AN EFFICIENT ITERATIVE LINING METHOD FOR THE MANAGEMENT OF CLOTHOID SPIRAL MANUFACTURES IN TECHNICAL DESIGNS

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Abstract: This paper presents a quick and accurate lining method, which confrontates the most common met problems in the manufacture management of several structures and machineries. An analysis is presented for manufactures within the field of clothoids spirals geometries. Proper solutions are presented for the examining problem in order to develop effective life cycle lining methods for the diagnosis and civil defence of the manufactures. In the end, useful conclusions are are made for the lining, assessment, construction and maintenance of machineries and infrastructure works.

Keywords: manufactures' diagnosis, sustainable development, manufactures' quality assurance, lining methods, infrastructure works, spatial analysis, environmental impact assessment.

1. INTRODUCTION

Nowadays, the necessities of our life are getting increased in time due to the nature of our civilization the associated environmental management problems are becoming more complex. Modern problems like the continuously increasing energy consumption, environmental pollution, climatic change, forest fires, ecoterrorism, industrial accidents, toxic emissions, bioterrorim and others. A confrontation to the latter hazards could be given by the right monitoring and quality assurance of manufactures and structures through the application of quick and accurate useful lining projects and the synchronization within the constraints of dynamic sustainable multivariable technical designs and norms on given time and space domains for several management cases, like civil rescue in emergencies; system' analysis control and biosafety; public health protection and other associated ones. Critical natural resources will be expended to support the relative demands and consumptions of goods for future human populations.

It is obvious that without taking the right measures in time, there will be an environmental collapse with unexpected consequences. Hence, enormous sustainable projects have to be generated our so as to confrontate the environmental crisis of our planet. In these circumstances, it is imperative necessary to find efficient ways in terms of timing and accuracy for the monitoring, project management and maintenance of manufactures using proper lining methods. Below is analysed an efficient iterative lining method for manufactures and structures which follow clothoid spiral geometrical characteristics. Clothoid spiral geometry has been investigated by several researchers in the literature, it is well known also as Cornu spiral or Euler's spiral (Bernoulli, 1967; Lawrence, 1972; Kaparis, 1993a,b, 1997). The lining method of a clothoid spiral, which is analysed below, can be applied in several monitoring, construction or maintenance applied projects or the management of curved manufacturies in several technical designs. This method can be used as an alternative linining method veryfying the final mapping out of particular mechanical or civil structures.

Special care should be given in the defined lining methods during the operation of particular machineries in complicate topographies. Infrastructure works and their manufactures are becoming more complex as the necessities of our life are getting increased in time. Quick and accurate operational manufacturies' support in emergencies due to several stochastic problems like fires, floods, storms or earthquakes crises could give a sense of the necessity of efficient use of lining methods for quick settlement of rehabilitation works. It is obvious that without taking any right measures in time, there will be a disaster with unexpected consequences. In these circumstances, it is imperative necessary to find efficient lining methods so as to settle the right technological equipments putting them in proper locations for the confrontation of any accidents. In such cases, a good timing of the project management and spatial monitoring is demanded in order to save civic populations, stream of goods, and to minimize any associate risks and environmental impacts to any receptors. Lining iterative methods should determine the right iterative steps focused on the particular engineering design characteristics or the results of associated numerical simulation risk assessment models for the right application of several environmental mitigation and reclamation projects (Aldrich *et al.* 1993; Kaparis, 1993a,b; Koliopoulos *et al.* 2007a,b,c).

2. PRINCIPLES OF CLOTHOID SPIRAL

Several formulas and tables have been analysed in the literature for the description of clothoid spiral (Bernoulli, 1967; Lawrence, 1972; Kaparis, 1993a,b, 1997). The most common used case and its geometrical principles of clothoid spiral in several technical works or in particular manufactures, i.e. hydrodynamic machineries; transportation designs of goods; hydraulic designs; hydro-turbines; air wings design; shells; aerodynamic designs; reclamation and bioremediation designs; sustainable hybrid multivariable systems etc. is presented in figures 1, 2, 3. In many cases of technical projects is ruled the clothoid spiral to be linked with circular arcs and then to be linked to the rest operational parts of a technical work with straight lines based on given vertexes of a polygonal line.

