THE SIMULATION BEHAVIOR OF BURGERS VISCOUS-ELASTIC PARAMETRICAL MODEL USED IN VIBRATION ACTIVE ISOLATION

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Abstract. Dynamic models used in shocks, vibration and seism calculus are based on rheologic configuration which are specifically used in isolation system. The BURGERS parametrical model presented in this paper is used for the behavior characterization of some of these special isolation materials. This is a parametrical model regarding the stiffness of two HOOKE elements (parameter N) and the damping level of two NEWTON elements (parameter M). The present paper gives some numerical results of the simulation of different values of damping and parameters N and M.

Keywords: BURGERS parametrical model, viscous elastic damping

1. MATHEMATIC MODEL

In respects of active isolation of vibration for mechanical structures with weight *m* under the action of harmonic perturbation force using the BURGERS parametrical element, the mathematical model consists in two serial models: one is the VOIGT KELVIN model with stiffness *k* and damping coefficient *c* and the other is the MAXWELL element with stiffness *Nk* and damping coefficient *Mc* [1],[2],[3]. The simulation of transfer factor *T* and amplifiers factor *A* with relative pulsation will be presented for different values of damping factor ζ and parameters *N, M.*

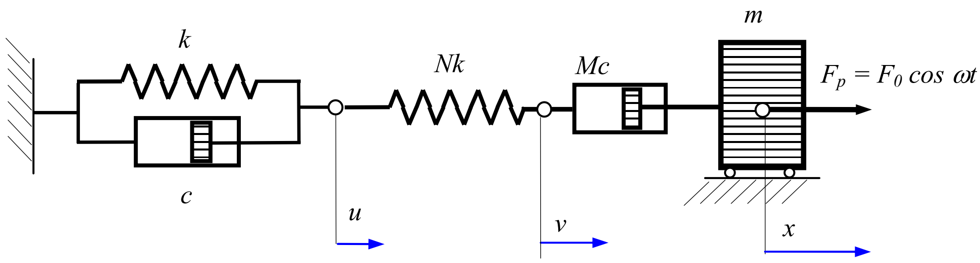

Fig.1. BURGERS viscous elastic parametrical model

The force transmitted to foundation in the case of isolation with BURGERS element according to figure 1 is the same for all three serial bound elements:

$$Q = k \cdot u + c \cdot \dot{u} = Nk \cdot (v - u) = Mc(\dot{x} - \dot{v}) \tag{1}$$

where *u* is the displacement of the end element VOIGT KELVIN and *v* is the displacement of the end element HOOKE as we see in figure 1.

The following differential equations of moves for the body *m* and differential equations for transmitted forces are written as follows:

$$\begin{cases} m\ddot{x} + Mc\cdot(\dot{x}-\dot{v}) = F_0 \cos \omega t \\ k\cdot u + c\cdot\dot{u} - Nk\cdot(v-u) = 0 \\ Nk\cdot(v-u) - Mc(\dot{x}-\dot{v}) = 0 \end{cases} \tag{2}$$

The complex expressions of variables are introduced in order to solve the equations (2) in a complex mode:

$$\tilde{z} = x + i\cdot y;\quad \tilde{w} = u + i\cdot s;\quad \tilde{t} = v + i\cdot q \tag{3}$$

The particular complex solutions of the equations (2) are written as follows:

$$\tilde{z} = \tilde{z}_0\cdot e^{i\cdot(\omega t-\varphi)};\qquad \tilde{w} = \tilde{w}_0\cdot e^{i\cdot(\omega t-\theta)};\qquad \tilde{t} = \tilde{t}_0\cdot e^{i\cdot(\omega t-\gamma)} \tag{4}$$

