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AN APPLICATION OF COMPLEX LEGENDRE TRANSFORMATION TO V-COHOMOLOGY GROUPS

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Abstract. In this paper, using the Lagrangian-Hamiltonian formalism ($\mathcal{L}$-dual proces) on the holomorhic tangent bundle of a complex Lagrange space (M, L), we obtain similar results as in [10] concerning to v-cohomology groups of a complex Hamilton space (M, H). Finally we study a relative vertical cohomology associated to complex Legendre transformation.

1. Introduction and preliminaries

In [10] are introduced the v-cohomology groups of a complex Finsler (Lagrange) space. The main purpose of this paper is to find a similar cohomology of a complex Hamilton space. In this sense, firstly we make a short review on the geometry of the holomorphic tangent and cotangent bundles of a complex manifold endowed with a complex regular Lagrangian and a complex regular Hamiltonian, respectively. Next, following [8], [9], using the complex Legendre transformation, we briefly recall the complex Lagrangian-Hamiltonian formalism (the $\mathcal{L}$-dual proces).

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In the last part of the paper, we define (p,s,r,q)-forms with complex values on $T^{'*}M$ as the image by complex Legendre transformation of the (p,q,r,s)-forms with complex values on $T^{'}M$, we prove a Grothendieck-Dolbeault type lema for these forms and we define the v-cohomology groups of complex Hamilton spaces. Finally, we study the relative vertical cohomology associated to complex Legendre transformation.

Let us consider a complex manifold M where $\dim_{\mathbb{C}} M = n$ and $(U, z^i), i = \overline{1,n}$ are the complex coordinates in a local chart. The complexification $T_{\mathbb{C}}M$ of the tangent bundle is decomposed in each point $z \in M$ after the $(1,0)$ vector fields and their conjugates of $(0,1)$ type, $T_{\mathbb{C}}M = T^{'}M \oplus T^{''}M$. As it is well-known [1], [2], [8], $T^{'}M$ is also a complex manifold of complex dimension $2n$ and the natural projection $\pi_T : T^{'}M \to M$ defines on $V(T^{'}M) = \{\xi \in T^{'}(T^{'}M)/\, \pi_{T*}(\xi) = 0\}$ a structure of holomorphic vector bundle of rank n over $T^{'}M$, called the holomorphic vertical bundle.

A given supplementary subbundle $H(T^{'}M)$ of $V(T^{'}M)$ in $T^{'}(T^{'}M)$ i.e. $T^{'}(T^{'}M) = H(T^{'}M) \oplus V(T^{'}M)$ defines a *complex nonlinear connection*, briefly c.n.c. on $T^{'}M$.

Considering also their conjugates $\overline{V(T^{'}M)}$ and $\overline{H(T^{'}M)}$, we obtain the following decomposition of the complexified tangent bundle $T_{\mathbb{C}}(T^{'}M) = H(T^{'}M) \oplus V(T^{'}M) \oplus \overline{H(T^{'}M)} \oplus \overline{V(T^{'}M)}$.

If $(\pi_T^{-1}(U), u = (z^i, \eta^i))$ are the complex local coordinates on $T^{'}M$ and if $N_i^j(z,\eta)$ are the coefficients of the c.n.c., then the following set of complex vector fields $\{\delta/\delta z^i = \partial/\partial z^i - N_i^j \partial/\partial \eta^j\}$, $\{\partial/\partial \eta^i\}$, $\{\delta/\delta \overline{z}^i = \partial/\partial \overline{z}^i - \overline{N_i^j} \partial/\partial \overline{\eta}^j\}$, $\{\partial/\partial \overline{\eta}^i\}$ are called the local adapted bases of $H(T^{'}M)$, $V(T^{'}M)$, $\overline{H(T^{'}M)}$ and $\overline{V(T^{'}M)}$, respectively. The dual adapted bases are given by $\{dz^i\}$, $\{\delta\eta^i = d\eta^i + N_j^i dz^j\}$, $\{d\overline{z}^i\}$ and $\{\delta\overline{\eta}^i = d\overline{\eta}^i + \overline{N_j^i} d\overline{z}^j\}$, respectively.