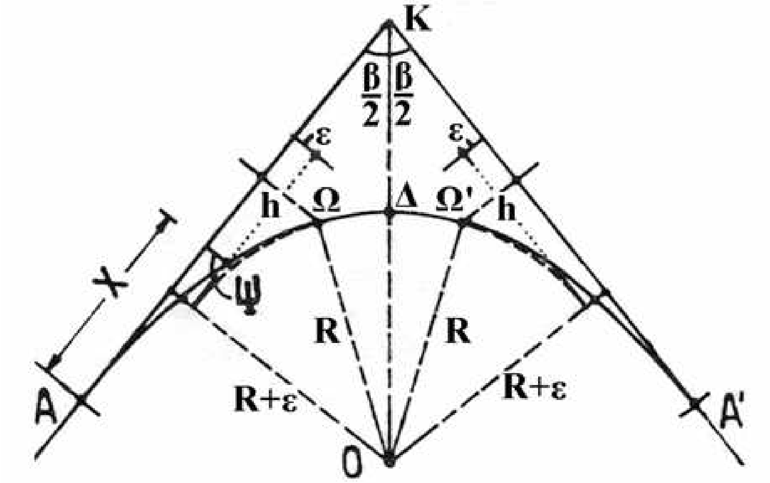
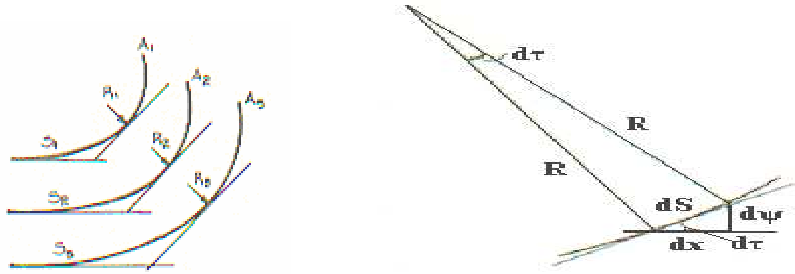


Fig. 1 Technical characteristics of the clothoid curve and application of a complex symmetrical arc AA' in technical works, where S=AΩ=AΩ' is the length of each clothoid spiral curve linked with the rest respective two circular arcs ΩΔ, Ω'Δ with radius R, based on a given polygonal line of a technical project.

However, in this paper, below are presented the necessary formulas for the mathematical investigation of the iterative clothoid arc's lining.

The geometrical principles and graphs of the latter equations are presented in figures 2, 3. Clothoid spiral lining method in combination with other lining methods could have several useful applications for the management of curved manufactures in technical designs (Kaparis, 1993a,b,1997; Koliopoulos *et al.* 2007a,b,c).



a. b.

Fig. 2a Geometrical similarity of clothoids spirals, $R_1/R_2/R_3=S_1/S_2/S_3=A_1/A_2/A_3$;
2b Curvature of a clothoid curve and its Cartesian coordinates

