

MECHANICAL SIMULATION OF THE HEAVY PLATES COLD – ROLL – BENDING

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Abstract In the elementary bending theory it is usual to admit the hypothesis of small strains, small enough to neglect the transversal stresses induced by severe curvature. It is also admitted that the neutral surface coincides, during deformation, with the central plane of the plate. To carry out a mechanical modeling of heavy plates bending, for the big diameter tube, it is necessary a general theory of plate bending, without restrictions regarding the magnitude of strains and curvature; it is also necessary to determine the neutral surface movement and the movement of each fiber across the plate thickness. The paper presents a model of strains and stresses calculus for both a rigid-plastic material and for a hardenable material. An important issue of these calculi is the width determination of the zone where the material suffer, during bending, both an elongation and a compression and so, the Bauschinger effect has an important influence on the mechanical proprieties of tubes. In this zone the material strength after bending is less than the plate strength, the strength diminishing being proportional to the width of mentioned zone.

Keywords: plate, cold forming, simulation, bending

Notations:

a – internal radius;
 b – external radius;
 h – plate thickness;
 σ_r -radial stress;
 σ_θ -tangential stress;
 c -the neutral surface radius;
 r – the bending current radius;
 k –the yielding strength;
 σ_c – yield stress by uniaxial tension;
 M - the bending moment;
 α -the bending angle;
 u - the internal radial component of displacement vector;
 v -the tangential component of the displacement vector;
 L_0 -initial length of the plate;
 θ - the angle of the radius and the symmetry plan;
 ε_r – the radial strain;
 ε_θ – the tangential strain;
 $\gamma_{r\theta}$ –the angular strain;
 m –coefficient which described the initial position of the fiber with L_0 length;
 r_0 –the bending radius correspondent to L_0 ;

T -tensile force (on width unit) which acts at the plate extremities, being normal to the extreme transversal sections;

q -uniform pressure applied on the internal surface of the plate;

s - the ratio $T/(2kh)$;

$\bar{\sigma}$ -the average stress;

$\bar{\epsilon}$ - the average strain.

1. STRESSES AND STRAINS DETERMINATION IN A RIGID – PERFECT PLASTIC MATERIAL SUBJECTED TO BENDING

I shall consider the bending produced by the moments applied at plate extremities, the state being a plane strain one; the material behaves rigid-plastic, without hardening.

The main stresses in the bending are radial and tangential oriented, as a result of the deformation symmetry, and I shall designate them by σ_r , σ_θ respectively.

The equilibrium equation can be written in the radial direction as:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_r - \sigma_\theta}{r} \quad (1.1)$$

I shall call by “c” the neutral surface radius, i.e. the radius of the cylindrical surface – including those fibers that do not modify their lengths when infinitesimal supplementary deformation takes place.

The fibers placed between the neutral surface and the external surface of the plate are subjected to elongation and those placed between the neutral surface and internal surface are subjected to compression.

The yielding condition for the plane strain state is:

$$\begin{aligned} \sigma_\theta - \sigma_r &= 2k \quad \text{for } c \leq r \leq b \\ \sigma_\theta - \sigma_r &= -2k \quad \text{for } a \leq r \leq c \end{aligned} \quad (1.2)$$

where:

$k = \frac{\sigma_c}{2}$ in the case of Tresca criterion, and

$k = \frac{\sigma_c}{\sqrt{3}}$, in the case of Mises criterion, [1]

Having in view that $\sigma_r = 0$ for $r = a, b$, from (1.1) and (1.2) one can obtain:

$$\begin{aligned} \sigma_r &= 2k \ln \frac{r}{b} \quad \text{for } c \leq r \leq b, \\ \sigma_r &= 2k \ln \frac{a}{r} \quad \text{for } a \leq r \leq c \end{aligned} \quad (1.3)$$

As the equilibrium condition required σ_r to be continuous across the neutral surface, one can obtain:

$$2k \ln \frac{c}{b} = 2k \ln \frac{a}{c}$$

Thus, the neutral surface radius is given by

$$c = \sqrt{ab} \quad (1.4)$$

The other main stress component may be obtained from (1.2) and (1.3), as:

$$\sigma_\theta = 2k \left(\ln \frac{r}{b} + 1 \right) \quad \text{if } c \leq r \leq b$$

$$\sigma_{\theta} = 2k \left(\ln \frac{a}{r} - 1 \right) \quad \text{if} \quad a \leq r \leq c \quad (1.5)$$

The variation of σ_r and σ_{θ} across the thickness of the plate is given in Fig.1. It can be noticed that σ_r attains a maximum value on the neutral surface and the resultant force acting upon a section is given by:

$$\int_a^b \sigma_{\theta} dr = \int_a^b \frac{d}{dr} (r\sigma_r) dr = r\sigma_r \Big|_a^b = 0$$

where the equilibrium equation (1.1) is used.

