OPTIMAL SIGNAL RECEPTION. THE MATHEMATICAL MODEL OF THE ADAPTIVE FILTER.

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Abstract: The authors take into discussion some of the issues specific to signal' reception. The received signal is transformed by the receiver into a time dependant function, a number or a Boolean variable. The study of such transformation requires the defiition of the functions group corresponding to the periodic signal, of the finite amplitude signals and the finite energy signals. The simplest model, and also the most important one for clearing the basic aspects of this issues, has imposed the assumption that the parasite influence of the received signal are additive. The method for the elimination of the additive noise and the extraction of the purified information will therefore be presented.

Keywords: communication, filter, signal, reception, model

1. THE MEANING OF THE RECEPTION CONCEPT

The communication systems are designed to transmit information. The information held by the electro-magnetic signal is affected by several factors, some known and some unknown, related to the information source or to the communication channel.
The modulation through with the information is being transmitted can be interpreted as the choosing of a signal from the possible ones.
The study of the transformations suffered by the signal is being done through the $L^2(T)$ multitude, which is the multitude of the functions who's square is integrate-able over the T interval, $T \in R$. The signal class represented by these functions is a more general one than the one of the signals represented through the multitude of continuous, derivate-able functions or the finite hops ones, and has the advantage that the transformations to the signals do not take them outside their initial space.
The transformation of a signal u(t) into another one y(t), also from $L^2(T)$, as the first one, is an application from a functions space to itself, meaning an operator. In the reception process this transformation is linear and limited.
The receiver has as mathematical model a linear and limited operator on a Hilbert space.
The main issue of reception is the extraction of the information from the noise affected received signal.
The simplest model and also the most important one for solving the basic aspects of the issue assumes the parasite influences in the received signal x(t) are additive, meaning:

$$x(t) = u(t) + n(t) \tag{1}$$

where u(t) is the signal transmitted by the source, n(t) is the additive noise and x(t) is the received signal, representing an implementation of an aleatory process.
The receiver's task is to extract from x(t) the u(t) component in the best way possible, using an optimal criteria according to what we know about the specifics of the signal and the noise.

2. THE LINEAR FILTER

We take into account the filter shown in figure1, on which we apply the x(t) input signal with a negligible τ delay.

We consider the receiver as being a linear filter. It makes the transformation of the input signal into the output one in the $L^2(T)$ space. The receiver's internal function can be expressed through an operator called L which acts on the x(t) function, resulting into the y(t) output signal as shown in figure 2. The transformation is given by the equation:

Fig. 1 Function of receiver

$$y(t) = L[x(t)] \tag{2}$$

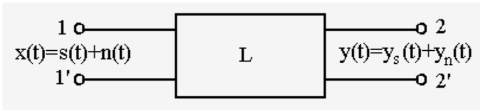

Fig. 2 The linear receiver

By applying the principle of overlapping effects we get:

$$y(t) = L[x(t)] = L[u(t) + n(t)] = L[u(t)] + L[n(t)] \tag{3}$$

We note:

$$L[u(t)] = y_u(t), L[n(t)] = y_n(t) \tag{4}$$

The general form of the L operator is being determined starting from the linear electrical line's response to a given signal $u(t)$, represented by the Duhamel integral

$$y(t) = A(0) \cdot u(t) + \int_0^t u(\tau) \cdot A'(t-\tau) d\tau \tag{5}$$

where A(t) is the index admittance, that is the response of the line to the step signal. Initially A(0)=0, we get:

$$y(t) = \int_0^\infty h(t-\tau) u(\tau) d\tau \tag{6}$$

where h(t-τ)=A'(t-τ), is called the ponder function , meaning the response to the Dirac impulse. Equation (6) shows that any linear operator is making a integral transformation of the signal function having as a nucleus the ponder function. The receivers function is defined in the frequency domain and for it the δ(t) spectral function of the signal is being computed:

$$U(f) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j2\pi ft} dt = 1 \tag{7}$$

the line's response to the δ(t) function, meaning the ponder function, has the following expression:

$$h(t) = \int_{-\infty}^{\infty} H(f) \cdot e^{-j2\pi ft} df \tag{8}$$

The inversed transformed function is :

$$H(f) = \int_{-\infty}^{\infty} h(t) \cdot e^{-j2\pi ft} dt \tag{9}$$

The H(f) function is called the frequency characteristic of the communication line and it is the Fourier transformation of the ponder function.
By the help of the previously presented equations one can determine the spectral function of the response as the product of the transfer characteristic and the spectral function of the input signal (figure 3).