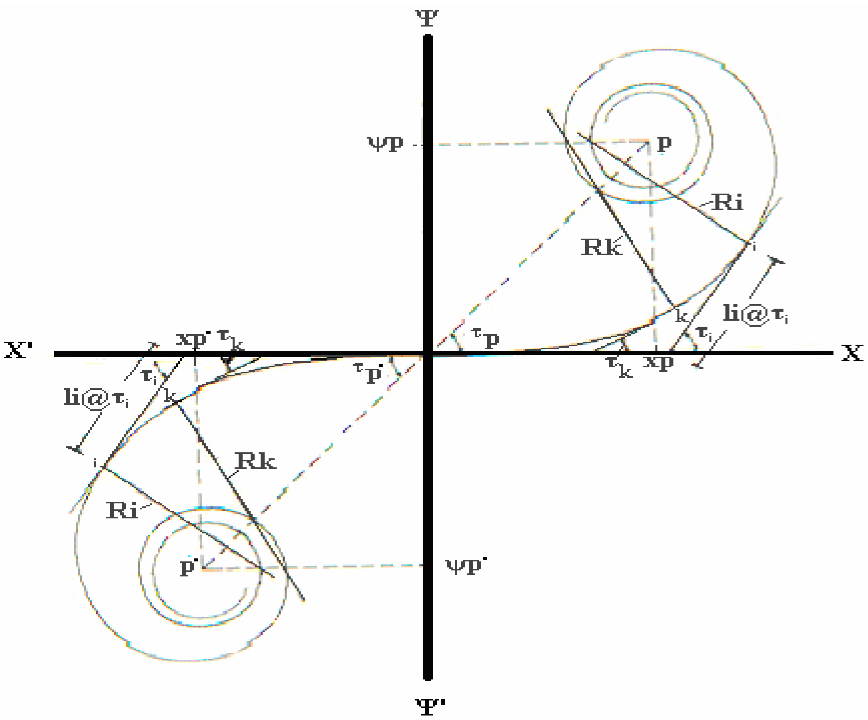


Fig. 3 Geometrical characteristics of a symmetrical clothoid curve
presented on a graph with Cartesian coordinates.

An iterative lining method

Initially for the iterative lining of a clothoid spiral have to be analysed its geometrical properties, as they are described in figures 1, 2, 3 and 4. For the examining clothoid arc is taken that it has length S and it presents an analogy to the curvature K, following the equation S = c * K (1), where c is a constant, K = 1 / R, $K = \frac{d\tau}{dS}$, τ is the angle between the tangent of clothoid spiral and the axis of x, c = R * S = A^2 . Hence, according the latter equations the differential equation of the clothoid curve is the following:

$$c * d\tau = S * dS \tag{2}$$

Integrating equation (2) it yields $\frac{S^2}{2} = c * \tau + k$ (3), but for S = 0 is τ =0 and it yields from (3) k = 0 and according to the above equations, $\tau = \frac{S}{2R} = \frac{S^2}{2c}$ (4). Also, according to figure 2b the next equations will operate in the examining spiral system:

$$dx = \cos\tau * dS = \cos\tau \frac{cd\tau}{S} = \cos\tau \frac{cd\tau}{\sqrt{2c\tau}} = \frac{A}{\sqrt{2}} \frac{\cos\tau}{\sqrt{\tau}} d\tau \quad (5)$$

$$dy = \sin\tau * dS = \sin\tau \frac{cd\tau}{S} = \sin\tau \frac{cd\tau}{\sqrt{2c\tau}} = \frac{A}{\sqrt{2}} \frac{\sin\tau}{\sqrt{\tau}} d\tau \quad (6)$$

Integrating equations (5) and (6) it yields:

$$X_{\Sigma i}(\tau) = \frac{A}{\sqrt{2}} \int_0^\tau \frac{\cos\tau}{\sqrt{\tau}} d\tau \quad (8) \qquad \text{and} \qquad \Psi_{\Sigma i}(\tau) = \frac{A}{\sqrt{2}} \int_0^\tau \frac{\sin\tau}{\sqrt{\tau}} d\tau \quad (9)$$

