

THERMAL SOLICITATIONS IN MONOLAYER ANNULAR PLATES

I. TEMPERATURE VARIATION ALONG THE RADIUS

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Abstract. This paper propose to show a number of variation lows of the temperature along the radius of an annular plate, keeping the constant values on its thickness, and having the purpose to evaluate the radial displacements and the radial and annular stresses. For the theoretical analyze, we will take into account that the plate is embedded on the external contour and the internal contour is free. In this sense, the expressions of the magnitudes necessary to determine the admissible maximal values of the developed temperature in diverse practical situations are offered, taking into account the complexity of the industrial processes on one side, and the conception of some adequate mechanical structures, on other side.

Keywords: annular plate, thermal gradient, thermal stresses

1. GENERALITIES

In the structure of the equipments of the processes industries, plane and circular plates and/or annular, monolayer are met, having both separation function of the processed or circulated mediums and for the transition for a working space into other [5, 6, 12, 13, 18, 20].

It is evidently the fact that in these situations the temperature (the temperatures) on a face or other face can have constant or different values along the considered plate radius. The regimes of thermal transfers can be stationary [5, 6, 12, 13, 18, 20] or unstationary [9 – 11, 16, 19]. The plates can be static or in rotate movement [7, 8, 21, 22], and others external loads (local or distributed about some lows) action over these plates, too.

We cannot neglect the fact the construction materials of the plates can modify the mechanical characteristics and/or thermal [1, 2, 14, 15], and they disadvantage the portent capacity of those mechanical structures.

In the case of a heat transfer in a circular or annular plate, function of diverse practical situations can be present situations as [3]:

- a) uniform heating or cooling on the thickness of the plate, but variable on radial direction about a certain low;
- b) stationary temperature field, axial symmetrically, but variable on the thickness of the plate and in radial direction;
- c) the heat transfer can be realized from a face of the plate toward the opposite face, or with simultaneous action from the lateral faces of the plates(the superposition of the effects).

Within this paper we take into discussion a stationary regime (heating or cooling), along the radius of the annular plate having constant thickness, embedded peripheral contour and free internal contour.

In this sense, starting from a general case, we realize corresponding particularizations for the accepted situation (about bibliographical sources, adaptations and suggestions), and we establish useful expressions for evaluate the radial displacement, the equivalent, circular and radial stresses.

2. GENERAL WORDING

Generally, for an annular plate, the radial displacement in a certain point of the median surface of the plate has the expression [3]:

$$u(r) = \frac{(1 + \nu_p) \cdot \alpha_T}{r} \cdot \int_{r_0}^r \Delta T(r) \cdot r \cdot dr + C_{1T} \cdot r + \frac{C_{2T}}{r}, \quad (1)$$

where r_0 is the internal radius of the plate, C_{1T} and C_{2T} - the integration constants, which are determined for the adequate conditions at limit.

The radial and annular stresses have the relations [3]:

$$\sigma_r(r) = -\frac{E_p \cdot \alpha_T}{r^2} \cdot I_1 + \frac{E_p}{1 - \nu_p} \cdot C_{1T} - \frac{E_p}{(1 + \nu_p) \cdot r^2} \cdot C_{2T}; \quad (2)$$

$$\sigma_\theta(r) = \frac{E_p \cdot \alpha_T}{r^2} \cdot I_1 - E_p \cdot \alpha_T \cdot \Delta T(r) + \frac{E_p}{1 - \nu_p} \cdot C_{1T} + \frac{E_p}{(1 + \nu_p) \cdot r^2} \cdot C_{2T}, \quad (3)$$

where the following notation is used:

$$I_1 = \int_{r_0}^r \Delta T(r) \cdot r \cdot dr. \quad (4)$$

The radial displacement becomes:

$$u(r) = \frac{(1 + \nu_p) \cdot \alpha_T}{r} \cdot I_1 + C_{1T} \cdot r + \frac{C_{2T}}{r}. \quad (5)$$