Now, let us consider $L : T^{'}M \to \mathbb{R}$ a complex regular Lagrangian, that is a function $L(z,\eta)$ defining a metric tensor $g_{i\overline{j}} = \partial^2 L/\partial\eta^i \partial\overline{\eta}^j$ which is Hermitian, i.e. $g_{i\overline{j}} = \overline{g_{j\overline{i}}}$ and $\det(g_{i\overline{j}}) \neq 0$ in any point $u = (z,\eta)$ of $T^{'}M$. By $g^{\overline{j}i}$ is denoted its inverse metric tensor. According to [8], a c.n.c. on $T^{'}M$ depending only of the complex Lagrangian L, is the Chern-Lagrange c.n.c., locally given by $\overset{CL}{N_i^j} = g^{\overline{k}j} \partial^2 L/\partial z^i \partial\overline{\eta}^k$.

Definition 1.1. *The pair (M, L) is called a complex Lagrange space.*

In the sequel, we consider $\pi_T^* : T^{'*}M \to M$ the holomorphic cotangent bundle of M. Likewise as above, $T^{'*}M$ has a natural structure of complex manifold of complex dimension $2n$ and a point is denoted by $u^* = (z^k, \zeta_k)$, $k = \overline{1,n}$. If we consider $V(T^{'*}M) = \ker \pi_{T*}^*$ the holomorphic vertical bundle over $T^{'*}M$ then, a c.n.c. on $T^{'*}M$ is defined by a supplementary distribution $H(T^{'*}M)$ of $V(T^{'*}M)$ in $T^{'}(T^{'*}M)$, i.e. $T^{'}(T^{'*}M) = H(T^{'*}M) \oplus V(T^{'*}M)$. By conjugation, we obtain a decomposition of the complexified tangent bundle,

$T_{\mathbb{C}}(T^{'*}M) = H(T^{'*}M) \oplus V(T^{'*}M) \oplus \overline{H(T^{'*}M)} \oplus \overline{V(T^{'*}M)}$.

If $N_{jk}(z, \zeta)$ are the coefficients of the c.n.c. on $T^{'*}M$, then the following set of complex vector fields $\{\delta^*/\delta z^i = \partial/\partial z^i + N_{ji}\partial/\partial\zeta_j\}$, $\{\partial/\partial\zeta_i\}$, $\{\delta^*/\delta\overline{z}^i = \partial/\partial\overline{z}^i + \overline{N_{ji}}\partial/\partial\overline{\zeta}_j\}$, $\{\partial/\partial\overline{\zeta}_i\}$ are called the local adapted bases of $H(T^{'*}M)$, $V(T^{'*}M)$, $\overline{H(T^{'*}M)}$ and $\overline{V(T^{'*}M)}$, respectively. The dual adapted bases are denoted by $\{d^*z^i\}$, $\{\delta\zeta_i = d\zeta_i - N_{ij}d^*z^j\}$, $\{d^*\overline{z}^i\}$ and $\{\delta\overline{\zeta}_i = d\overline{\zeta}_i - \overline{N_{ij}}d^*\overline{z}^j\}$, respectively.

A complex regular Hamiltonian is a function $H : T^{'*}M \to \mathbb{R}$ such that $h^{\bar{j}i} = \partial^2 H/\partial\zeta_i\partial\overline{\zeta}_j$ defines a Hermitian metric tensor on $T^{'*}M$, i.e. $h^{\bar{j}i} = \overline{h^{i\bar{j}}}$ and $\det(h^{\bar{j}i}) \neq 0$ on $T^{'*}M$. Let $h_{i\bar{j}}$ be its inverse. A c.n.c. connection on $T^{'*}M$ depending only of the complex Hamiltonian H is the Chern-Hamilton c.n.c., locally given by $\overset{CH}{N}_{ij} = -h_{i\bar{k}}\partial^2 H/\partial z^j\partial\overline{\zeta}_k$.

Definition 1.2. *The pair (M, H) is called a complex Hamilton space.*

In the real case is well-known the Lagrangian-Hamiltonian formalism from the clasical mechanics, this being possible via Legendre transformation. An excelent solution in the study of real geometry of the correspondent spaces was given by R. Miron [5].