Applying on equations (8) and (9) the series form of the functions cos τ and sin τ respectively after calculations it yields:

$$X_{\Sigma i} = S_{\Sigma i} - \frac{S_{\Sigma i}^5}{30R^2S^2} + \frac{S_{\Sigma i}^9}{3456R^4S^4} - \ldots \quad (10) \quad \text{and} \quad \Psi_{\Sigma i} = \frac{S_{\Sigma i}^3}{6R\ S} - \frac{S_{\Sigma i}^7}{336R^3S^3} + \ldots \quad (11)$$

where $S_{\Sigma i}$ is the length of the clothoid arc on the point $\Sigma i, \ldots \Sigma n, \forall i = 1(1)n$, and for i=1 it yields the point $\Sigma 1$ on the clothoid spiral or for the nth last point of the lined clothoid curve there will be $\Sigma n = \Omega$, where in this case could be found after calculations that $\Psi_\Omega \cong \frac{S^2}{6R}$ or on the points E, E' (figure 1) for S/2 length of the clothoid curve t could be found after calculations that there will be $\Psi_\Omega \cong \frac{S^2}{48R}$. According to figure 4 the iterative lining and thickening of a klothoid spiral could be achieved by measuring the x coordinates on the line $\Pi 1 \Pi 2$ from $\Pi 1$ point for x = 0 and taking perpendiculars on the respective $X_{\Sigma i}$ points, lining up the respective $\Psi_{\Sigma i}$ points in order to line and map out the respective points $\Sigma i, \ldots \Sigma n, \forall i = 1(1)n$ of the clothoid spiral. The above formulas and results could be found in the literature in summarized tables from several relative technical books (Kaparis, 1993a,b, 1997).

The mathematical and respective technical problem is transferred on how could be increased the accuracy and accelerated efficiency of the above presented iterative method. A useful solution of the latter problem is analysed in the following section.

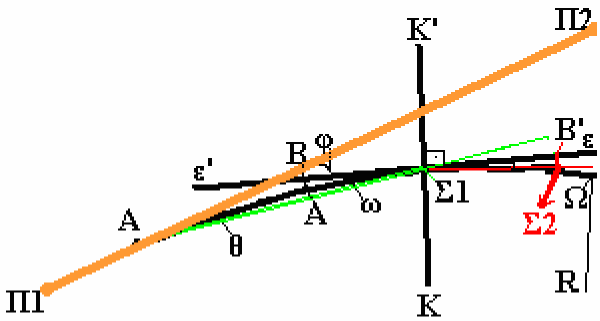


Fig. 4 Geometrical characteristics and iterative lining steps of a clothoid arc

3. A LINING METHOD OF THE CLOTHOID CURVE BASED ON ITERATIVE DETERMINED TANGENT LINES

Based on the above presented iterative method a function f(x) can be developed for given geometrical properties of a clothoid spiral , i.e. S, R, K etc. based on the $X_{\Sigma i}$ and $\Psi_{\Sigma i}$ respective points, applying the method least squares. By taking proper small x intervals and calculating the respective y coordinates, a satisfied thickening of the points $\Sigma i, ...\Sigma n, \forall i = 1(1)n$ is achieved. In the next step applying the method of least squares to the calculated x, y coordinates it results a satisfied determination of function f(x), which describes the function of clothoid curve. A least-square modelling could be developed in order to determine the function f(x) as it is described below. Least squares' mathematical formulation principles are based on the following mathematical model construction. For the determination of the f(x), expressed by a n-order polynomial function, using the Method of Least Squares, should be followed the next methodology's steps:
Firstly, the multitude m of the measurements has to be greater than the multitude n+1 of the determinable parameters a_k (k=0 (1) n) of the n-order polynomial function

$$Y = P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 = \sum_{k=0}^{n} a_k x^k \ , \quad a_n \neq 0 \quad \wedge \ n < m-1 \qquad (12)$$

Is considered that U_i (i=1 (1) m) are the corresponding errors of the measurement equations. From the system of observation equations we will have the system of error equations. The system of observation equations and the system of error equations are the following respectively:

$$Y_i = P(x_i) \ . \ i = 1\ (1)\ m \quad \wedge \ n < m-1 \quad (\ a_n \neq 0\ ) \qquad (13)$$

and

$$Y_i + U_i = \sum_{k=0}^{n} a_k x_i^k \ . \ i = 1\ (1)\ m \quad \wedge \ n < m-1 \quad (\ a_n \neq 0\ ) \qquad (14)$$

