ANALYTICAL RELATION FOR DESIGN OF PROTECTIVE ZONES AT VERTICAL LIGHTNING RODS

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Abstract: This paper presents some relations established by authors, for analytical design of protective zones at vertical lightning rods, according to standard NTE 001/03/00. There are presented relations for usually cases in the energetic installations: a vertical lightning rod, two lightning rods with equal heights, and two lightning rods with different heights. The analytical models proposed by authors permit easy design and verification of protective surface for complex electrical installations. For example, using these relations it can be easy design the protective surface of substation.

Keywords: lightning arrester, lightning rods, analytical relation

1. INTRODUCTION

Application of methodology corresponding standard NTE 001/03/00 for lightning protective zones design is simply and easy to apply when height of installations is constant and heights of lightning rods are equal. When installations are in the variable plan, like connecting line, protective zones design is difficult. In these cases, it the installation must be divided in many subdivisions and design of protective zones must be made for these. For example, for the connecting lines with phase circuits in the different planes, it must be design protective zones for every 0.5 m, for every phase. These mean 150 subdivisions for 20-30 m connecting line.

This reason has determined the authors of this paper to develop an analytical methodology for design of protective surface for entire analyzed zone.

2. CASE OF A VERTICAL LIGHTNING ROD

We will represent surface which delimited protective zones of lightning rod, as a spatial function, z(x,y).

For a vertical lightning rod with height h, standard gives protective radius, r_x, according to altitude of the protected object, h_x [1]:

$$\frac{r_x}{h_a} = \frac{1,6}{1 + \dfrac{h_x}{h}} \cdot p \tag{1}$$

where

$$h_a = h - h_x \tag{2}$$

$$p = \frac{\sqrt{30}}{\sqrt{h}} = \frac{5,5}{\sqrt{h}} \tag{3}$$

if h > 30, else p=1.

If we consider arrangement of lightning rod in a point with x_0, y_0 coordinates, from relation (1) result successively:

$$h_x = \frac{1.6 \cdot p \cdot h - r_x}{r_x + 1.6 \cdot p \cdot h} \cdot h \tag{4}$$

$$z(x,y) = h_x \tag{5}$$

$$r_x = \sqrt{(x - x_0)^2 + (y - y_0)^2} \tag{6}$$

$$z(x,y) = \frac{1.6 \cdot p \cdot h - \sqrt{(x - x_0)^2 + (y - y_0)^2}}{\sqrt{(x - x_0)^2 + (y - y_0)^2} + 1.6 \cdot p \cdot h} \cdot h \tag{7}$$

A installation with h_x height and x_1, y_1 coordinates is protective if:

$$h_x < z(x_1, y_1) \tag{8}$$

Figure 1 show an example for a protective surface which is designed analytically

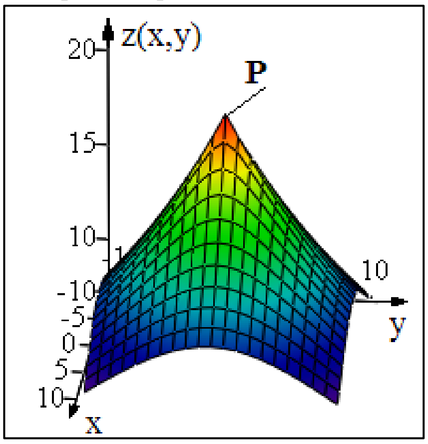

Figure 1
Protective zone surface of
vertical lightning rod P

3. CASE OF TWO VERTICAL LIGHTNING RODS

In case of two vertical lightning rods with equal heights, $h=z_1=z_2$, and x_1, y_1 and x_2, y_2 coordinates, it must be calculated R radius, figure 2 [1]. This is radius of circle which reach the protective rods peak, P_1, P_2 and point P_0 with x_0, y_0 coordinates and h_0 height, at the a/2 distance by protective rods.