The bending moment, corresponding to a width unit, is obtained as:

$$M = \int_b^a \sigma_{\theta} r dr = kc^2 \ln \frac{ab}{c^2} + \frac{k}{2} (a^2 + b^2 - 2c^2)$$

and, using (1.4), on can write:

$$M = \frac{1}{2} k (b - a)^2 = \frac{1}{2} kh^2 \quad (1.6)$$

Let $u d\alpha$ be the radial component, and $v d\alpha$ the tangential component of the displacement vector due to an infinitesimal strain; the bending angle α , calculated for the initial length L_0 , increases by $d\alpha$.

Neglecting the elastic compressibility and taking into account that the associated deformation is an elongation for $r > c$, and a compression for $r < c$, we can consider the following expressions [3]:

$$u = \frac{1}{2\alpha} \left(r + \frac{c^2}{r} \right), \quad v = \frac{r\theta}{\alpha} \quad (1.7)$$

where θ is the angle between the radius and the symmetry plan. Than, the corresponding increments of strains have the following components:

$$d\varepsilon_{\theta} = -d\varepsilon_r = \frac{1}{2\alpha} \left(1 - \frac{c^2}{r^2} \right) d\alpha, \quad d\gamma_{r\theta} = 0 \quad (1.8)$$

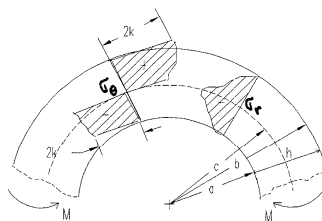


Fig.1. σ_r and σ_{θ} distribution in a plate subjected to bending without hardening

obtained from the relations [1]:

$$d\varepsilon_\theta = \frac{1}{r} \frac{\partial}{\partial \theta} (du_\theta) + \frac{du_r}{r}, \quad d\varepsilon_r = \frac{\partial}{\partial r} (du_r)$$

$$d\gamma_{r\theta} = \frac{1}{r} \frac{\partial}{\partial \theta} (du_r) + \frac{\partial}{\partial r} (du_\theta) - \frac{du_\theta}{r}$$

Where using (1.7) relations, we get

$$(du_r)_{r=a} = -\frac{1}{2\alpha} \left(a + \frac{c^2}{a} \right) d\alpha = -\frac{1}{2\alpha} (a+b) d\alpha$$

$$(du_r)_{r=b} = -\frac{1}{2\alpha} \left(b + \frac{c^2}{b} \right) d\alpha = -\frac{1}{2\alpha} (b+a) d\alpha$$

And

$$dh = (du_r)_{r=b} - (du_r)_{r=a} = 0$$

As a result, one can consider h being constant in every configuration, for a non-hardenable material; thus, according to (1.6), the moment of plastically bending does not depend on plastically deformation, for such a material.

From the equality of the final area and the initial area:

$$L_0 h = \frac{1}{2} (b^2 - a^2) L_0 \alpha$$

the following relation is obtained (for the internal, external radii and the bending angle)

$$\alpha = \frac{2}{a+b} \quad (1.9)$$

The mechanical work, corresponding to the unit of width, is:

$$ML_0 \alpha = 2k \frac{h^2}{4} \cdot \frac{2L_0}{a+b} = k \frac{b-a}{b+a} \quad (1.10)$$

In spite of the constant thickness of the plate, the fibers are subjected to complex deformations. In (1.8), one can notice that the fibers having $r < c$ are compressed, and those having $r > c$ are subjected to an elongation.

If we consider a fiber in a non – bended plate, at a distance of $\frac{mh}{2}$ to the central plane, the following relation can be written, taking into account the equality of areas before and after bending:

$$\frac{1+m}{1-m} = \frac{r^2 - a^2}{b^2 - r^2}$$

or

$$r = \sqrt{\frac{1}{2} (a^2 + b^2) + \frac{1}{2} (b^2 - a^2) m} \quad (1.11)$$

where $|m| \leq 1$ and $m > 0$ for a fiber placed between the central plane and the convex part of the plate.