Substituting the derivative functions of the particular complex solutions in the equations (2) we obtain:

$$\begin{cases} -m\cdot\omega^2\cdot\tilde{z}_0\cdot e^{i\cdot(\omega t-\varphi)} + Mc\cdot i\cdot\omega\cdot\tilde{z}_0\cdot e^{i\cdot(\omega t-\varphi)} - Mc\cdot i\cdot\omega\cdot\tilde{t}_0\cdot e^{i\cdot(\omega t-\gamma)} = F_0\cdot e^{i\cdot\omega t} \\ k\cdot\tilde{w}_0\cdot e^{i\cdot(\omega t-\theta)} + i\cdot c\cdot\omega\cdot\tilde{w}_0\cdot e^{i\cdot(\omega t-\theta)} - Nk\cdot\tilde{t}_0\cdot e^{i\cdot(\omega t-\gamma)} + Nk\cdot\tilde{w}_0\cdot e^{i\cdot(\omega t-\theta)} = 0 \\ Nk\cdot\tilde{t}_0\cdot e^{i\cdot(\omega t-\gamma)} - Nk\cdot\tilde{w}_0\cdot e^{i\cdot(\omega t-\theta)} - Mc\cdot i\cdot\omega\cdot\tilde{z}_0\cdot e^{i\cdot(\omega t-\varphi)} + Mc\cdot i\cdot\omega\cdot\tilde{t}_0\cdot e^{i\cdot(\omega t-\gamma)} = 0 \end{cases} \tag{5}$$

By solving the system (5) the amplitudes of the particular complex solutions result:

$$\begin{cases} \tilde{w}_0 = F_0\cdot\dfrac{i\cdot c\cdot\omega\cdot M}{\left(-m\cdot\omega^2+i\cdot c\cdot\omega\cdot M\right)(k+i\cdot c\cdot\omega)+i\cdot c\cdot\omega^2\cdot m\cdot M\left(1+\dfrac{1}{N}+\dfrac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\theta} \\ \tilde{t}_0 = F_0\cdot\dfrac{-\dfrac{M}{Nk}\cdot c^2\cdot\omega^2+i\cdot c\cdot\omega\cdot M\left(1+\dfrac{1}{N}\right)}{\left(-m\cdot\omega^2+i\cdot c\cdot\omega\cdot M\right)(k+i\cdot c\cdot\omega)+i\cdot c\cdot\omega^2\cdot m\cdot M\left(1+\dfrac{1}{N}+\dfrac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\gamma} \\ \tilde{z}_0 = F_0\cdot\dfrac{k-\dfrac{M}{Nk}\cdot c^2\cdot\omega^2+i\cdot c\cdot\omega\cdot\left(1+M+\dfrac{M}{N}\right)}{\left(-m\cdot\omega^2+i\cdot c\cdot\omega\cdot M\right)(k+i\cdot c\cdot\omega)+i\cdot c\cdot\omega^2\cdot m\cdot M\left(1+\dfrac{1}{N}+\dfrac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\varphi} \end{cases} \tag{6}$$

The particular complex solutions (4) become:

$$\begin{cases} \tilde{w} = F_0\cdot\dfrac{i\cdot c\cdot\omega\cdot M}{\left(-m\cdot\omega^2+i\cdot c\cdot\omega\cdot M\right)(k+i\cdot c\cdot\omega)+i\cdot c\cdot\omega^2\cdot m\cdot M\left(1+\dfrac{1}{N}+\dfrac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\cdot\omega t} \\ \tilde{t} = F_0\cdot\dfrac{-\dfrac{M}{Nk}\cdot c^2\cdot\omega^2+i\cdot c\cdot\omega\cdot M\left(1+\dfrac{1}{N}\right)}{\left(-m\cdot\omega^2+i\cdot c\cdot\omega\cdot M\right)(k+i\cdot c\cdot\omega)+i\cdot c\cdot\omega^2\cdot m\cdot M\left(1+\dfrac{1}{N}+\dfrac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\cdot\omega t} \\ \tilde{z} = F_0\cdot\dfrac{k-\dfrac{M}{Nk}\cdot c^2\cdot\omega^2+i\cdot c\cdot\omega\cdot\left(1+M+\dfrac{M}{N}\right)}{\left(-m\cdot\omega^2+i\cdot c\cdot\omega\cdot M\right)(k+i\cdot c\cdot\omega)+i\cdot c\cdot\omega^2\cdot m\cdot M\left(1+\dfrac{1}{N}+\dfrac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\cdot\omega t} \end{cases} \tag{7}$$

