

THEORETICAL STUDY ABOUT THE WAVES MOVING ACROSS THE OPEN CHANNEL FLOW

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Abstract: Unsteady flows in open channels occur frequently. Examples include waves moving across the free surfaces of lakes and reservoirs, changes in flow rates and water levels downstream from reservoir outlet works, sudden releases of water from burst dams and floods moving down rivers. Many of these problems can be modeled by assuming one-dimensional velocity distributions and pressure distributions that are hydrostatic along lines normal to the channel bottom.

Keywords: Unsteady open channel, pressure gradient, velocity distribution, the Saint-Venant equations

1. INTRODUCTIONS

The enounced problem may be resolved used a set of partial differential equations known as the Saint-Venant equations. The equations are hyperbolic, so the method of characteristics can be used in their solution. The use of characteristics is considerably more difficult for flow in open channels than for flow in pipes.

This is because the equations are nonlinear, so that the slope of the characteristic curves is not constant but is a function of the dependent variables. As a result, shocks or surges can appear spontaneously in these solutions, causing difficulties with numerical accuracy and stability.

In this chapter we will obtain the Saint-Venant equations for unsteady flow in a prismatic rectangular channel, figure 1.

Then the method of characteristics will be used to show the types of boundary and initial conditions that must be imposed and to illustrate an effective numerical technique that is sometimes used to solve these equations.

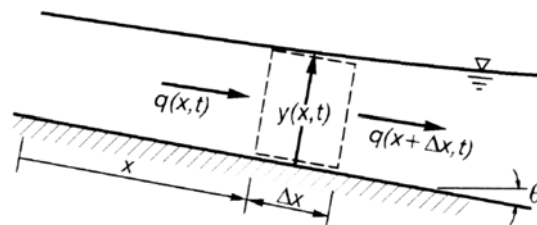


Figure.1. A control volume for unsteady flow in an open channel

2. THE SAINT-VENANT EQUATIONS

A control volume bounded by the free surface, the channel bottom and two surfaces normal to the channel bottom is shown with dashed lines in figure 1. The two surfaces normal to the channel bottom are fixed, which

means that fluid passes through them. The top boundary however, moves in the vertical direction as the free surface geometry changes with time.

Let $q(x, t)$ be the flow rate per unit width. Since the continuity principle states that the difference between flow rates entering and leaving the control volume must be balanced by the rate at which fluid is stored within, we obtain:

$$q(x, t) - q(x + \Delta x, t) = \frac{\partial(y\Delta x)}{\partial t} \quad (1)$$

Since x and t are independent variables, dividing (1) by Δx and putting all terms on the same side of the equation gives

$$\frac{q(x, t) - q(x + \Delta x, t)}{\Delta x} + \frac{\partial y}{\partial t} = 0 \quad (2)$$

Finally, letting $\Delta x = 0$ gives the continuity equation for flow in a prismatic rectangular channel with zero lateral inflow.

$$\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad (3)$$

Since $q = Uy$ in which U is the one-dimensional flux velocity. The equation (3) can be written with U and y as its two unknowns.

$$\frac{\partial(Uy)}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad (4)$$

Probably the simplest way to recover the momentum equation is to take the partial derivative of equation

$$h + \frac{v^2}{2g} + \frac{1}{g} \frac{\partial \Phi}{\partial t} = H$$

with respect

$$\frac{\partial h}{\partial t} + \frac{v^2}{g} \frac{\partial V}{\partial x} + \frac{1}{g} \frac{\partial^2 \Phi}{\partial x \partial t} = \frac{\partial H}{\partial x} \quad (5)$$

If pressures are hydrostatic along any vertical line, then

$$h = z(x) + y(x, t)$$

in which $z(x)$ channel bed elevation and the channel slope, θ , has been assumed small enough to make the approximations

$$\begin{aligned} \cos \theta &= 1; \\ \sin \theta &= \tan \theta = \theta = S_0 \end{aligned}$$

means bed slope.