As an example, the final radius of the central fiber, in a non-deformed state, which corresponds to $m = 0$ (see Fig. 1.2), is given by:

$$r_f = \sqrt{\frac{1}{2}(a^2 + b^2)} > \frac{a+b}{2}$$

Considering the formula (1.11) and $r = c$, one can notice that the fiber which, in the final configuration, coincides with the neutral surface, has “ m ” given by:

$$m = \frac{a-b}{a+b}$$

Thus, the neutral surface, which before the bending has coincided with the central plane, is moving towards the plate inner plane during the plastically bending.

All the fibers having $m \geq 0$ are subjected to elongation and the fibers for which $r < c$ are subjected to compression; the fibers for which:

$$0 > m > \frac{a-b}{a+b}$$

$$c < r < \sqrt{\frac{1}{2}(a^2 + b^2)}, \quad (1.12)$$

have been surpassed by the neutral surface, so the fibers being subjected firstly to compression and afterwards to an elongation. In this zone, the Bauschinger effect can have a remarkable importance.

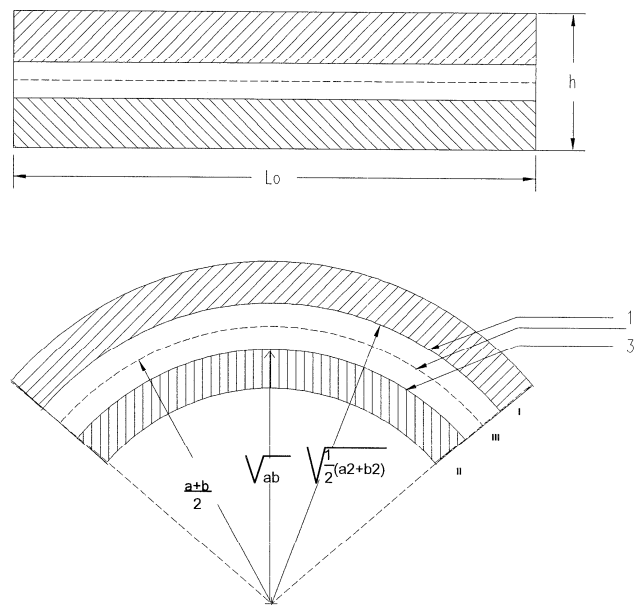


Fig.2. The relative movement of longitudinal fibers, during the bending:
1 – The central fiber (initial; 2 – The fiber having L_0 length; 3 – The neutral surface.

Every moment, there is a fiber subjected to a compression followed by an elongation; so, the present length equals L_0 (the initial length). The radius of this fiber is obtained using (1.9) and is given by

$$r_0 = \frac{1}{\alpha} = \frac{a+b}{2} = a + \frac{h}{2} \quad (1.13)$$

and its initial position-obtained using (1.11) – is:

$$m = \frac{1}{2} \left(\frac{a-b}{a+b} \right)$$

Now, we shall consider that tensile forces T (on width unit) are acting at the plate extremities, being normal to the extreme transversal sections.

The resultant forces must be balanced by an uniform pressure, q , applied on the internal surface of the plate; from the equality of the internal components we can write:

$$T = aq \quad (1.14)$$

The stress components are given, in this case, by:

$$\sigma_r = 2k \ln \frac{r}{b}$$

for

$$c \leq r \leq b,$$

$$\sigma_\theta = 2k \left(\ln \frac{r}{b} + 1 \right)$$

$$\sigma_r = 2k \ln \frac{a}{r} - q$$

for

$$a \leq r \leq c \quad (1.15)$$

$$\sigma_\theta = 2k \left(\ln \frac{r}{b} - 1 \right) - q$$

Taking into account the continuity of σ_r across the neutral surface, it is obtained:

$$c = \sqrt{abe^{-\frac{q}{2k}}} \quad (1.16)$$

So, the neutral surface is moving more, in this case, towards the plate inner planes. Further, the displacements are given by the relations (1.7)_{1,2}, but the plate thickness does not remain constant because:

$$(du_r)_{r=a} = -\frac{1}{2\alpha} \left(a + \frac{c^2}{a} \right) d\alpha = -\frac{1}{2\alpha} \left(a + be^{-\frac{q}{2k}} \right) d\alpha,$$

$$(du_r)_{r=b} = -\frac{1}{2\alpha} \left(b + \frac{c^2}{b} \right) d\alpha = -\frac{1}{2\alpha} \left(b + ae^{-\frac{q}{2k}} \right) d\alpha,$$

from where it can be written

$$\frac{dh}{da} = \frac{(du_r)_{r=b} - (du_r)_{r=a}}{da} = \frac{b + ae^{-\frac{q}{2k}} - a - be^{-\frac{q}{2k}}}{a + be^{-\frac{q}{2k}}},$$

$$\frac{dh}{da} = \frac{h(e^{\frac{q}{2k}} - 1)}{h + a(e^{\frac{q}{2k}} + 1)} \quad (1.17)$$

The equation (1.17) expresses the fact that $\frac{dh}{da} > 0$, so-during plastically bending-the plate thickness is decreasing.