Plate embedded on the external circumference, having free the internal contour

To establish the expressions of the integration constants the following limit conditions are carried into effects:

$$r = r_{cr}; \quad u(r_{cr}) = 0; \quad r = r_0; \quad \sigma_r(r_0) = 0, \quad (6)$$

resulting from (1) and (2):

$$C_{1T} = -\frac{(1 + \nu_p) \cdot \alpha_T \cdot I_2}{R^2} \cdot \left[1 - \frac{(1 + \nu_p) \cdot r_0^2}{(1 + \nu_p) \cdot r_0^2 + (1 - \nu_p) \cdot R^2} \right]; \quad (7)$$

$$C_{2T} = -\frac{(1 + \nu_p)^2 \cdot \alpha_T \cdot r_0^2 \cdot I_2}{(1 + \nu_p) \cdot r_0^2 + (1 - \nu_p) \cdot R^2}, \quad (8)$$

with notation:

$$I_2^\bullet = \int_{r_0}^{r_{cr}} \Delta T(r) \cdot r \cdot dr ; \quad (9)$$

Particularizations:

Note: Further, we will determine only the expressions of the $I_1^\bullet$ - (4) equality - and the $I_2^\bullet$ - (8) equality - integrals, for every low of variation of the temperature, the stresses will be evaluated with the (2) and (3) equalities and the corresponding radial displacement with the (5) relation. The integration constants have the (7) and (8) expressions.

► **THE I CASE.** *Stationary thermal field, irrespective of the current radius of the plate*

The low of variation of the temperature along the radius of the plate is:

$$\Delta T(r) = T_e - T_0 = \Delta T = \text{constant}, \quad (10)$$

therefore:

$$I_1^\bullet = \frac{1}{2} \cdot (r^2 - r_0^2) \cdot \Delta T ; \quad I_2^\bullet = \frac{1}{2} \cdot (R^2 - r_0^2) \cdot \Delta T . \quad (11)$$

► **THE II CASE.** *Stationary thermal field, linear depending of the current radius of the plate*

The low of variation of the temperature is:

$$\Delta T(r) = \Delta T_{ec} + (T_{ep} - T_{ec}) \cdot \frac{r}{R} , \quad (12)$$

carrying at:

$$I_1^\bullet = \frac{1}{2} \cdot \Delta T_{ec} \cdot (r^2 - r_0^2) + \frac{1}{3 \cdot R} \cdot (T_{ep} - T_{ec}) \cdot (r^3 - r_0^3) ; \quad (13)$$

$$I_2^\bullet = \frac{1}{2} \cdot \Delta T_{ec} \cdot (R^2 - r_0^2) + \frac{1}{3 \cdot R} \cdot (T_{ep} - T_{ec}) \cdot (R^3 - r_0^3) . \quad (14)$$

► **THE III CASE.** *Central warmed or cooled plate. Stationary thermal field, with parabolic variation [17]*

This time the low of variation of the temperature along the radius of the plate is:

$$\Delta T(r) = \Delta T_{ec} \cdot \left(1 - \frac{r^2}{R^2} \right) \quad (15)$$

which permit to conclude the expressions:

$$I_1^\bullet = \frac{\Delta T_{ec}}{2} \cdot (r^2 - r_0^2) \cdot \left[1 - \frac{1}{2 \cdot R^2} \cdot (r^2 + r_0^2) \right] ; \quad (16)$$

$$I_2^\bullet = \frac{\Delta T_{ec}}{4} \cdot (R^2 - r_0^2) \cdot \left(1 - \frac{r_0^2}{R^2} \right) . \quad (17)$$

► **THE IV CASE.** *Peripheral warmed or cooled plate. Stationary thermal field, with exponential variation*

