

$$L := 3 \qquad \Omega := \pi$$

$$l := 1 \qquad b = 1.571 S \qquad c = 4.712 S$$

$$t := 0, 0.01 .. 5 \qquad R1 := \sqrt{L^2 + (1 - R)^2} \qquad R2 := \sqrt{L^2 + (1 + R)^2}$$

$$x(t) := L\cdot\cos(\omega 1\cdot t)\cdot\sin(\omega 3\cdot t) - (1 + R\cdot\sin(\Omega\cdot t))\cdot\sin(\omega 1\cdot t)$$

$$y(t) := L\cdot\sin(\omega 1\cdot t)\cdot\sin(\omega 3\cdot t) + (1 + R\cdot\sin(\Omega\cdot t))\cdot\cos(\omega 1\cdot t)$$

$$z(t) := L\cdot\cos(\omega 3\cdot t) + h$$

$$x1(u,v) := R1\cdot\sin(v)\cdot\cos(u) \qquad x2(u,v) := R2\cdot\sin(v)\cdot\cos(u)$$

$$y1(u,v) := R1\cdot\sin(v)\cdot\sin(u) \qquad y2(u,v) := R2\cdot\sin(v)\cdot\sin(u) \qquad A(t) := \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \quad A(0) = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} \quad A(1) = \begin{pmatrix} -1 \\ 2.121 \\ 4.121 \end{pmatrix}$$

$$z1(u,v) := R1\cdot\cos(v) + h \qquad z2(u,v) := R2\cdot\cos(v) + h$$

$$D := \mathrm{CreateSpace}(A, 0, 5, 600) \qquad B := \mathrm{CreateMesh}(x1, y1, z1, 45)$$

$$C := \mathrm{CreateMesh}(x2, y2, z2, 40)$$

3. THE REPRESENTATION OF THE PATH OF MOTION IN SPHERICAL FIELD

Using the MathCAD program the path of motion of the characteristic point M is shows in figure 3.

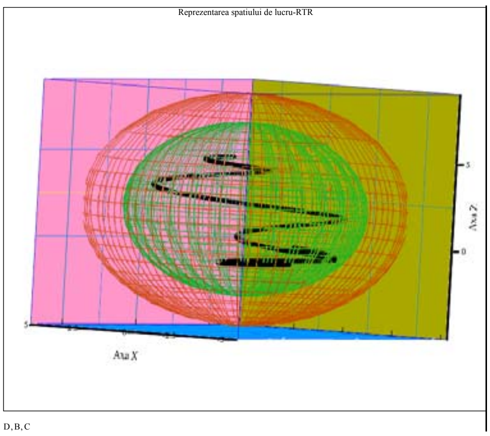


Fig. 3. The solid representation of the spherical field for an RTR robot

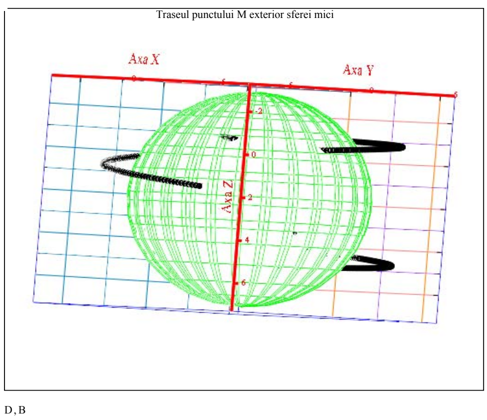
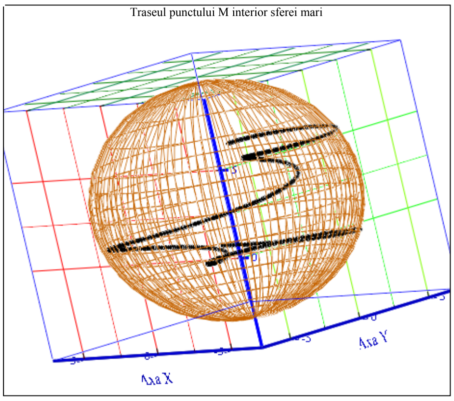


Fig. 4. The path of motion for the M point outside the small sphere

Figure 3 shows the path of motion for the characteristic point *M*, in solid space, when the translations law are replaced with sinusoidal law: the black colour represents the path of motion in inner space; the red colour represents the big sphere; the green colour represents the small sphere. The space is defined by two concentring spheres, the origin of the spheres are placed on Oz axe, in point Q(0,0,h). In function of the robot using, suck example in medical filed and the work jobs the dimensional space of work will be modifying using the convenient constructive dates: *h, l, L*.
In figure 4 is shows the representative path of motion for the point named *M*, for o better observations at the unfolding in space work, without covering the superior limit of the "spherical upper crust".



D,C
Fig. 5. The path of motion of the point M, inner sphere with big semi diameter

In figure 5 is represented the path of motion for point *M*, in inner space of the bigger sphere, from which are excluded the smaller sphere.

3. CONCLUSIONS

■ If it is necessary the bigger work space, like medicine field (the introduction of the boiling tubes at the test tube centrifuge, the perforation in hide fibrous tissue shallow), it is limited the minimum semi diameter and it is scale up the maximum semi diameter of the spheres.

■ When the working space must be superficial (the penetration of the tissue when it is used in operations) it is necessary to scale up the minimum semi diameter and limited the maximum semi diameter, until to require a certain wall thickness to the "spherical upper crust", namely

$$r_{max} - r_{min} = \Delta r\,. \qquad (22)$$

Because of the time periodicity of the *M* coordinate it is sufficient to studies the movies in definite time equal to T period, which is analytically defined.

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