If $\frac{h}{a}$ is less than $\frac{1}{5}$, for example, the thickness variation can be approximated by neglecting σ_r as well as the modification of the state from compression to elongation, for the fibers placed between the initial position and the present position of the neutral surface.

By putting “s” as the ratio $T/(2kh)$, it is obtained

$$\frac{q}{2k} = \frac{T}{2ka} = \frac{sh}{a},$$

with $0 \leq s \leq 1$.

Using (1.16) and Taylor serial developments for $e^{-sh/a}$ and for $\sqrt{1 + (1-s)\frac{h}{a}}$, the neutral surface results to be at a distance of $sh/2$ to the central surface.

Identically, from (1.17) it is obtained:

$$\frac{dh}{da} \cong \frac{sh^2}{2a^2}$$

If “s” is kept constant during the bending, one can obtain:

$$\Delta h = -\frac{sh^2}{2a},$$

relation which allows the calculation of Δh in the considered approximation.

2. THE DETERMINATION OF STRAINS AND STRESSES IN A RIGID PLASTIC HARDENABLE MATERIAL SUBJECTED TO BENDING

I consider the plate made of a rigid plastic hardenable material. As I already mentioned previously, the fibers can be subjected to complex deformations. According to the nature of these deformations, 3 zones can be marked on the plate thickness (Fig. 1.2) i.e.: the I zone in which the fibers are elongated, the II zone where the fibers are subjected to a compression and the III zone in which the fibers are subjected firstly to a compression, and afterwards to an elongation.

To calculate the deformation intensity, the way of charging of each fiber must be studied; the stress intensity is calculated using the hardening law, which we presume to be of the general form:

$$\sigma_c = \bar{\sigma} = \bar{\sigma}(\bar{\varepsilon}),$$

where

$$\bar{\sigma} = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta} \quad (\text{von Mises}) \quad (1.19)$$

and

$$d\varepsilon = \frac{2}{\sqrt{3}} |d\varepsilon'_\theta|, \quad \bar{\varepsilon} = \int d\varepsilon$$

For the elongation itself or for the compression itself, one can write:

$$\varepsilon'_\theta = \int d\varepsilon'_\theta = \ln \frac{r}{r_0},$$

where r_0 is the radius of the fiber having L_0 length. So, the strain intensity (for the fibers in I and II zones, suffering an one-direction deformation) is calculated by:

$$\bar{\varepsilon} = \bar{\varepsilon}(r) = \frac{2}{\sqrt{3}} \left| \ln \frac{r}{r_0} \right| \quad (\text{von Mises}) \quad (1.20)$$

The yield condition can be written, for a hardenable material, as:

$$\sigma_\theta - \sigma_r = 2k(\bar{\sigma}) \quad \text{for } c \leq r \leq b \quad (1.21)$$

$$\sigma_\theta - \sigma_r = -2k(\bar{\sigma}) \quad \text{for } a \leq r \leq c$$

where $k = \frac{\bar{\sigma}}{2}$ for Tresca's criterion and $k = \frac{\bar{\sigma}}{\sqrt{3}}$ for von Mises' criterion.

The σ_r component is obtained from (1.1) and (1.21) by integration, as follows:
-in I zone

$$\sigma_r^I(r) = \int_r^b \frac{d\sigma_r}{dr'} dr' = \int_r^b \frac{\sigma_r - \sigma_\theta}{r'} dr' = 2 \int_r^b \frac{k(\bar{\sigma})}{r'} dr' = 2 \int_r^b \frac{\hat{k}(\bar{\varepsilon})}{r'} dr' = 2 \int_r^b k(\bar{\varepsilon}) d\bar{\varepsilon} \quad (1.22)$$

where $c_0 \leq r \leq b$, and
-in II zone

$$\sigma_r^{II}(r) = \int_a^r \frac{d\sigma_r}{dr'} dr' = \int_a^r \frac{\sigma_r - \sigma_\theta}{r'} dr' = -2 \int_a^r \frac{k(\bar{\sigma})}{r'} dr' = -2 \int_a^r \frac{\hat{k}(\bar{\varepsilon})}{r'} dr' = -2 \int_a^r k(\bar{\varepsilon}) d\bar{\varepsilon} \quad (1.23)$$

where $a \leq r \leq c$ and $\bar{\varepsilon}$ is calculated according to (1.20) in both zones.