In the complex case, a solution of complex Lagrangian-Hamiltonian formalism is recently given by ([8], Ch. VI.7), by using a complex Legendre morphism. By complex Legendre transformation (the $\mathcal{L}$-dual proces) the image of a complex Lagrange space is (at least locally) a complex Hamilton space. The complex Legendre transformation pushes-forward and its inverse pulls-back the various described geometric objects of a complex Hamilton space, respectively.

Let us consider L a local Lagrangian on $U \subset T^{'}M$. Then the map $\phi : U \subset T^{'}M \to \overline{U^*} \subset \overline{T^{'*}M}$ given by $\phi(z^k, \eta^k) = (z^k, \overline{\zeta}_k = \partial L/\partial\overline{\eta}^k)$ is a local diffeomorphism. Since the sections of $V(T^{'}M)$ are identified with those of $T^{'}M$, we can extend ϕ to the open set of

$V(T^{'}M)$. By conjugation, the local diffeomorphism $\phi \times \overline{\phi}$ sends the sections of the complexified bundle $V(T^{'}M) \times \overline{V(T^{'}M)}$ into sections of $V(T^{'*}M) \times \overline{V(T^{'*}M)}$. This (local) morphism is called *the complex Legendre transformation*, briefly c.L.t.

Then, locally the function $H = \zeta_k \eta^k + \overline{\zeta}_k \overline{\eta}^k - L$ defines a regular (local) Hamiltonian on $T^{'*}M$. By the inverse $\phi^{-1} : \overline{U^*} \to U$, $\phi^{-1}(z^k, \overline{\zeta}_k) = (z^k, \eta^k = \partial H/\partial \zeta_k)$ from a Hamiltonian structure on $T^{'*}M$, a Lagrangian structure on $T^{'}M$ is obtained by $L = \zeta_k \eta^k + \overline{\zeta}_k \overline{\eta}^k - H$.

The properties obtained by c.L.t. are called $\mathcal{L}$-dual one to other. As in [8], [9], in the following, with " $*$ " will be designed the image of an object by ϕ and with " $\circ$ " their image by ϕ^{-1}.

According to [8], the unique pair of c.n.c. on $T^{'}M$ and on $T^{'*}M$ which correspond by $\mathcal{L}$-duality is given by Chern-Lagrange c.n.c. and Chern-Hamilton c.n.c., i.e. $(\overset{CL}{N_i^k})^* = \overset{CH}{N_{ki}}$ and $(\overset{CH}{N_{ki}})^\circ = \overset{CL}{N_i^k}$. In the sequel we consider the simply notations: $\partial/\partial \zeta^k := h_{k\bar{j}} \partial/\partial \overline{\zeta}_j$, $\partial/\partial \overline{\zeta}^k := h_{j\bar{k}} \partial/\partial \zeta_j$, $\delta \zeta^k := h^{\bar{j}k} \delta \overline{\zeta}_j$ and $\delta \overline{\zeta}^k := h^{\bar{k}j} \delta \zeta_j$. We have

Proposition 1.1. ([8]). *If the adapted bases and cobases are considered with respect to Chern-Lagrange c.n.c. and Chern-Hamilton c.n.c., the following equalities hold by $\mathcal{L}$-duality*

- (i) $(f^*)^\circ = f$, $\forall\, f \in \mathcal{F}(U)$, $(g^\circ)^* = g$, $\forall\, g \in F(U^*)$;
- (ii) $(\delta/\delta z^k)^* = \delta^*/\delta z^k$, $(\partial/\partial \eta^k)^* = \partial/\partial \zeta^k$, $(\delta/\delta \overline{z}^k)^* = \delta^*/\delta \overline{z}^k$, $(\partial/\partial \overline{\eta}^k)^* = \partial/\partial \overline{\zeta}^k$;
- (iii) $(\delta^*/\delta z^k)^\circ = \delta/\delta z^k$, $(\partial/\partial \zeta^k)^\circ = \partial/\partial \eta^k$, $(\delta^*/\delta \overline{z}^k)^\circ = \delta/\delta \overline{z}^k$, $(\partial/\partial \overline{\zeta}^k)^\circ = \partial/\partial \overline{\eta}^k$;
- (iv) $(dz^k)^* = d^* z^k$, $(\delta \eta^k)^* = \delta \zeta^k$, $(d\overline{z}^k)^* = d^* \overline{z}^k$, $(\delta \overline{\eta}^k)^* = \delta \overline{\zeta}^k$;
- (v) $(d^* z^k)^\circ = dz^k$, $(\delta \zeta^k)^\circ = \delta \eta^k$, $(d^* \overline{z}^k)^\circ = d\overline{z}^k$, $(\delta \overline{\zeta}^k)^\circ = \delta \overline{\eta}^k$.

