THE LINEAR AND ANGULAR LOGARITHMIC STRAIN AND THEIR UTILIZATION IN THE CONTINUUM MEDIA PROBLEMS

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Abstract: The focus of this paper is the presentation of linear and angular components of logarithmic strain tensor determination that are useful in the description of large plastic deformation. The definition of logarithmic strain is obtained from law of energy transformation and conservation. The linear and angular components of logarithmic strain tensor are compared with the conventional strain. The differences between the conventional and logarithmic (true or natural strain) are presented in some obtained diagrams. The components of logarithmic strain are useful in the experimental analysis of strain field in the cold forming processes, in the mechanical characteristics of the materials determination, and other application in the mechanics of continuum media.

Key words: linear logarithmic strain, angular logarithmic strain, displacements.

Nomenclature: α – change of a rectangular angle; ε_x, ε_y, ε_z – linear logarithmic strain; ε_{xy},= ε_{yx} , ε_{yz} = ε_{zy}, $\varepsilon_{zx} = \varepsilon_{xz}$ – angular logarithmic strain.

1. INTRODUCTION

The utilization of logarithmic strain for the description of large plastic deformation, was first introduced by Ludwig [1…8]. Usually, the logarithmic strain is determined for principal direction of the strain. If the principal directions of the strain are unknown, first are determined the specific strain (Lagrangean strain), then principal direction and finally are determined the correspondingly linear logarithmic strain. The logarithmic strains may be represented as components of logarithmic strain tensor in respect with principal directions. But, as any tensor, the tensor of logarithmic strain should submit to the law of tensor transformations in respect with the change of coordinate system. In other words, the tensor of logarithmic strain should have a correspondingly shearing components.

To analyse the difference between the conventional (engineering) strain and logarithmic strain, let see the figure 1 and try to find some characteristics of stress and strain measurement. Consider a bar of length l_0 submitted to some axial forces. If at a moment we apply an incremental force *dF* which produces an differential change in length *dl* of momentary length *l*, the work done by the force *dF* well be transformed in to elastic potential energy of a stressed bar. The work *dW* done by the incremental force *dF* may be written as:

$$dw = \frac{dW}{A.l} = 1/2.\frac{dF}{A}.\frac{dl}{l}.$$

If the volume of the bar is *V= A.l*, we can calculate the unit potential energy *w* from:

$$dW = 1/2.dF.dl.$$

If the applied force varies from zero to *F*, the length changes from l_0 to *l* and the cross sectional area *A* is considered constant, the unit potential energy *w* may be written:

$$w = 0,5.\frac{1}{A}\int_0^F dF.\int_{l_0}^{l}\frac{dl}{l} = 0,5.\frac{F}{A}.ln\frac{l}{l_0}.$$

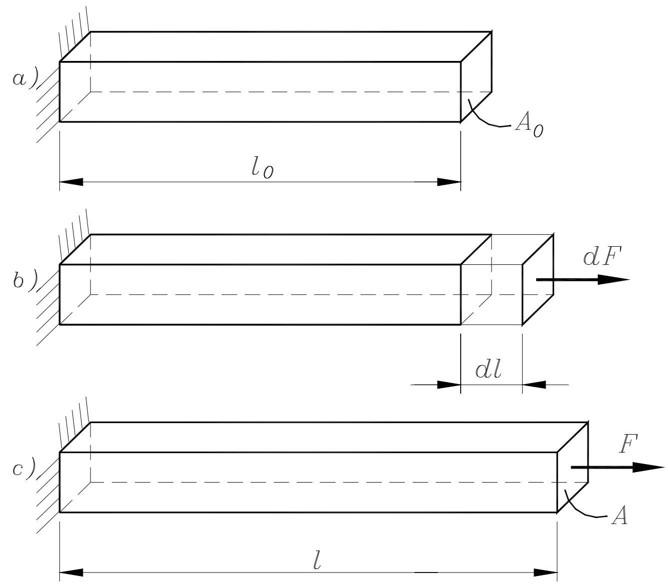

Fig.1. A bar submitted to axial force.

In the above relationship, obtained from law of energy transformation and conservation, $F/A=\sigma$ is the true stress and $ln(l/l_0)=\varepsilon$ is the true or linear logarithmic strain. Accordingly to the Marina [9], the definition of logarithmic strain may be obtained from first principle of thermodynamics.