The expression of force transmitted to the foundation in the complex mode is:

$$\widetilde{Q} = Nk\cdot(\widetilde{t} - \widetilde{w}) = F_0 \cdot \frac{(-c\cdot\omega + i\cdot k)\cdot c\cdot\omega\cdot M}{(-m\cdot\omega^2 + i\cdot c\cdot\omega\cdot M)(k + i\cdot c\cdot\omega) + i\cdot c\cdot\omega^2\cdot m\cdot M\left(1 + \frac{1}{N} + \frac{i\cdot c\cdot\omega}{kN}\right)}\cdot e^{i\cdot\omega t} \tag{8}$$

By noting $p^2 = \frac{k}{m};\ n = \frac{c}{2m};\ \ \eta = \frac{\omega}{p};\ \ \zeta = \frac{n}{p} = \frac{c}{c_{cr}}$, the expression (8) becomes (9)

$$\widetilde{Q} = F_0 \frac{2M\cdot\zeta\cdot\eta\cdot(i - 2\cdot\zeta\cdot\eta)}{\eta^2\left[-1 - 4M\cdot\zeta^2 + \frac{4M}{N}\zeta^2\eta^2\right] + 2i\cdot\eta\cdot\zeta\cdot\left[M - \eta^2\left(1 + M + \frac{M}{N}\right)\right]}\cdot e^{i\cdot\omega t} \tag{10}$$

The transfer factor T represents the ratio between the amplitude of real force transmitted to foundation and the amplitude of perturbation force:

$$T = \frac{Q_0}{F_0} = \frac{2M\cdot\zeta\cdot\sqrt{1 + 4\eta^2\zeta^2}}{\sqrt{\eta^2\left[1 + 4M\cdot\zeta^2 - \frac{4M}{N}\zeta^2\eta^2\right]^2 + 4\cdot\zeta^2\cdot\left[M - \eta^2\left(1 + M + \frac{M}{N}\right)\right]^2}} \tag{11}$$

The complex amplitude of vibrations' displacement obtained in (6) may also be written as:

$$\widetilde{z}_0 = \frac{F_0}{k}\cdot\frac{1 - \frac{M}{N}\cdot 4\eta^2\zeta^2 + 2i\cdot\eta\zeta\cdot\left(1 + M + \frac{M}{N}\right)}{\eta^2\left[-1 - 4M\cdot\zeta^2 + \frac{4M}{N}\zeta^2\eta^2\right] + 2i\cdot\eta\cdot\zeta\cdot\left[M - \eta^2\left(1 + M + \frac{M}{N}\right)\right]}\cdot e^{i\cdot\varphi} \tag{12}$$

The amplifier factor A of vibrations is defined as the ratio between the amplitude of real displacement vibrations and the deformation of spring k under a force F_0:

$$A = \frac{x_0}{F_0/k} = \frac{\sqrt{\left(1 - \frac{M}{N}\cdot 4\eta^2\zeta^2\right)^2 + 4\eta^2\zeta^2\left(1 + M + \frac{M}{N}\right)^2}}{\sqrt{\eta^4\left[1 + 4M\cdot\zeta^2 - \frac{4M}{N}\zeta^2\eta^2\right]^2 + 4\cdot\zeta^2\cdot\eta^2\left[M - \eta^2\left(1 + M + \frac{M}{N}\right)\right]^2}} \tag{13}$$

2. PARTICULAR CASES

The transfer factor T (11) relation and the amplifier factor A relation can be particularized in order to obtain the following particular cases [3],[4]:

- For $N\to\infty$ VOIGT KELVIN - NEWTON particular case is obtained:

$$T = \frac{2M\cdot\zeta\sqrt{1 + 4\eta^2\zeta^2}}{\sqrt{\eta^2(1 + 4M\cdot\zeta^2)^2 + 4\zeta^2(M - \eta^2(1 + M))^2}}$$
$$A = \frac{\sqrt{1 + 4\eta^2\zeta^2(1 + M)^2}}{\sqrt{\eta^4(1 + 4M\cdot\zeta^2)^2 + 4\zeta^2\eta^2(M - \eta^2(1 + M))^2}} \tag{14}$$

- For $M\to\infty$ VOIGT KELVIN- HOOKE particular case is obtained:

$$T=\frac{N\cdot\sqrt{1+4\eta^2\zeta^2}}{\sqrt{\left(N-\eta^2(1+N)\right)^2+4\eta^2\zeta^2\left(N-\eta^2\right)^2}}$$
$$A=\frac{\sqrt{(1+N)^2+4\eta^2\zeta^2}}{\sqrt{\left(N-\eta^2(1+N)\right)^2+4\eta^2\zeta^2\left(N-\eta^2\right)^2}} \tag{15}$$

- For $N\to\infty$ *and* $M\to\infty$ VOIGT KELVIN particular case is obtained:
-

$$T=\sqrt{\frac{1+4\zeta^2\eta^2}{\left(1-\eta^2\right)^2+4\zeta^2\eta^2}};\quad A=\frac{1}{\sqrt{\left(1-\eta^2\right)^2+4\zeta^2\eta^2}} \tag{16}$$

3. THE SIMULATION OF TRANSFER FACTOR *T*

- The simulation of transfer factor T with pulsation for different values of the damping factor. Figure 2 shows the transfer factor $T(\eta)$ for the following damping factors: $\zeta_1=0,1;\ \zeta_2=0,3;\ \zeta_3=0,7;\ \zeta_4=0,9$ and for the following parameters values: $N=10$, $M=5$.
- The simulation of transfer factor T with pulsation for different values of N parameter. Figures 3 shows the transfer factor $T(\eta)$ for the following parameters: $N_1=1;\ N_2=2;\ N_3=10;\ N_4=100;\ (M=10)$ in case of low damping $\zeta_1=0,1$.
- The simulation of transfer factor T with pulsation for different values of M parameter. Figures 4 shows the transfer factor $T(\eta)$ for the following parameters: $M_1=1;\ M_2=2;\ M_3=5;\ M_4=10;\ (N=5)$ in case of low damping $\zeta_1=0,1$.