2. V-cohomology groups of complex Hamilton spaces

At the begining of this section following [10], we make a short review on v-cohomology groups of a complex Lagrange (Finsler) space (M, L).

Let us consider $\mathcal{A}^{p,q,r,s}(T^{'}M)$ the set of all (p,q,r,s)-forms with complex values on $T^{'}M$ locally defined by,

$$\omega = \sum \omega_{I J \overline{H}\, \overline{K}} dz^I \wedge \delta \eta^J \wedge d\overline{z}^H \wedge \delta \overline{\eta}^K \tag{2.1}$$

where $I = (i_1, \ldots, i_p)$; $J = (j_1, \ldots, j_q)$; $H = (h_1, \ldots, h_r)$; $K = (k_1, \ldots, k_s)$ and the sum is after the indices $i_1 \leq \ldots \leq i_p$; $j_1 \leq \ldots \leq j_q$; $h_1 \leq \ldots \leq h_r$ and $k_1 \leq \ldots \leq k_s$, respectively.

The conjugated vertical differential operator $d^{''v} : \mathcal{A}^{p,q,r,s} \to \mathcal{A}^{p,q,r,s+1}$ is locally defined by

$$d^{''v}\omega = \sum \frac{\partial \omega_{I\,J\,\overline{H}\,\overline{K}}}{\partial \overline{\eta}^k} \delta\overline{\eta}^k \wedge dz^I \wedge \delta\eta^J \wedge d\overline{z}^H \wedge \delta\overline{\eta}^K. \tag{2.2}$$

This operator has the property $(d^{''v})^2 = 0$ and satisfies a Dolbeault type lemma (for details see [10]). Also, the v-cohomology groups of a complex Lagrange space with coefficients in the sheaf $\Phi^{p,q,r}$ of germs of $(p, q, r, 0)$-forms $d^{''v}$-closed, are given by

$$H^s(M, L, \Phi^{p,q,r}) = Z^{p,q,r,s}/d^{''v}\mathcal{A}^{p,q,r,s-1}(T^{'}M) \tag{2.3}$$

where $Z^{p,q,r,s}$ is the space of $d^{''v}$-closed (p, q, r, s)-forms.

In the sequel, using the $\mathcal{L}$-dual proces we obtain the v-cohomology groups of complex Hamilton spaces.

For $\omega \in \mathcal{A}^{p,q,r,s}(T^{'}M)$ locally given by (2.1) we denote

$$\omega^* := \varphi(\omega)$$

the image of ω by c.L.t. and we consider

$$\mathcal{A}^{p,s,r,q}(T^{'*}M) = \varphi(\mathcal{A}^{p,q,r,s}(T^{'}M)) = \{\varphi(\omega) | \omega \in \mathcal{A}^{p,q,r,s}(T^{'}M)\}. \tag{2.4}$$

Since c.L.t is a diffeomorphism, $\varphi : \mathcal{A}^{p,q,r,s}(T^{'}M) \to \mathcal{A}^{p,s,r,q}(T^{'*}M)$ is bijective and

$$\varphi^{-1}(\omega^*) = (\varphi(\omega))^{\circ} = \omega.$$

According to Proposition 1.1., the local expression of $\varphi(\omega)$ is

$$\varphi(\omega) = \sum \omega^*_{I\,\overline{K}\,\overline{H}\,J} d^*z^I \wedge \delta\overline{\zeta}^K \wedge d^*\overline{z}^H \wedge \delta\zeta^J \tag{2.5}$$

