COMPUTER ANALYSIS OF DYNAMIC RESPONSE OF GEAR TRANSMISSIONS WITH BACKLASH CONSIDERATION

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Abstract. The paper presents an analytical and computer-aided analysis of the dynamic characteristics of spur gear transmissions. The dynamic analysis of geared systems is difficult to accurately investigate due to the nonlinear characteristics accounting for periodic changes of tooth stiffness and gear backlash. An improved model to calculate the cyclic mesh stiffness of gear pairs is developed in order to obtain reliable data for the prediction of gear dynamic behavior. The relation between gear ratio and stiffness function is presented. A comparative study has been performed to investigate the effects of the mesh stiffness variation and the backlash between contacting teeth on the dynamic characteristics of gear systems.

Keywords: spur gears , mesh stiffness, gear backlash, dynamic factor, dynamic transmission error, addendum modifications.

1. INTRODUCTION

Gear transmissions are used in specific robotic mechanisms with small size actuators to achieve high driving torques at the end-effector. The dynamic characteristics of spur gear pairs are significant for the design and control of these mechanisms [4], [5], [9]. Reported studies of gear dynamics include Band and Velex [3], Litvin et al. [7], Yang and Sun [10].

The position accuracy in a motion control system is affected by vibration due to nonlinear effects such mesh stiffness or backlash. The time-varying mesh stiffness represents the main cause of undesired vibrations in the case of gear transmissions with high manufacturing precision. The backlash between meshing teeth can generate instability and position errors in gear pairs under dynamic conditions.

In order to obtain reliable data for the prediction of gear dynamic behavior, the dynamic model accounts the non-linear time varying mesh stiffness and backlash.

2. THE MATHEMATICAL MODEL

The mechanical model for a gear pair in mesh is shown in Figure 1. In this model, the teeth are considered as springs and the gear blanks as inertia masses. The differential equations of motion can be expressed as

$$J_1\ddot{\theta}_1 + F_d \cdot r_{b1} = M_{t1} \tag{1}$$

$$J_2\ddot{\theta}_2 - F_d \cdot r_{b2} = M_{t2} \tag{2}$$

where θ_1, θ_2 are the rotation angle of the pinion and the driven gear, respectively. J_1 and J_2 are the mass moments of inertia of the gears.

M_{t1} and M_{t2} denote the external torques applied on the gear system and r_{b1}, r_{b2} are the base circle radii of the gears.

The dynamic load is expressed as

$$F_d = \sum_{i=1}^{N} F_{di}(t) \tag{3}$$

where

$$F_{di} = k_i(t)\left[r_{b1}\theta_1 - r_{b2}\theta_2\right] + c(r_{b1}\dot{\theta}_1 - r_{b2}\dot{\theta}_2) \tag{4}$$

By introducing the composite coordinate

$$x_d = r_{b1}\theta_1 - r_{b2}\theta_2 \tag{5}$$

Eqs. (1) and (2) yield an equation of motion in the following form

$$m\ddot{x}_d + c\dot{x}_d + \sum_{i=1}^{N} F_{di}(t) = F_n \tag{6}$$

where

$$\begin{aligned} F_{di}(t) &= k_i(t)\ \left[x_d + e_i(t)\right] \\ F_{di}(t) &= 0\ , \quad \text{if} \quad F_{di}(t) < 0 \end{aligned} \tag{7}$$

where c is the damping coefficient calculated by

$$c = 2\xi\sqrt{m_e k_m} \tag{8}$$

and N represents the number of simultaneous tooth pairs in mesh.

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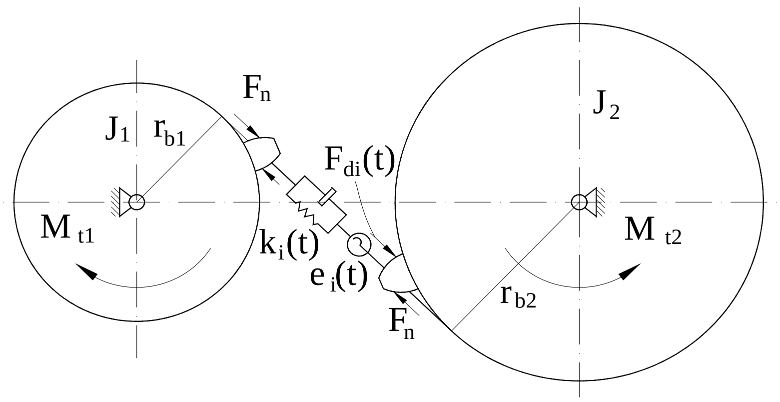