The expression of the thermal gradient according with this case is:

$$\Delta T(r) = \Delta T_{ep} \cdot \left(\frac{r}{R} \right)^n \quad (18)$$

$$I_1^\bullet = \frac{1}{n+2} \cdot \frac{\Delta T_{ep}}{r_{cr}^n} \cdot (r^{n+2} - r_0^{n+2}); \quad (19)$$

$$I_2^\bullet = \frac{1}{n+2} \cdot \frac{\Delta T_{ep}}{r_{cr}^n} \cdot (r_{cr}^{n+2} - r_0^{n+2}). \quad (20)$$

► **THE V CASE.** *Combined thermal field (one field irrespective of radius and one field with parabolic variation which decrease from the centre to the outline of the plate)*

The low of the unfolding of the thermal transfer is:

$$\Delta T(r) = \Delta T_{ec} \cdot \left(1 - \frac{r^2}{R^2} \right) + \Delta T_{ep}, \quad (21)$$

the magnitudes which serve to the calculation being:

$$I_1^\bullet = \frac{1}{2} \cdot \left[\Delta T_{ec} + \Delta T_{ep} - \frac{\Delta T_{ec}}{2 \cdot R^2} \cdot (r^2 + r_0^2) \right] \cdot (r^2 - r_0^2); \quad (22)$$

$$I_2^\bullet = \frac{1}{2} \cdot \left[\Delta T_{ec} + \Delta T_{ep} - \frac{1}{2} \cdot \Delta T_{ec} \cdot \left(1 + \frac{r_0^2}{R^2} \right) \right] \cdot (R^2 - r_0^2). \quad (23)$$

► **THE VI CASE.** *Combined thermal field (one field irrespective of radius and one field with exponential variation which increase from the centre to the outline of the plate) [4]*

The variation of the temperature along the radius of the plate is given of the expression:

$$\Delta T(r) = \Delta T_{ec} + (T_{ep} - T_{ec}) \cdot \left(\frac{r}{R} \right)^n, \quad (24)$$

which permit the deduction of the necessary magnitudes of this study:

$$I_1^\bullet = \frac{1}{2} \cdot \Delta T_{ec} \cdot (r^2 - r_0^2) + \frac{1}{n+2} \cdot \frac{1}{R^n} \cdot (r^{n+2} - r_0^{n+2}) \cdot (T_{ep} - T_{ec}); \quad (25)$$

$$I_2^\bullet = \frac{1}{2} \cdot \Delta T_{ec} \cdot (R^2 - r_0^2) + \frac{1}{n+2} \cdot \frac{1}{R^n} \cdot (R^{n+2} - r_0^{n+2}) \cdot (T_{ep} - T_{ec}). \quad (26)$$

► **THE VII CASE.** *Combined thermal field (one field irrespective of radius and one field with exponential variation)*

Respect this situation we accept the following low:

$$\Delta T(r) = \Delta T_{ep} + \Delta T_{ec} \cdot \left(1 - \frac{r}{R} \right)^n, \quad (27)$$

used for the magnitudes:

$$I_1^\bullet = \frac{1}{2} \cdot \Delta T_{ep} \cdot (r^2 - r_0^2) - \Delta T_{ec} \cdot R^2 \cdot \left\{ \frac{1}{n+1} \cdot \left(1 - \frac{r}{R} \right)^{n+1} - \left(1 - \frac{r_0}{R} \right)^{n+1} \right\} + \frac{1}{n+2} \cdot \frac{r - r_0}{R} \Bigg\}; \quad (28)$$

$$I_2^\bullet = \frac{1}{2} \cdot \Delta T_{ep} \cdot (R^2 - r_0^2) + \Delta T_{ec} \cdot R^2 \cdot \left(1 - \frac{r_0}{R} \right) \cdot \left[\frac{1}{n+1} \cdot \left(1 - \frac{r_0}{R} \right)^n - \frac{1}{n+2} \right]. \quad (29)$$