where $\omega^*_{I\,\overline{K}\,\overline{H}\,J}(z, \zeta) = (\omega_{I\,J\,\overline{H}\,\overline{K}}(z, \eta))^*$, $d^*z^I = d^*z^{i_1} \wedge \ldots \wedge d^*z^{i_p}$, $\delta\overline{\zeta}^K = \delta\overline{\zeta}^{k_1} \wedge \ldots \wedge \delta\overline{\zeta}^{k_s}$, $d^*\overline{z}^H = d^*\overline{z}^{h_1} \wedge \ldots \wedge d^*\overline{z}^{h_r}$ and $\delta\zeta^J = \delta\zeta^{j_1} \wedge \ldots \wedge \delta\zeta^{j_q}$.

We consider the following diagram

$$\begin{array}{ccc}
\mathcal{A}^{p,q,r,s}(T^{'}M) & \xrightarrow{d^{''v}} & \mathcal{A}^{p,q,r,s+1}(T^{'}M) \\
\downarrow \varphi & & \downarrow \varphi \\
\mathcal{A}^{p,s,r,q}(T^{'*}M) & \longrightarrow & \mathcal{A}^{p,s+1,r,q}(T^{'*}M)
\end{array}$$

and we define $d^{'*v}:\mathcal{A}^{p,s,r,q}(T^{'*}M)\to\mathcal{A}^{p,s+1,r,q}(T^{'*}M)$ by

$$d^{'*v}=\varphi\circ d^{''v}\circ\varphi^{-1}. \tag{2.6}$$

Proposition 2.1. *The operator $d^{'*v}$ satisfies $(d^{'*v})^2=0$.*

Proof. According to (2.6) we have

$$(d^{'*v})^2=(\varphi\circ d^{''v}\circ\varphi^{-1})^2=\varphi\circ(d^{''v})^2\circ\varphi^{-1}$$

and taking into account $(d^{''v})^2=0$ and φ is bijective we get $(d^{'*v})^2=0$. □

Let us consider $\Phi^{p,r,q}(U^*)=\{\omega^*\in\mathcal{A}^{p,0,r,q}(U^*)/d^{'*v}\omega^*=0\}$ the set of all $d^{'*v}$-closed $(p,0,r,q)$-forms on U^*. We have

Theorem 2.1. *Let ω^* be a $d^{'*v}$-closed (p,s,r,q)-form defined on a neighborhood U^* on $T^{'*}M$ and $s\geq 1$. Then there exists a $(p,s-1,r,q)$-form θ^* on some neighborhood $U^{'*}\subset U^*$ and such that $d^{'*v}\theta^*=\omega^*$ on $U^{'*}$.*

Proof. Let ω^* be a (p,s,r,q)-form on U^* such that $d^{'*v}\omega^*=0$. Then

$$(\varphi\circ d^{''v}\circ\varphi^{-1})\omega^*=\varphi(d^{''v}(\varphi^{-1}\omega^*))=0$$

and since φ is bijective we get $d^{''v}(\varphi^{-1}\omega^*)=0$, for $\varphi^{-1}\omega^*=\omega$ a (p,q,r,s)-form on $U=\phi^{-1}(U^*)$. Here $\phi=\phi\times\overline{\phi}$ and $U=U\times\overline{U}$. By Theorem 1 from [10], there exists a $(p,q,r,s-1)$-form θ on $U^{'}\subset U$ such that $d^{''v}\theta=\omega$ on $U^{'}$. But, for this θ exists θ^* a $(p,s-1,r,q)$-form on $U^{'*}=\phi(U^{'})$ such that $\theta=\varphi^{-1}\theta^*$. Thus, for $\omega=\varphi^{-1}\omega^*$, $\theta=\varphi^{-1}\theta^*$ and $\omega=d^{''v}\theta$ we have

$$\omega^*=\varphi(\omega)=\varphi(d^{''v}\theta)=\varphi(d^{''v}(\varphi^{-1}\theta^*))=(\varphi\circ d^{''v}\circ\varphi^{-1})\theta^*=d^{'*v}\theta^*$$