$$a = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{9}$$

$$x_0 = \frac{x_1 + x_2}{2} \tag{10}$$

$$y_0 = \frac{y_1 + y_2}{2} \tag{11}$$

$$h_0 = h - \frac{a}{7 \cdot p} \tag{12}$$

$$z_0 = h_0 \tag{13}$$

In the lightning rods plane, it is calculated l_1, l_2, l_3 triangle lengths and s_p half perimeter:

$$l_1 = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} \tag{14}$$

$$l_2 = \sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2 + (z_2 - z_0)^2} \tag{15}$$

$$l_3 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \tag{16}$$

$$s_p = \frac{l_1 + l_2 + l_3}{2} \tag{17}$$

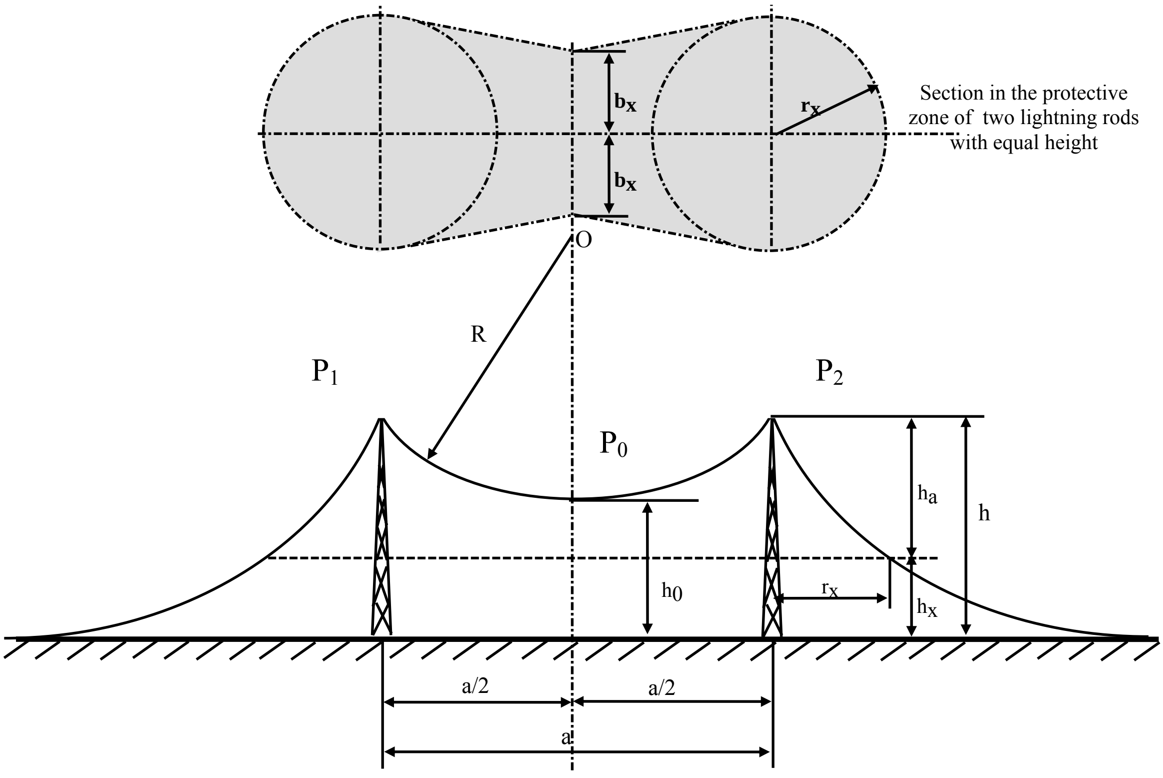

Figure 2. Protective zones of two vertical lightning rods

It is resulted triangle surface (Heron relation) and R radius:

$$A = \sqrt{s_p \cdot (s_p - l_1) \cdot (s_p - l_2) \cdot (s_p - l_3)} \tag{18}$$

$$R = \frac{l_1 \cdot l_2 \cdot l_3}{4 \cdot A} \tag{19}$$

Circle center has x_0, y_0 and z_0+R coordinate.

For assessment the equation of protective zone peak between both lightning rods it must be written x_m, y_m coordinates according to x,y current coordinates (figure 3). They are determined at the intersection of P_1, P_2 line with vertical line which goes through x, y point.