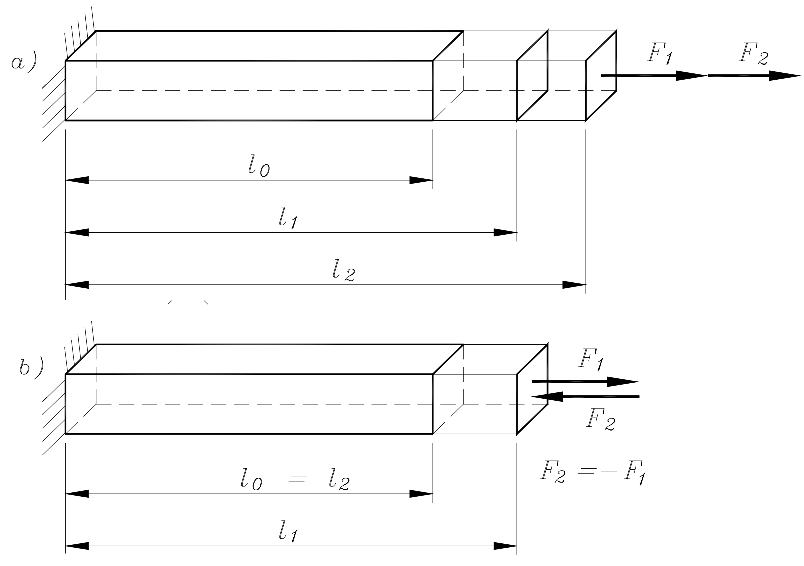

Fig.2. A bar submitted to some axial forces.

If place of logarithmic strain is used conventional (or engineering) strain, are obtained some incorrect results. For example, if we apply two forces F_1 and F_2 (figure 2a) thou that initial length changes from l_0 to l_1 and then to l_2, we obtain overall stress $\sigma_t=\sigma_1+\sigma_2$, but the overall conventional strain ε_t is not equal with sum $\varepsilon_1+\varepsilon_2$. If logarithmic strain is used, we can write $\varepsilon_t=\varepsilon_1+\varepsilon_2$ i.e. overall strain is equal with the sum of partial strains. In another example, if we apply two forces F_1 and F_2 thou that $F_2= -F_1$ (figure 2b), that is a neutral force system, the overall stress is equal with zero but the overall conventional strain is not equal with zero. If logarithmic strain is used, the overall strain will be equal with zero too. The above situation is presented succinct in table 1.

Table 1. The correspondence between the stress and strain state.

Case	Stress state	Conventional strain	Logarithmic strain
$F_1+F_2\neq 0$ (figure2a).	$\sigma_t=\sigma_1+\sigma_2$	$\varepsilon_t\neq\varepsilon_1+\varepsilon_2$	$\varepsilon_t=\varepsilon_1+\varepsilon_2$
$F_1+F_2=0$ (figure 2b).	$\sigma_t=0$	$\varepsilon_t\neq 0$	$\varepsilon_t=0$

Thous consideration shows that the linear logarithmic strains are more suited for strain description in the mechanics of deformable media. In order to make possible a complete application of logarithmic strain, results the necessity of angular logarithmic strain determination. In this way, the tensor of logarithmic strain may be written for any situation, not only for principal direction.

2. THE ANGULAR LOGARITHMIC STRAIN DETERMINATION

It is well known, that if a square plate element (figure 3.a) is submitted to a tensile stress σ in a Ox direction and to a compression stress –σ in a Oy direction, it is in the planar pure share stress state, and $\tau = [\sigma - (-\sigma)] / 2 = \sigma$.

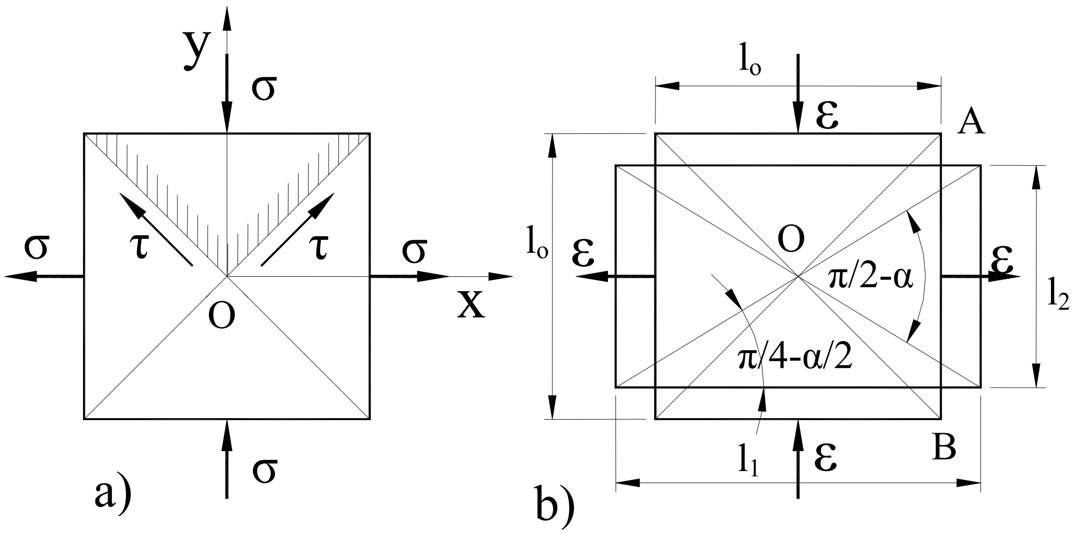

Fig. 3. The pure share stress and pure share strain.