which ends the proof. □

Let $\mathcal{F}^{p,s,r,q}$ be the sheaf of germs of (p,s,r,q)-forms on $T^{'*}M$ and we denote by $i:\Phi^{p,r,q}\to\mathcal{F}^{p,0,r,q}$ the natural inclusion. The sheaves $\mathcal{F}^{p,s,r,q}$ are fine and taking into account Theorem 2.1, it follows that the sequence of sheaves

$$0\to\Phi^{p,r,q}\overset{i}{\to}\mathcal{F}^{p,0,r,q}\overset{d^{'*v}}{\to}\mathcal{F}^{p,1,r,q}\overset{d^{'*v}}{\to}\dots\overset{d^{'*v}}{\to}\mathcal{F}^{p,s,r,q}\overset{d^{'*v}}{\to}\dots$$

is a fine resolution of $\Phi^{p,r,q}$, and we denote by $H^s(M,H,\Phi^{p,r,q})$ the cohomology groups of M with coefficients in the sheaf $\Phi^{p,r,q}$, called v-cohomology groups of (M,H). Then we have a de Rham type theorem, namely

Theorem 2.2. *The v-cohomology groups of the complex Hamilton space (M, H) are given by*

$$H^s(M, H, \Phi^{p,r,q}(T^{'*}M)) \approx Z^{p,s,r,q}(T^{'*}M)/d^{'*v}\mathcal{A}^{p,s-1,r,q}(T^{'*}M) \tag{2.7}$$

where $Z^{p,s,r,q}(T^{'*}M)$ is the space of $d^{'*v}$-closed (p, s, r, q)-forms globally defined on $T^{'*}M$.

Now, from the above discussion we have

Proposition 2.2. *$H^s(M, L, \Phi^{p,q,r}(T^{'}M))$ and $H^s(M, H, \Phi^{p,r,q}(T^{'*}M))$ are isomorphic by the map $[\omega] \mapsto [\omega^*]$, $\forall\, \omega \in \mathcal{A}^{p,q,r,s}(T^{'}M)$.*

Finally, following [3] pag. 78 and [11], we define a relative vertical cohomology with respect to complex Legendre transformation ϕ.

Define the differential complex

$$0 \longrightarrow \mathcal{A}^{p,q,r,0}(\phi) \xrightarrow{\widetilde{d}^{''v}} \mathcal{A}^{p,q,r,1}(\phi) \xrightarrow{\widetilde{d}^{''v}} \dots$$

where $\mathcal{A}^{p,q,r,s}(\phi) = \mathcal{A}^{p,q,r,s}(T^{'}M) \oplus \mathcal{A}^{p,s-1,r,q}(T^{'*}M)$ and

$$\widetilde{d}^{''v}(\omega, \theta) = (d^{''v}\omega, \varphi\omega - d^{'*v}\theta).$$

Taking into account $(d^{''v})^2 = (d^{'*v})^2 = 0$ and (2.6) we easily verify that $(\widetilde{d}^{''v})^2 = 0$. Denote the cohomology groups of this complex by $H^{p,q,r,*}(\phi)$.

If we regraduate the complex $\mathcal{A}^{p,s,r,q}(T^{'*}M)$ as $\widetilde{\mathcal{A}}^{p,s,r,q}(T^{'*}M) := \mathcal{A}^{p,s-1,r,q}(T^{'*}M)$, then we obtain an exact sequence

$$0 \longrightarrow \widetilde{\mathcal{A}}^{p,s,r,q}(T^{'*}M) \xrightarrow{\alpha} \mathcal{A}^{p,q,r,s}(\phi) \xrightarrow{\beta} \mathcal{A}^{p,q,r,s}(T^{'}M) \longrightarrow 0 \tag{2.8}$$

with the obvious mappings α and β given by $\alpha(\theta) = (0, \theta)$ and $\beta(\omega, \theta) = \omega$, respectively. From (2.8) we have an exact sequence in cohomologies, see for instance [12] p. 69, namely

$$\dots \longrightarrow H^{s-1}(M, H, \Phi^{p,r,q}(T^{'*}M)) \xrightarrow{\alpha^*} H^{p,q,r,s}(\phi) \xrightarrow{\beta^*}$$
$$H^s(M, L, \Phi^{p,q,r}(T^{'}M)) \xrightarrow{\delta^*} H^s(M, H, \Phi^{p,r,q}(T^{'*}M)) \longrightarrow \dots.$$