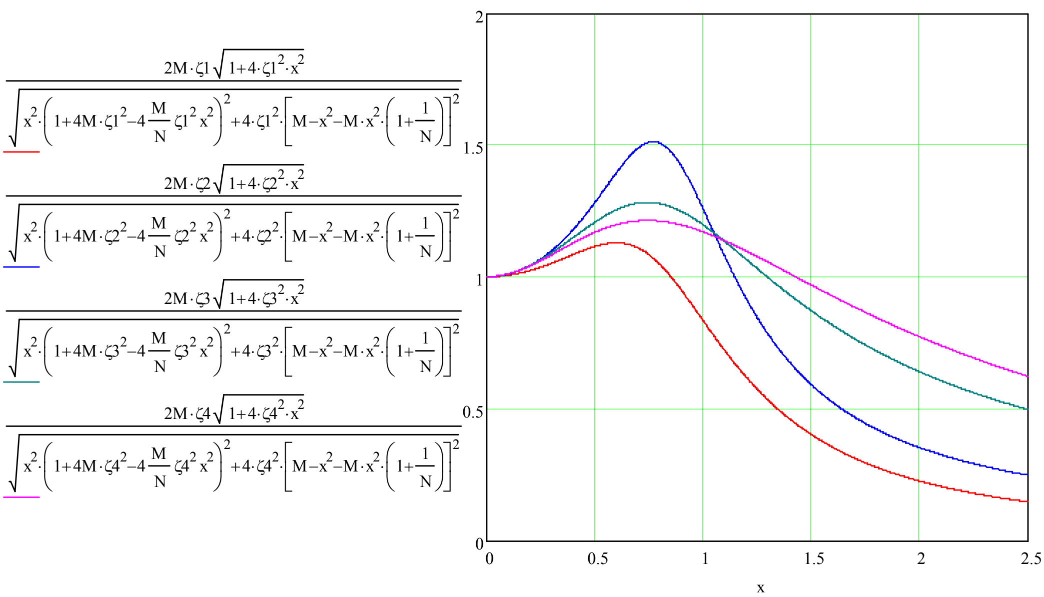

Fig.2. The simulation of transfer factor with pulsation for different values of the damping factor.

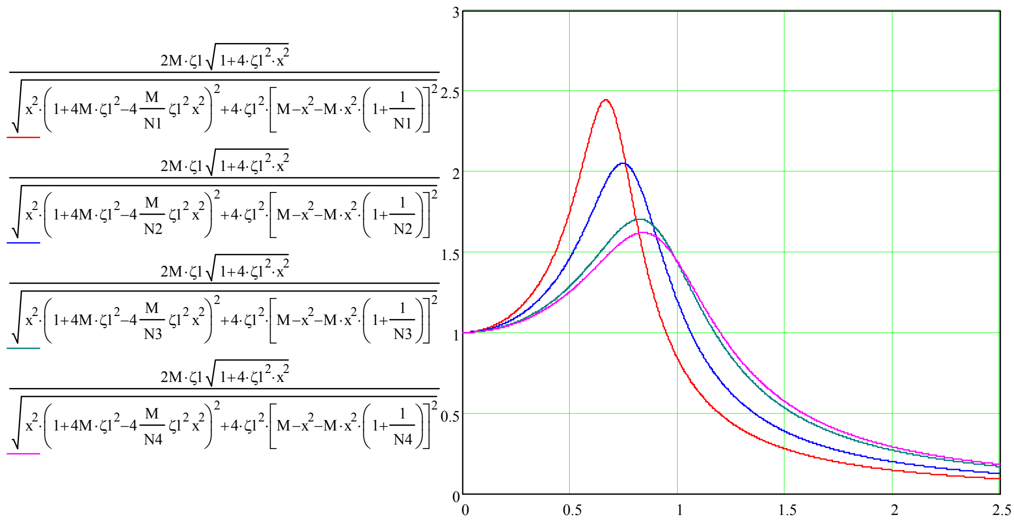

Fig.3. The simulation of transfer factor with pulsation for different values of the N factor - low damping