For a hardenable material, c is not known, and for the integration of the following relations

$$\frac{d\sigma_r}{dr} = \frac{\sigma_r - \sigma_\theta}{r} = \frac{2k(\bar{\sigma})}{r}, \quad c \leq r \leq c_0 \quad (1.24)$$

each fiber deformation must be determined. The integration is carried out starting from the continuity condition of σ_r across the surface $r = c_0$, σ_r being given in (1.22).

The procedure is identically applied starting from (1.23). The calculation finishes if the continuity of σ_r for $r = c$ is verified (see [3]).

It is necessary to notice that some geometrical relations, established in part (1) are not yet valid. Thus, considering the constancy of the area and h as variable, one can write:

$$L_0 h_0 = \frac{1}{2}(b^2 - a^2)L_0 \alpha,$$

And

$$\alpha h^2 + 2a\alpha h - 2h_0 = 0 \quad (1.25)$$

The relation (1.25) allows the calculus of plate thickness in the present configuration, if one knows α , a and h_0 . For a rigid-perfect plastic material, where $h = h_0$, the (1.25) relation implies (1.9) which can be written:

$$a = a(\alpha) = \frac{1}{\alpha} - \frac{h_0}{2} \quad (1.26)$$

This is true for α as the only parameter of plastically bending. Using (1.25), one can obtain: $h < h_0$, if:

$$\alpha h_0 + 2a\alpha - 2 > 0 \quad (1.27)$$

The (1.27) condition must be observed in all the cases where a decrease of plate thickness takes place.

3. NUMERICAL RESULTS

Firstly, we shall calculate the tube radii, the stresses and strains produced after the bending of a plate having $L_0 = 3780$ mm and $h_0 = 14.3$ mm, the plate is made of steel with the following characteristics: $E = 205$ GPa, $\nu = 0.29$ and $\sigma_c = 475$ MPa (a perfect plastic rigid material). If we use Tresca's criterion, then we can write, $\sigma_c = 2k$ and $\bar{\epsilon} = \left| \ln \frac{r}{r_0} \right|$. At the end of the bending, $\alpha = \frac{2\pi}{L_0} = 0.0017$ the final internal radius is $a_{fin} = 594.45$ mm, according to (1.26); from $b_{fin} = a_{fin} + h_0$, one obtain $b_{fin} = 608.75$ mm.

The final radius of central fiber in a non – deformed state, is $r_f = 601.64$ mm. The neutral fiber is described by (1.4) and one can obtain $c_{fin} = 601.56$ mm, and the radius of the fiber having the initial length L_0 is $r_0 = 601.6$ mm.

The stress has the following components, at the end of the bending (the values are given in MPa):

$$\begin{aligned} \sigma_r = & \begin{aligned} & 475 \ln \frac{r}{608.75} \quad \text{if } 601.56 \leq r \leq 608.75 \\ & 475 \ln \frac{594.45}{r} \quad \text{if } 594.45 \leq r \leq 601.56 \end{aligned} \\ \sigma_\theta = & \begin{aligned} & 475 \left(\ln \frac{r}{608.75} + 1 \right) \quad \text{if } 601.56 \leq r \leq 608.75 \\ & 475 \left(\ln \frac{594.45}{r} - 1 \right) \quad \text{if } 594.45 \leq r \leq 601.56 \end{aligned} \end{aligned}$$

Taking into account the $\bar{\varepsilon}$ expression and using Tresca's criterion, for $r = b_{fin}$ one obtains:

$$\bar{\varepsilon} = \bar{\varepsilon}_0 = \ln \frac{b_{fin}}{r_0} = 0.011$$

and, for $r = a_{fin}$

$$\bar{\varepsilon} = -\bar{\varepsilon}_0 = -\ln \frac{a_{fin}}{r_0} = 0.012$$

The I zone, in which the fibers are subjected to elongation, is described by $601.64 \text{ mm} \leq r \leq 608.75 \text{ mm}$. The II zone, of compression, is described by $594.45 \text{ mm} \leq r \leq 601.56 \text{ mm}$, and the zone III, of mixed stresses, delimited by $601.56 \text{ mm} \leq r \leq 601.64 \text{ mm}$, has a very little width, of 0.08 mm, i.e. 0.5% of h_0 .