It is easily seen that $\delta^* = \phi^*$. Here ϕ^* denotes the corresponding map between cohomology groups. Let $\omega \in \mathcal{A}^{p,q,r,s}(T^{'}M)$ be a $d^{''v}$-closed form, and $(\omega, \theta) \in \mathcal{A}^{p,q,r,s}(\phi)$. Then $\widetilde{d}^{''v}(\omega, \theta) = (0, \varphi\omega - d^{'*v}\theta)$ and by the definition of the operator δ^* we have

$$\delta^*[\omega] = [\varphi\omega - d^{'*v}\theta] = [\varphi\omega].$$

Hence we finally get a long exact sequence

$$\ldots \longrightarrow H^{s-1}(M,H,\Phi^{p,r,q}(T^{'*}M)) \xrightarrow{\alpha^*} H^{p,q,r,s}(\phi) \xrightarrow{\beta^*}$$
$$H^{s}(M,L,\Phi^{p,q,r}(T^{'}M)) \xrightarrow{\phi^*} H^{s}(M,H,\Phi^{p,r,q}(T^{'*}M)) \xrightarrow{\alpha^*} \ldots$$

which leads to

Proposition 2.3. *If* $\dim_{\mathbb{C}} M = n$ *then*

(i) $\beta^* : H^{p,q,r,n+1}(\phi) \to H^{n+1}(M,L,\Phi^{p,q,r}(T^{'}M))$ *is an epimorphism;*

(ii) $\alpha^* : H^{n}(M,H,\Phi^{p,r,q}(T^{'*}M)) \to H^{p,q,r,n+1}(\phi)$ *is an epimorphism;*

(iii) $\beta^* : H^{p,q,r,s}(\phi) \to H^{s}(M,L,\Phi^{p,q,r}(T^{'}M))$ *is an isomorphism for* $s > n+1$*;*

(iv) $\alpha^* : H^{s}(M,H,\Phi^{p,r,q}(T^{'*}M)) \to H^{p,q,r,s+1}(\phi)$ *is an isomorphism for* $s > n$*;*

(v) $H^{p,q,r,s}(\phi) = 0$ *for* $s > n+1$*.*

References

[1] M. Abate, G. Patrizio, **Finsler metrics-A global approach**, Lectures Notes in Math.,1591, Springer-Verlag, 1994.

[2] T. Aikou, **The Chern-Finsler connection and Finsler-Kähler manifolds**, Adv. Stud. in Pure Math., 48 (2007), 343-373.

[3] R. Bott, L. W. Tu, **Differential Forms in Algebraic Topology**, Graduate Text in Math., 82 Springer-Verlag, Berlin, 1982.

[4] J.J. Faran, **The equivalence problem for complex Finsler Hamiltonians**, Cont. Math., 196 (1996), 133-143.

[5] R. Miron, **Hamilton Geometry**, An. Şt. ale Univ. "Al. I. Cuza" din Iaşi, S. I., Mat., 35 (1989), 38-85.

[6] R. Miron, M. Anastasiei, **The Geometry of Lagrange Spaces. Theory and Applications**, Kluwer Acad. Publ. 59 1994.

[7] R. Miron, D. Hrimiuc, H. Shimada, S. Sabău, **The geometry of Hamilton and Lagrange spaces**, Kluwer Acad. Publ., 118 2001.

[8] G. Munteanu, **Complex spaces in Finsler, Lagrange and Hamilton Geometries,** Kluwer Acad. Publ., 141 FTPH 2004.

[9] G. Munteanu, **Gauge field theory in terms of complex Hamilton geometry**, Balkan J. of Geom. and Appl., 12(1) (2007), 107-121.

[10] G. Pitiş, G. Munteanu, **V-cohomology of complex Finsler manifolds**, Studia Univ., "Babeş-Bolyai", 43(3) (1998), 75-81.

[11] Z. Tevdoradze, **Vertical cohomologies and their application to completely integrable Hamiltonian systems**, Georgian Math. J. 5 (5) (1998), 483-500.

[12] I. Vaisman, **Cohomology and differential forms**, M. Dekker Publ. House, New York, 1973.

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