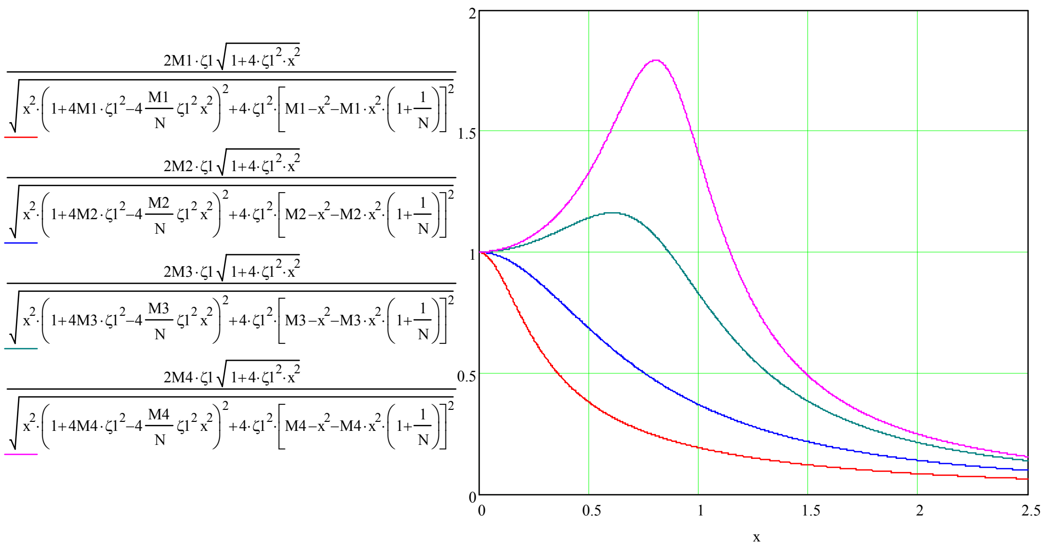

Fig.4. The simulation of transfer factor T with pulsation for different values of the M factor - low damping

4. THE SIMULATION OF AMPLIFIER FACTOR *A*

- The simulation of amplifier factor *A* with pulsation for different values of the damping factor. Figure 5 shows the amplifier factor $A(\eta)$ for the following damping factors: $\zeta_1=0,1;$ $\zeta_2=0,3;$ $\zeta_3=0,7;$ $\zeta_4=0,9$ and for the following parameters values: $N=10$, $M=5$.
- The simulation of amplifier factor *A* with pulsation for different values of *N* parameter. Figures 6 shows the amplifier factor $A(\eta)$ for the following parameters: $N_1=1;$ $N_2=2;$ $N_3=10;$ $N_4=100;$ $(M=10)$ in case of low damping $\zeta_1=0,1$.
- The simulation of amplifier factor *A* with pulsation for different values of *M* parameter. Figures 7 shows the amplifier factor $A(\eta)$ for the following parameters: $M_1=1;$ $M_2=2;$ $M_3=5;$ $M_4=10;$ $(N=5)$ in case of low damping $\zeta_1=0,1$.

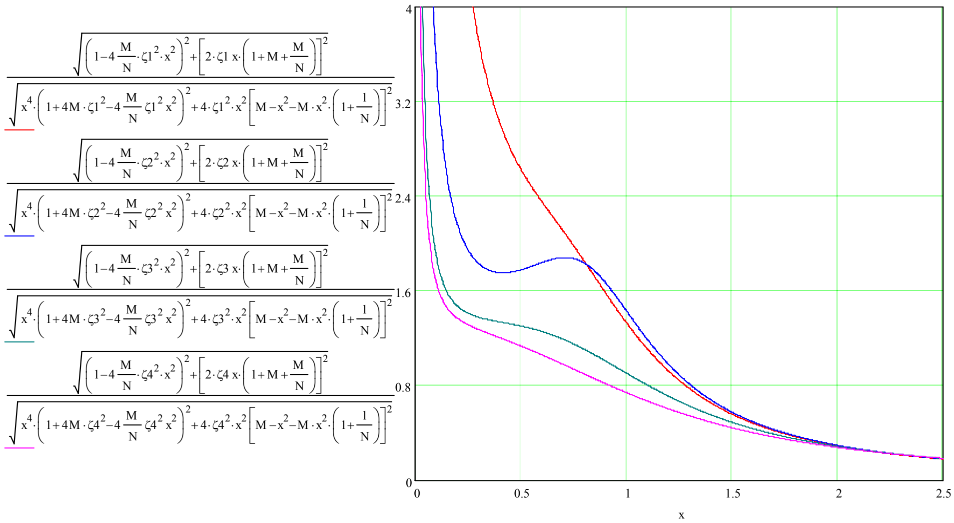

Fig.5. The simulation of amplifier factor with pulsation for different values of the damping factor.