Fig.1. Dynamic model of gear pair

If gear *1* is the driving gear and δ is linear the backlash, the following conditions can occur:

(i) $$x_{d_i} > 0$$

This is the normal operating case and the dynamic tooth load

(ii) $$x_{d_i} \le 0 \text{ and } \left|x_{d_i}\right| \le \delta$$

In this case the gear will separate and the contact between gears will be lost. Hence, $F_{di} = 0$.

(iii) $$x_{d_i} < 0 \text{ and } \left|x_{d_i}\right| > \delta$$

In this case gear 2 will collide with gear 1 on the back side.
The meshing resonance frequency of the gear pair is determined as follows

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_e}} \tag{9}$$

where m_e m represents the equivalent inertia mass and k_m is the average mesh stiffness of the gear pair.

3. MESH STIFFNESS

For a pair of contacting teeth i, the time-varying mesh stiffness $k_i(t)$ acts as a parameter excitation. The gear tooth is modeled to be a nonuniform cantilever beam supported by a flexible fillet region and foundation [1] as shown in Figure 2. The total deflection f_j of a pair of meshing teeth is expressed as

$$f_j = \sum_{j=1}^{2} f_{bj} + \sum_{j=1}^{2} f_{fj} + f_H \tag{10}$$

where f_b - the deflection due to bending, shear and axial deformation of the tooth corresponding to the involute profile; f_f - the deflection due to the flexibility of the tooth foundation and fillet; f_H - the local compliance of the Hertzian contact.

The individual tooth mesh stiffness is defined in the normal direction to the contact surface as

$$k_j = \frac{F_n}{f_j} \tag{11}$$

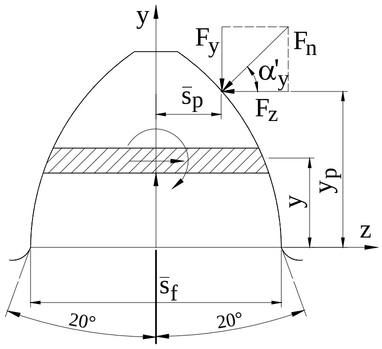
where F_n is the normal tooth load.

The teeth pairs in contact act like parallel springs. Therefore, the total mesh stiffness during each engagement cycle can be written as a function of the position of contact point on the action line

$$k_t = k_s^{I} + k_s^{II} \quad , \quad \text{for double-tooth contact}$$

$$k_t = k_s^{I} \quad , \quad \text{for simple - tooth contact}$$

where I and II are the mating points of the teeth pairs (Fig. 3).



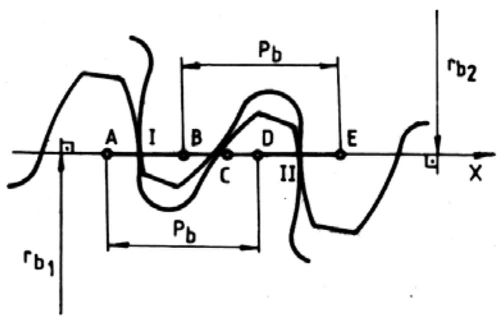
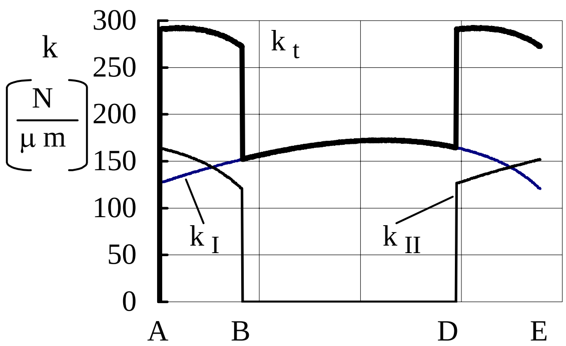


Fig.2. Gear tooth deflection model *Fig.3. Specific contact points of mesh cycle*

Referring to Figure 3, the following mesh points were used to represent the successive positions of contact point of a tooth as it passes through the zone of loading: the initial point of engagement, A; the lowest point of single-tooth contact, B; the highest point of single-tooth contact, D; and the final point of engagement, E. Section AB and DE are double - tooth contact zone and section BD is the single tooth - contact zone.