If the material is hardenable, following the law:

$$\sigma_c = 826.22(\bar{\varepsilon})^{0.15}$$

then, assuming that the radii a_{fin} , b_{fin} and c_{fin} are those previously calculated, σ_r can be expressed by:
-in the I zone

$$\begin{aligned} \sigma_r^I(r) &= -\int_r^{b_{fin}} \frac{d\sigma_r}{dr'} dr' = -\int_r^{b_{fin}} \frac{\sigma_r - \sigma_\theta}{r'} dr' = -826.22 \int_r^{b_{fin}} \frac{1}{r'} \left(\ln \frac{r'}{r_0} \right)^{0.15} dr' = \\ &= -\frac{826.22}{1.15} \int_r^{b_{fin}} \frac{d}{dr'} \left(\ln \frac{r'}{r_0} \right)^{1.15} = 718.45 \left[\left(\ln \frac{r}{r_0} \right)^{1.15} - \left(\ln \frac{b_{fin}}{r_0} \right)^{1.15} \right]; \end{aligned}$$

- in the II zone

$$\sigma_r^{II}(r) = \int_{a_{fin}}^r \frac{d\sigma_r}{dr'} dr' = 718.45 \left[\left(\ln \frac{a_{fin}}{r_0} \right)^{1.15} - \left(\ln \frac{r}{r_0} \right)^{1.15} \right].$$

Now, we shall consider how will vary the third zone dimension function of plate length (L_0) and thickness (h_0), for some examples done in Table 1. and Figure 3.

The variation of third zone thickness (III) divided by h_0 function of L_0 and h_0 - Table 1

L_0 [mm]	h_0 [mm]	(III/ h_0)*100
3780	14.3	0.59
2780	14.3	0.81
1780	14.3	1.26
780	14.3	2.88
3780	25	1.04
2780	25	1.41
1780	25	2.21
780	25	5.03

From Table 1 and Figure 3 it is obvious that the third zone thickness increases with plate length decreasing and plate thickness increasing. So, the contribution of this zone, where the Bauschinger effect takes place, to the mechanical properties of rolled tubes becomes important for small tubes diameter and large plate thickness.

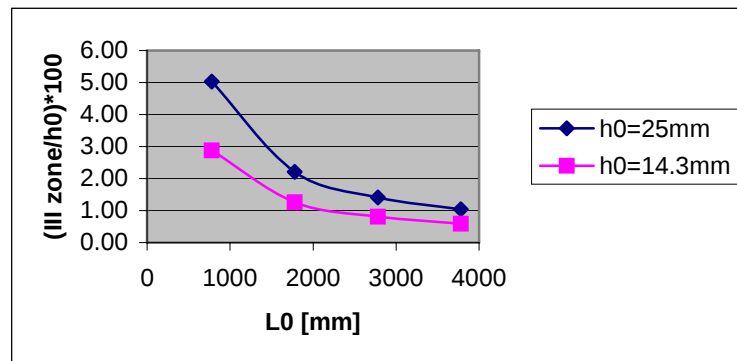


Fig.3 Third zone variation for different plates, $L_0 = 780-3780\text{mm}$ and $h_0 = 14.3$ and 25mm

4. CONCLUSIONS

1. It is proposed a model for strains and stresses calculus during the bending a heavy plates. The strains and stresses are calculated by considering two types of materials: a rigid-plastic one and a hardenable one.
2. The model allows the plate fibers movement description in every moment of the processing. So, is described the evolution of three zones across the plate thickness : the I zone in which the fibers are elongated, the II zone where the fibers are subjected to a compression and the III zone in which the fibers are subjected firstly to a compression, and afterwards to an elongation, in this zone, the Bauschinger effect can have a remarkable importance.
3. The third zone thickness increases with plate length decreasing and plate thickness increasing. So, the contribution of this zone, where the Bauschinger effect takes place, to the mechanical proprieties of rolled tubes becomes important for small tubes diameter and large plate thickness.

REFERENCES

- [1]. Hill, R., The mathematical theory of plasticity, Oxford, Clarendon Press, 1950.
- [2]. Katchanov, L., Elements de la theorie de la plasticite, Editions Mir, Moscou, 1975.
- [3]. Verguts, H., Sowerby, R., The pure plastic bending of laminated sheet metals, Int. J. Mech. Sci, 17, 1975.