► **THE VIII CASE.** *Stationary thermal field, with exponential variation*

The expression that offer the way of variation of the temperature along the radius of the plate:

$$\Delta T(r) = \Delta T_{ec} \cdot e^{K_T r}; \quad K_T = \pm \frac{1}{R} \cdot \ln \frac{\Delta T_{ep}}{\Delta T_{ec}}, \quad (30)$$

will carry to the obtaining of the equalities:

$$I_1^\bullet = \frac{\Delta T_{ec}}{K_T^2} \cdot \left[e^{K_T r} \cdot (K_T \cdot r - 1) - e^{K_T r_0} \cdot (K_T \cdot r_0 - 1) \right]; \quad (31)$$

$$I_2^\bullet = \frac{\Delta T_{ec}}{K_T^2} \cdot \left[e^{K_T R} \cdot (K_T \cdot R - 1) - e^{K_T r_0} \cdot (K_T \cdot r_0 - 1) \right]. \quad (32)$$

► **THE IX CASE.** *Stationary thermal field, with logarithmic variation*

For this case, we consider a thermal gradient like that:

$$\Delta T(r) = \Delta T_{ep} - \left[\Delta T_{ep} - \Delta T(r_0) \right] \cdot \frac{\ln(R/r)}{\ln(R/r_0)}. \quad (33)$$

The (4) and (9) integrals have the expressions:

$$I_1^\bullet = \frac{1}{2} \cdot \left\{ \Delta T_{ep} \cdot (r^2 - r_0^2) - \frac{\Delta T_{ep} - \Delta T(r_0)}{\ln(R/r_0)} \cdot \left[(r^2 - r_0^2) \cdot \left(\ln R + \frac{1}{2} \right) - r^2 \cdot \ln r + r_0^2 \cdot \ln r_0 \right] \right\}; \quad (34)$$

$$I_2 = \frac{1}{2} \cdot \left\{ \Delta T_{ep} \cdot (R^2 - r_0^2) - \frac{\Delta T_{ep} - \Delta T(r_0)}{\ln(R/r_0)} \cdot \left[(R^2 - r_0^2) \cdot \left(\ln R + \frac{1}{2} \right) - R^2 \cdot \ln R + r_0^2 \cdot \ln r_0 \right] \right\}. \quad (35)$$

Notations:

n – exponent having a corresponding value to the concrete case of thermal transfer; r – current radius of the plate; r_0 – radius of the internal contour of the annular plate; $u(r)$ – radial displacement of the current points of the median surface of the considered plate; C_{1T} , C_{2T} – integration constants; E_p – elasticity longitudinal modulus of the material of the plate; I_1 , I_2 – helping magnitudes of calculation; K_T – exponent with thermal influence; R – radius of the external contour of the annular plate; T_e – temperature of exploitation of the plate, having the thermal gradient irrespective of radius; T_{ec} – temperature at level of the internal contour of the plate; T_{ep} – temperature of exploitation at level of the external contour of the plate; T_0 – temperature of the external medium; α_T – factor of the thermal deformation; ν_p – coefficient of the transversal contraction of the material of the annular plate; σ_r , σ_θ – radial and annular stresses; ΔT_{ec} , ΔT_{ep} – thermal gradient at the level at centre of the plate, respectively at the external level of the plate; $\Delta T(r)$, $\Delta T(r_0)$ – thermal gradient at the level at one current radius, respectively at the level of the internal contour of the annular plate.

3. CONCLUSIONS

Taking into account that in industrial practice there are many situations of warmth transfer, which are imposed by the processing of different substances, this paper present nine lows of change of the temperature along of an annular plate. We consider that its thickness is constant, irrespectively of its circumference.

We did not take into account, in this analyze, the change of elastic characteristics of the material of the plate. This theory can tackle this state.

Starting of o general presentation, using corresponding particularizations, the necessary expressions for the radial displacements of the points, which belong of the median surface of the annular plate and the expressions of the radial and annular stresses are determined.

Taking into consideration these magnitudes, the evaluation of the maximal solicitation and therefore the relocation of the acceptable maximal values of the working temperature are possible.

Because the previous study had been established for a plate having the external contour embedded and the internal circumference free, in a next paper a plate having the internal contour embedded and the external contour free, will be studied.

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