The time-varying mesh stiffness is mainly caused by the following factors: (i) the variation of the single mesh stiffness along the line of action; (ii) the fluctuation of the total number of total pairs in contact during the engagement cycle.



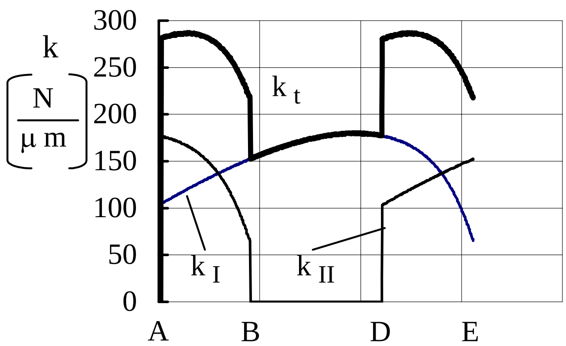
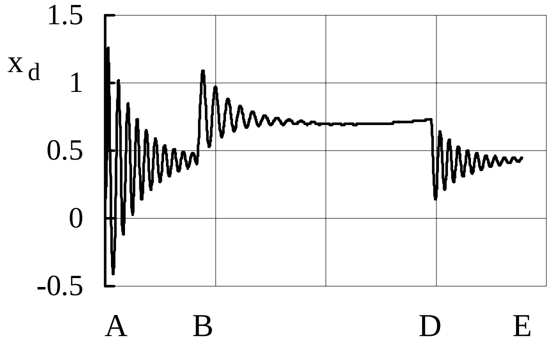


Fig.4. Variation of mesh stiffness components of gear pair GP1 *Fig.5. Variation of mesh stiffness components of gear pair GP2*

Two gear pairs (Table 1) are chosen to illustrate in Figures 4 and 5 the variation of individual and total mesh stiffness during a mesh cycle. For a gear pair, average mesh stiffness k_m decreases with decreasing contact ratio.

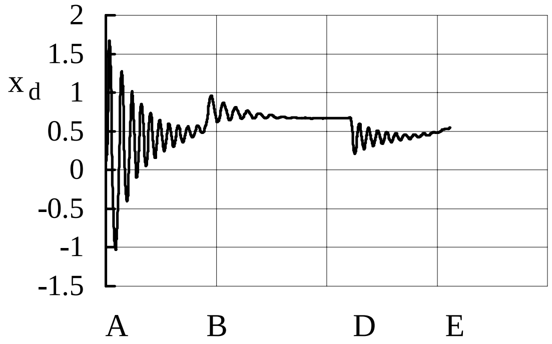
4. DYNAMIC SIMULATION

Specifications of the pertinent geometrical and kinematics parameters of the analyzed gear pairs are shown in Table 1, where z_1, z_2 represent the tooth number of a gear pair, x_1, x_2 are the addendum modification coefficients and ε_α is the transverse contact ratio. These parameters are for spur gear pairs having: face-width of gears, b=12 [mm] and center distance, a = 70 [mm]. Additionally, the damping ratio ξ =0.12 is considered in the dynamic analysis.



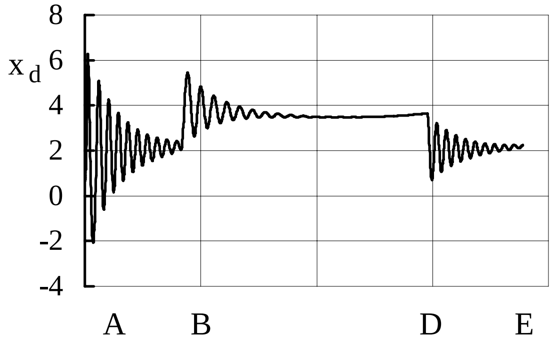
$F_n / b = 10\left[N/m\right] \quad \omega_1 = 50\left[s^{-1}\right]$