The determinable parameters, have to verify the fundamental formula of the Method of Least Squares:

$$F = U_1^2 + U_2^2 + \ldots + U_m^2 = \sum_{i=1}^{m} U_i^2 = \text{minimum} \qquad (15)$$

For the simplification of symbols and calculations is used the Matrix Algebra, assuming the following matrices:

$$A = \begin{bmatrix} x_1^n & x_1^{n-1} \ldots x_1 & 1 \\ x_2^n & x_2^{n-1} \ldots x_2 & 1 \\ \ldots & \ldots & \ldots \\ x_m^n & x_m^{n-1} \ldots x_m & 1 \end{bmatrix}, \quad X = \begin{bmatrix} a_n \\ a_{n-1} \\ . \\ . \\ a_1 \\ a_0 \end{bmatrix}, \quad B = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_{m-1} \\ Y_m \end{bmatrix}, \quad U = \begin{bmatrix} U_1 \\ U_2 \\ . \\ . \\ U_{m-1} \\ U_m \end{bmatrix} \qquad (16)$$

Finally, we have for solution the system of the following Matrix Equations

$$A.X = B + U, \quad F = U^T . U = \text{minimum} \qquad (17)$$

where, U^T is the transpose matrix of U matrix and for the solution of the latter system is known that exists the following condition

$$X = (\ A^T . A\ )^{-1} . A^T . B \qquad (18)$$

Therefore, based on the above manipulations, the function f(x) of the clothoid curve can be defined. The polynomial function f(x) has continuity and it is differentiable in the domain where the clothoid curve is investigated for the length S of the clothoid arc. As the function f(x) has been determined satisfactory, the first derivative of function f'(x) can be derived for the respective examining x,y coordinates. Therefore, having defined the equations f(x) and f'(x) can be calculated the total arc length S of the examining clothoid spiral or the arc length L as a part of it (L≤S) for the respective clothoid's curve x,y coordinates between the points $\Sigma i,...\Sigma n, \forall i = 1(1)n$. Hence, according to the integral calculus can be found the requesting arc length L by the use of the following equation:

$$L = \int_a^b \sqrt{\left(f'(x)\right)^2 + 1}dx \tag{19}$$

where, a, b are the selected xi intervals with which is determined the tangent line i.e. selection of X1,X2 intervals for the lining of $\Sigma 1, \Sigma 2$ points in figure 4. As the f'(x) and the arc length L are known and according to clothoid's geometrical characteristcs in figure 4, then can be lined the respective perpendiculars on the points $\Sigma i,...\Sigma n, \forall i = 1(1)n$, determining iteratively the respective tanget lines and utilizing them for the lining up of all the $\Sigma i,...\Sigma n, \forall i = 1(1)n$ points in small xi intervals or selected ones on the clothoid arc, as it can be shown in figure 4 for the known defined clothoid's arc geometrical properties. In case that the function f(x) is a group of separated polynomial functions for particular x,y intervals, describing complicate combinations between clothoids curves and other complex arc links, then for the effective complex arc's lining a new function h(x) should be defined instead of the f(x).

The new above presented, in the literature, iterative lining method of clothoids curves, except its accuracy and effective acceleration, provides the benefit that it can be achieved this method close to the axis of a clothoid curve, making the above presented lining method not only a robust one but also a stable one. The above useful lining principles for f(x) clothoid arc function can be applied on any respective applied technical projects for the lining, monitoring and maintenance of several manufactures.

4. CONCLUSIONS

The development of robust lining methods and their combination with dynamic analysis and numerical simulation models are useful for efficient spatial data manipulation not only of particular topographies but also of particular manufactures' automation control system analysis. Effective software, hardware tools should be developed by students or graduates based on efficient lining methods so as to be easy readable, comprehensive and easily accessible to any interest party within project management of manufactures and lining of infrastructure technical works. The presented iterative lining method is robust and applicable on several clothoids' spiral manufactures and it can be used efficiently in several linings and monitoring techniques of sustainable hybrid projects.

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