Therefore, the first term in (5) can be rewritten as

$$\frac{\partial h}{\partial t} = \frac{dz}{dx} + \frac{dy}{dx} = -S_0 + \frac{dy}{dx} \quad (6)$$

in which the negative sign in front of S_0 , indicate that z decreases with x when S_0 is positive. (θ is positive when the channel slopes downward in the direction of flow.) In one-dimensional flow the velocity magnitude, V , is given by the velocity component in the x direction, U . Since x and t are independent variables, the third term becomes

$$\frac{\partial^2 \Phi}{\partial x \partial t} = \frac{\partial}{\partial t} \left(\frac{\partial \Phi}{\partial x} \right) = \frac{\partial U}{\partial t} \quad (7)$$

in which use has been made of $w = \frac{\partial \Phi}{\partial z}$.

Finally,

$$S_f = -\frac{\partial H}{\partial x} = \frac{f}{8} \frac{U^2}{gR}$$

allows us to rewrite the last term in (5) in terms of the friction slope.

$$-S_f = \frac{\partial H}{\partial x} \quad (8)$$

Then use of (6)-(8) in (5) and a bit of algebra gives the momentum equation for unsteady one-dimensional flow.

$$g \frac{\partial y}{\partial x} + U \frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} = g(S_0 - S_f) \quad (9)$$

Equations (4) and (9) are known as the Saint-Venant equations of open channel flow. When the right side of

$$S_f = -\frac{\partial H}{\partial x} = \frac{f}{8} \frac{U^2}{gR}$$

is used to approximate S_f , they contain U and y as their only unknowns.

3. CHARACTERISTIC FORM OF THE SAINT-VENANT EQUATIONS

Expansion of the first term in (4) gives

$$y \frac{\partial U}{\partial x} + U \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad (10)$$

Multiplication of (10) by an unknown parameter, λ and addition to (9) gives

$$(g + \lambda U) \frac{\partial y}{\partial x} + \lambda \frac{\partial y}{\partial t} + (U + \lambda y) \frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} = g(S_0 - S_f) \quad (11)$$

which can be altered slightly to

$$\lambda \left(\frac{g + \lambda U}{\lambda} \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} \right) + (U + \lambda y) \frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} = g(S_0 - S_f) \quad (12)$$

The directional derivative defined by $\frac{\partial \Phi}{\partial t} = \frac{\partial \Phi}{\partial x} + \frac{\partial \Phi}{\partial t} \frac{dt}{dx}$ allows (12) to be rewritten as a set of simultaneous ordinary differential equations that hold along a characteristic curve in the (x, t) plane.

$$\lambda \frac{\partial y}{\partial t} + \frac{dU}{dt} = g(S_0 - S_f) \quad (13)$$

The time derivatives in (13) are calculated along characteristic curves that satisfy the differential equation

$$\frac{dx}{dt} = \frac{g + \lambda U}{\lambda} = U + \lambda y \quad (14)$$

The second of the two equations in (14) gives $\lambda = \pm \sqrt{\frac{g}{y}}$ and use of this result in (13)-(14) gives two equations for each of two separate and distinct families of characteristic curves in the (x, t) plane,

$$\begin{aligned} \frac{d(U + 2c)}{dt} &= g(S_0 - S_f) \quad \text{along} \quad \frac{dx}{dt} = U + c \\ \frac{d(U - 2c)}{dt} &= g(S_0 - S_f) \quad \text{along} \quad \frac{dx}{dt} = U - c \end{aligned} \quad (15 \text{ a, b})$$

in which

$$c = \sqrt{gy} \quad (16)$$

4. CONCLUSIONS

Equations (15) show that c is the speed of a disturbance or wave when $U = 0$. These equations also show that, when $U \neq 0$, a disturbance travels with a speed of $U + c$ along one family of characteristics and with a speed $U - c$ along the other family of characteristics.

When the flow is *sub critical*, with $U < c$, one wave travels downstream and another wave travels upstream.

When the flow is *supercritical*, with $U > c$, $\frac{dx}{dt}$ is positive for both families of characteristics and waves travel only in the downstream direction.

Another way of stating this result that conditions at a point in *sub critical* flow are influenced by boundary conditions *both upstream* and *downstream* from the point in question.

On the other hand, conditions at a point in *supercritical* flow are influenced only by boundary conditions *upstream* from the point under consideration.

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