Fig.6. Variation of dynamic transmission error of gear pair GP1



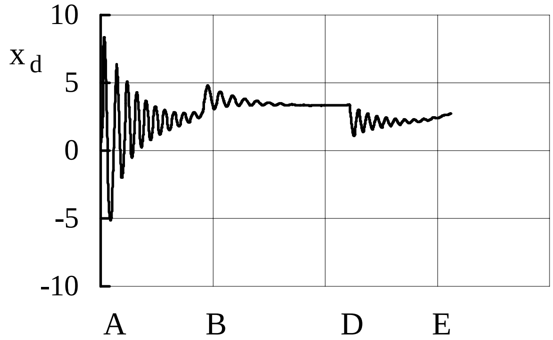
$F_n / b = 10\left[N/m\right] \quad \omega_1 = 50\left[s^{-1}\right]$

Fig.7. Variation of dynamic transmission error of gear pair GP2



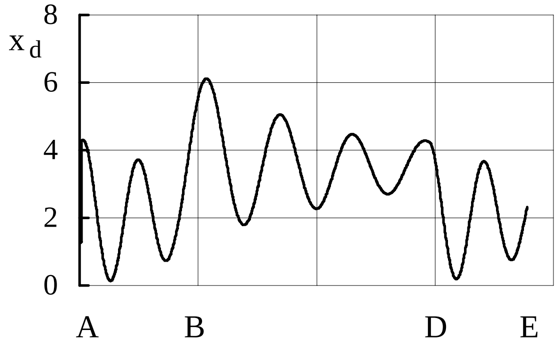
$F_n / b = 50\left[N/m\right] \quad \omega_1 = 50\left[s^{-1}\right]$

Fig.9. Variation of dynamic transmission error of gear pair GP1



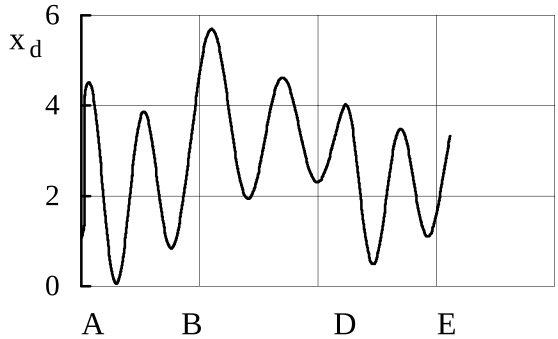
$F_n / b = 50\left[N/m\right] \quad \omega_1 = 50\left[s^{-1}\right]$

Fig.10. Variation of of dynamic transmission error of gear pair GP2



$F_n / b = 50\left[N/m\right] \quad \omega_1 = 280\left[s^{-1}\right]$

F*ig.11. Variation of dynamic transmission error of gear pair GP1*



$F_n / b = 50\left[N/m\right] \quad \omega_1 = 280\left[s^{-1}\right]$

Fig.12. Variation of of dynamic transmission error of gear pair GP2

Table 1

Gear pair	z_1	z_2	m	x_1	x_2	ε_α	k_m N/μm]	f_n [Hz]
GP1	18	29	3	0.8	-0.66	1.54	216.4	4419
GP2	18	76	1.5	0.8	-0.82	1.56	249.8	3591

In the analysis of dynamic loads, the transmitting load is defined as F_n / b, where F_n represents the external static load transmitted by the gear teeth that is proportional to the applied torque M_t.

A computer program was developed for simulating the dynamic characteristics of spur gear pairs. The equations of motion are solved by the fourth-order Runge-Kutta method. Computer analysis of dynamic characteristics includes different gear pairs with combination of addendum modifications and gear ratio.

The dynamic transmission error x_d are considered in the analysis. Figures 6 through 11 show the variation of dynamic response of spur gear systems with different operation conditions. The effects of backlash and variable mesh stiffness on the amplitude and the variation of dynamic displacement can be analyzed from these data. The successive impacts correspond to the initial period of operating time in relation to the initial velocity of the pinion.

5. CONCLUSIONS

Dynamic transmission error is a result of the instantaneous contact conditions between active tooth profiles. An analytical procedure for calculating dynamic characteristics of spur gear pairs with addendum modifications and backlash between contacting teeth is presented. This procedure predicts the effect of the time-varying mesh stiffness and backlash between meshing teeth on the variation of the dynamic transmission error during the meshing cycle of the gear pairs. The time-varying mesh stiffness along the path of contact is found by using an exact analytical model.

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