In similar way, if the direction Ox and Oy are principal directions and the correspondingly logarithmic strain are $\varepsilon_x = \varepsilon$, $\varepsilon_y = -\varepsilon$ and $\varepsilon_z = 0$, we have pure planar share strain (figure 3b), and the angular logarithmic strain is numerically equal with the linear logarithmic strain. From definition of logarithmic strain, we have:

$$\ln\frac{l_1}{l_o} = \varepsilon \;\; or \;\; l_1 = l_o.e^{\varepsilon}, \;\; and \;\; \ln\frac{l_2}{l_o} = -\varepsilon \;\; or \;\; l_2 = l_o.e^{-\varepsilon} \;\; , \tag{1}$$

e being the natural logarithms basis.

If the rectangle AOB changes with ***α***, from geometrical considerations (figure 3b), we have:

$$tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)=\frac{l_2}{l_1}=\frac{e^{-\varepsilon}}{e^{\varepsilon}} \; .$$

The above expression may be written as:

$$e^{2\varepsilon} = \frac{1+tg(\alpha/2)}{1-tg(\alpha/2)} \quad ,$$

or

$$\varepsilon = \frac{1}{2}\ln\frac{1+tg(\alpha/2)}{1-tg(\alpha/2)} = \varepsilon_{xy} = \varepsilon_{yx}. \tag{2}$$

In the relationship (2), $\varepsilon_{xy} = \varepsilon_{yx}$ are the angular logarithmic strain numerically equal with linear logarithmic strain ε. The relationship (2) may be used for the determination of angular logarithmic strain knowing the change of right angle of some concurrent elements. The linear and angular logarithmic strain may be expressed as a function of displacements [10…14], resulting relationships that may be used in experimental determination of strain field.

Observation: If in place of logarithmic strain, in figure 3b, is used conventional strain $\varepsilon = \Delta l/lo$ than for angular strain will be obtained expression $\varepsilon_{xy} = \varepsilon_{yx} = tg(\alpha/2)$

3. A COMPARISON BETWEEN THE LOGARITHMIC AND CONVENTIONAL STRAIN

The logarithmic strain are preferable in the large plastic deformations like sheet metal forming because it seems that it described better the change of shape. If, instead of logarithmic strain are used conventional strain, are expected some differences in the values of determined strain. In table 2 are presented some numerical values of angular logarithmic strain and *tg(α/2)* corresponding to the conventional *α/2* strain.

Table 2. Numerical value for angular logarithmic strains.

α/2	0,0	0,01	0,02	0,05	0,1	0,2	0,3	0,4	0,5
tg(α/2)	0,0	0,0100006	0,020003	0,05004	0,1003	0,2027	0,3093	0,4228	0.5463
rel. (2)	0,0	0,010001	0,020006	0,05008	0,1006	0,2055	0,3197	0,4511	0,6131

The difference between the linear logarithmic strain and conventional linear strain is presented in figure 4, and difference between the angular logarithmic strain and half of change of rectangle *α/2* is presented in figure 5. In figure 5 is presented also the difference between *tg(α/2)*, which corresponds to the conventional linear strain, and angle *α/2* expressed in radians.

Fig. 4. The difference Δε between the linear logarithmic strain and conventional strain.

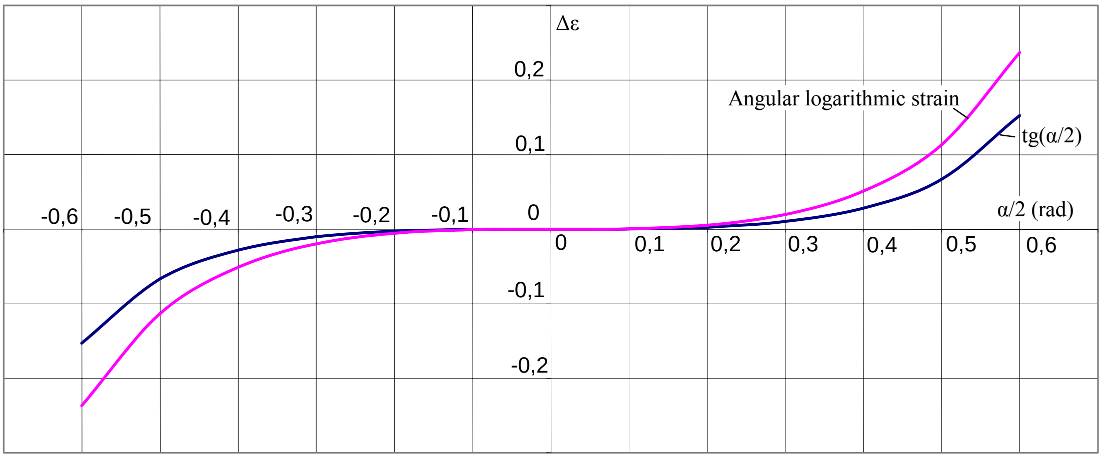

Fig. 5. The difference between the angular logarithmic strain and and *tg(α/2)* in respect with *α/2.*

4. CONCLUSIONS

The relationship for calculation of angular (1) logarithmic strain is proposed. The determined relationships are more suited for the large strain description compared with the conventional strain tensor. Like for any tensor, may be calculated the eigenvalues and its invariants. Also the tensor of logarithmic strain may be represented as a sum of a spherical tensor and the strain deviator. The expressed in term of displacements, the established relationship for logarithmic strain may be used in the experimental determination of the strain fields [15-18].

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