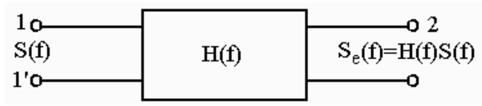

Fig. 3 Spectral function of the input signal

$$U_e(t) = H(f)\cdot U(f) \tag{10}$$

By using y(t) as time and frequency dependant we get the time dependant response as a convolution integral of the signal and the ponder function.

$$y(t) = \int_{-\infty}^{\infty} H(f)U(f)\cdot e^{j2\pi ft}df = \int_{0}^{\infty} h(t-\tau)u(\tau)d\tau \tag{11}$$

3. ADAPTIVE FILTER

In order to determine the linear x function, corresponding to the truth-like fraction we is the adaptive filter for the x(t) signal or a correlator. By a adapted filter to the u(t) signal we mean a linear filter who's ponder function is :

$$h(t) = u(T-t) \tag{12}$$

From such a filter's response to signal *x(t)* we get the x linear function:

$$\int_0^T x(t)\cdot u(t)dt = \frac{N_0}{2}\cdot \ln[l(x)] + \frac{1}{2}\cdot\int_0^T u^2(t)dt \tag{13}$$

Where N_0 is the noise spectral density and $\int_0^T u^2(t)dt = E$ is the signal energy.

Admitting u(t)=0 for t<0, t>T and the filter described in equation 12, the filter's response is

$$y(t) = \int_0^T x(\tau)h(t-\tau)d\tau = \int_0^T x(\tau)\cdot u(T-t+\tau)d\tau \tag{14}$$

from which

$$y(T) = \int_0^T x(\tau)u(\tau)d\tau \tag{15}$$

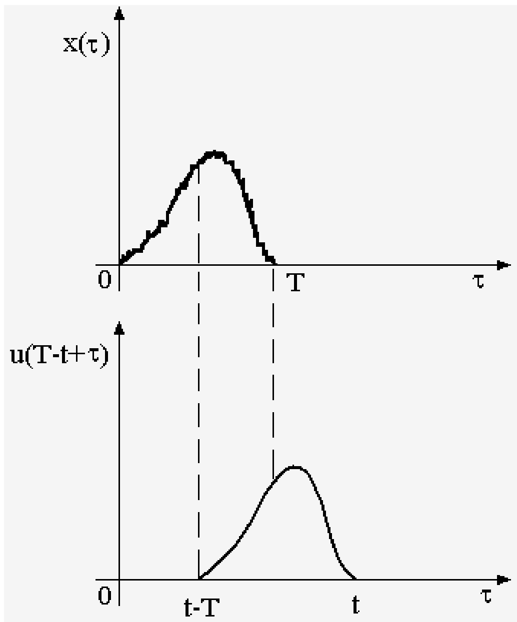

Fig. 4 The limits of the integral

Figure 5 shows the time dependant operations. The Fourier transformation of equation 12 is:

$$F[u(T-t)] = U^*(f)\cdot e^{-j2\pi fT} \tag{16}$$

therefore the frequency characteristic of the filter is equal to the conjugated spectral function of the signal to which the filter is adapted to, multiplied by a fazer representing the T delay.
The u(t) signal, together with the white noise, is passing through a linear filter with the H(f) transfer function. At the filter exit the noise spectral density is:

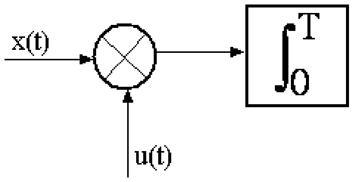

Fig. 5 Operations realize by filter

$$U_n(f) = \frac{1}{2} N_0 |H(f)|^2 \tag{16}$$

and the power:

$$N = \int_{-\infty}^{+\infty} U(f) \cdot H(f) e^{j2\pi ft} df \tag{17}$$

By knowing that at the t_M moment y(t) is reaching a maximum (y_M), and also the $P_i = ky_M^2$ instantaneous peak power, we can calculate the signal to noise fraction:

$$r = \frac{P_i}{N} \tag{18}$$

We wish to find a filter with a transfer function called H(f), so that the ratio above would be maximum. The condition for this extreme is found through variation calculus.

We conclude that if y_M, N, N_0 are not frequency dependant and are therefore considered constants. H(f) has the following form:

$$H(f) = a \cdot U^*(f) \cdot e^{-j2\pi ft_M} \tag{19}$$

In this equation we recognize the definition of the adaptive filter in the frequency domain.
In figure 6 is presented the frequencies characteristic of the adaptive filter.

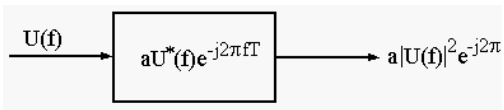

Fig. 6 The frequencies characteristic

4. CONCLUSIONS

As a conclusion the adaptive filter allows, in the case of having inputted the exacted adapted signal, to obtain a linear function for an output; also the filter allows us to obtain a maximum signal to noise ratio.
Going back to the adaptive filter definition in equation 12 we note that for T=0,

$$h(t) = u(-t) \tag{20}$$

If the signal is symmetrical u(t)=u(-t) therefore:

$$h(t) = u(t) \tag{21}$$

Hence in the case of symmetrical signals, the adaptive filter response to the Dirac impulse is exactly the signal the filter is adapted to. In relation to equation 6 and 14 we also note that a filter's response corresponds to a convolution integral, and the filter's response is adapted to a correlation.

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