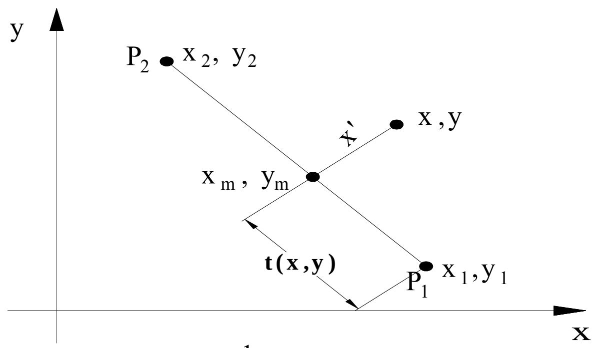

Figure 3
Assessment of t(x,y) and x' distances

$$x_m(x,y) = \frac{1}{a^2} \cdot \left[\left(x_2 - x_1\right)^2 \cdot x + \left(y - y_1\right) \cdot \left(x_2 - x_1\right) \cdot \left(y_2 - y_1\right) + \left(y_2 - y_1\right)^2 \cdot x_1 \right] \tag{20}$$

$$y_m(x,y) = y_1 + \left(y_2 - y_1\right) \cdot \frac{x_m(x,y) - x_1}{x_2 - x_1} \tag{21}$$

The equation of protective zone peak between both lightning rods, written from P_1 to P_2, in its plane, is:

$$zv(x,y) = \begin{cases} z_0 + R - \sqrt{R^2 - \left(t(x,y) - \dfrac{a}{2}\right)^2}\,, t(x,y) \le a,\ y_1 \le y_m(x,y) \le y_2 \\ where \quad t(x,y) = \sqrt{\left(x_m(x,y) - x_1\right)^2 + \left(y_m(x,y) - y_1\right)^2} \end{cases} \tag{22}$$

For assessment a spatial representation of protective zone between two lightning rods it must be evaluated x' (r_x) distance (fig.3) according to x, y current coordinates and (x_1,y_1), (x_2,y_2) lightning rod coordinates.

$$x'(x,y) = \sqrt{\left(x - x_m(x,y)\right)^2 + \left(y - y_m(x,y)\right)^2} \tag{23}$$

The equation of protective surface between both lightning rods is determined with relation as (1) where lightning rod height is replaced with relation (22) and r_x by x':

$$zo(x,y) = \frac{1.6 \cdot p \cdot zv(x) - x'(x,y)}{x'(x,y) + 1.6 \cdot p \cdot zv(x)} \cdot zv(x) \tag{24}$$

Finally, it results protective surface of two lightning rods:

$$z_1(x,y)=\frac{1.6\cdot p\cdot h-\sqrt{(x-x_1)^2+(y-y_1)^2}}{\sqrt{(x-x_1)^2+(y-y_1)^2}+1.6\cdot p\cdot h}\cdot h \tag{25}$$

$$z_2(x,y)=\frac{1.6\cdot p\cdot h-\sqrt{(x-x_2)^2+(y-y_2)^2}}{\sqrt{(x-x_2)^2+(y-y_2)^2}+1.6\cdot p\cdot h}\cdot h \tag{26}$$

$$z(x,y)=\max\left\{z_1(x,y),z_2(x,y),zo(x,y)\right\} \tag{27}$$

Figure 4 show the graphic.

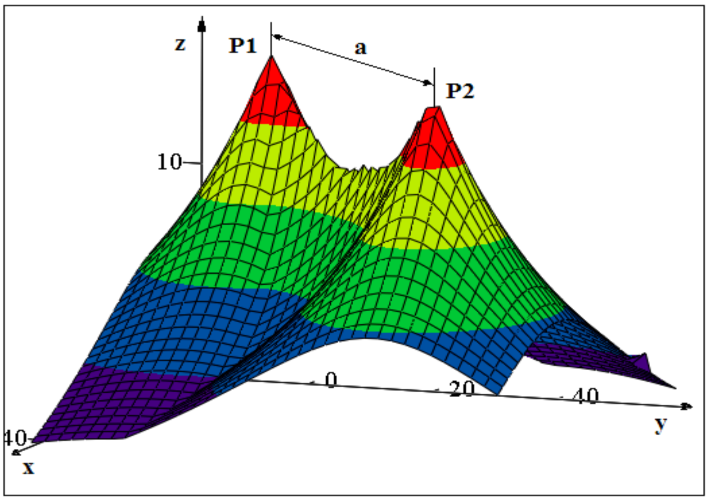

Figure 4
Protective zone surface
of two lightning rods
with equal heights

In case of two vertical lightning rods with different heights, $h_1=z_1$, $h_2=z_2$, $h_1 > h_2$, it must be assessed location of fictive lightning rod, near by P_1. This is situated at r_f distance by P_1 and a_1 by P_2.