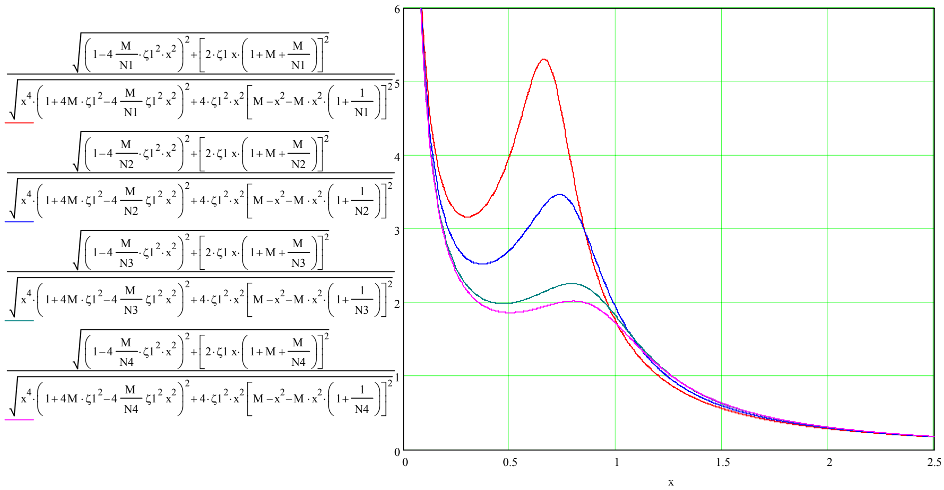

Fig.6. The simulation of amplifier factor with pulsation for different values of the N factor - low damping

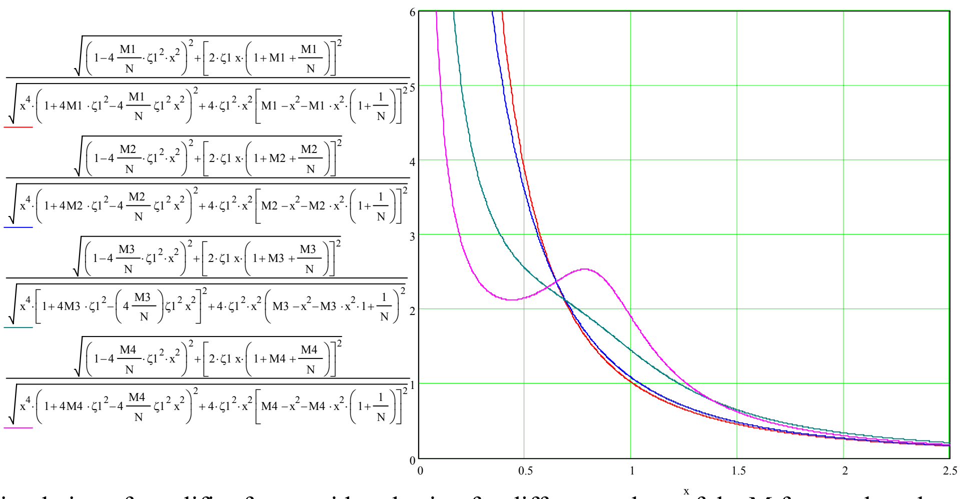

Fig.7. The simulation of amplifier factor with pulsation for different values of the M factor - low damping

5. CONCLUSIONS

The simulation of the BURGERS parametrical model shows a specific behavior to the value modification of the elastic and damping parameters of the basic elements as well as the relative values modification of elasticity and damping of the VOIGT KELVIN and MAXWELL elements [3]. Therefore:

- *The transfer factor of vibration* in the resonant area is higher for average level of damping, for lower values of *N* parameter for weak as well as for strong damping and for higher values of *M* parameters (fig. 2, 3, 4).
- *The amplifying factor of vibrations* in the low frequencies area is huge for each situation, and higher in the resonant area for small values of damping, for even smaller values of *N* parameter for low as well as for high damping and for low values of the *M* parameter (fig. 5, 6, 7).

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