For assessment of x_f, y_f coordinates of fictive lightning rod which is situated at $a-a_1$ distance by P_1, it is used line equation between P_1 and P_2 and circle equation by $r_f = a-a_1$ radius:

$$\begin{gathered}\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}\\(x-x_1)^2+(y-y_1)^2=r_f^2\end{gathered} \tag{28}$$

From here results:

$$x_f=x_1\pm\sqrt{\frac{r_f^2}{1+\left(\frac{y_2-y_1}{x_2-x_1}\right)^2}},\ x_1\le x_f\le x_2 \tag{29}$$

$$y_f=y_1+\frac{y_2-y_1}{x_2-x_1}\cdot\left(x_f-x_1\right) \tag{30}$$

$$z_f = h_2 \tag{31}$$

To achieve conditions $x_1 \le x_f \le x_2$, $y_1 \le y_f \le y_2$, in relation (29) appear sign + or –. Assessment equation is made similar with case of two lightning rods with equal heights, replacing in these relations x_1 with x_f, y_1 with y_f and a with a_1. Thus, coordinates of minimal point between lightning rods are calculated with relations:

$$h_0 = h_2 - \frac{a_1}{7 \cdot p} \tag{32}$$

$$x_0 = \frac{x_f + x_2}{2} \tag{33}$$

$$y_0 = \frac{y_f + y_2}{2} \tag{34}$$

Triangle lengths which determine R radius are:

$$l_1 = \sqrt{\left(x_f - x_0\right)^2 + \left(y_f - y_0\right)^2 + \left(z_f - z_0\right)^2} \tag{35}$$

$$l_2 = \sqrt{\left(x_2 - x_0\right)^2 + \left(y_2 - y_0\right)^2 + \left(z_2 - z_0\right)^2} \tag{36}$$

$$l_3 = \sqrt{\left(x_f - x_2\right)^2 + \left(y_f - y_2\right)^2 + \left(z_f - z_2\right)^2} \tag{37}$$

It is calculated R radius, using relations (17),(18), (19) and $x_m(x,y)$, $y_m(x,y)$ using relations (20), (21).
The equation of protective zone peak between both lightning rods, written from P_f to P_2, in its plane, is similar with relation (22), replacing x_1, y_1 with x_f, y_f.
Equation of protective surface between both lightning rods is similar with (24) and of protective surface is similar with (27). Figure 5 show an example for protective surface of two lightning rods with different heights.

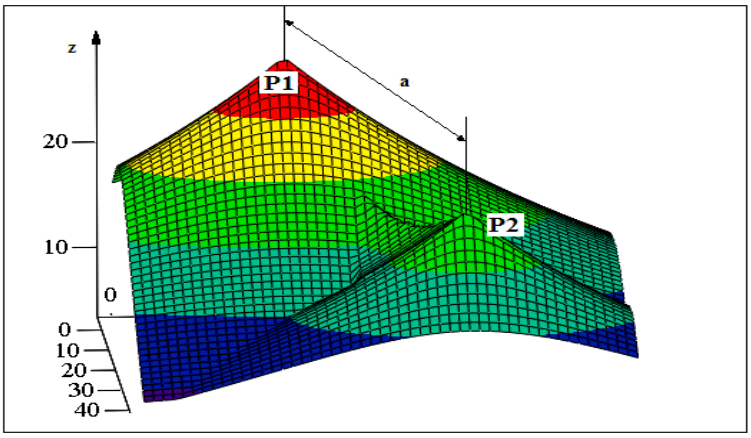

Figure 5
Protective zone surface of two lightning rods with different heights

For three or many vertical lightning rods, are written equations of protective zone for every couple of lightning rods and are overlapped.

4. CONCLUSIONS

Results presented up lead to the following conclusions:

- Using of analytical relations for design of protective surface of lightning rods is necessary for check out of protection of installations with variable height. Using of classical technologies according to standard NTE 001/03/00 is difficult [4].
- Analytical description of protective surface of lightning rods according to its coordinates and to its heights permits graphical design of them. This leads to good understanding, to easier check out and to detection of errors.
- Using the relations presented above for lightning rods couple permit design of protective zone surface for complex installations, like substations.

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