

THEORETICAL ANALYSIS OF A CIRCULAR THIN PLATE UNDER PRESSURE CLAMPED TO RIM IN THE PLASTIC RANGE DEFORMATION

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Abstract: This work presents a computational method for solving the plane-plastic-stress and strains with axial symmetry problems applied in the sheet hydro forming technology. The sheet hydro formed may be approximate to a circular membrane under pressure clamped at the rim.

Keywords: stress, strain hardening, circular membrane, sheet hydro forming

1. NOMENCLATURE

ε – the logarithmic strain;
 b – initial outside radius of membrane;
 γ – octahedral shear strain;
 k – constant having a dimension of length
 σ – normal stress;
 θ – angular coordinate;
 p – pressure on membrane;
 r – radial coordinate of un deformed membrane;
 s – arc length;
 u – radial displacement;
 w – axial displacement;
 z – axial coordinate;

1.1. Subscripts:

o – at center for member;
 $1, 2, 3$ – principal directions in general;
 r, θ, z – principal directions: radial, tangential and axial directions.

2. INTRODUCTION

In the design of the hydro-formed sheet, it is desirable to know the detailed stress and strain distributions and the increase in load that can be sustained between the onset of yielding and failure. If the sheet is thin, it can be analyzed on the basis of plane stress. For materials with strain-hardening characteristics, a solution of plane-stress problems by an iterative procedure with a good first approximate solution has been obtained. A simple method of solving plane-plastic-stress problems with axial symmetry in the strain-hardening range that is obtained in a sheet hydro-forming process is based on the deformation theory.

In the present problems with axial symmetry, the directions of the axes of the principal stresses remain fixed during loading and the ratios of principal stresses and strains remain approximately constant. Therefore the deformation theory is used. The equations of equilibrium, strain, and plastic law are reduced to two simultaneous nonlinear differential equations involving three variables, one independent and two dependent, that can be integrated numerically to any desired accuracy.

3. STRESS-STRAIN RELATIONS IN PLASTIC DEFORMATION

In the deformation theory of plasticity for ideally plastic materials, the general stress-strain relations are the following:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \quad (1)$$

$$\frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} = \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} = \frac{\sigma_3 - \sigma_1}{\varepsilon_3 - \varepsilon_1} \quad (2)$$

$$r = r(\gamma) \quad (3)$$

where:

$$r = \frac{1}{3} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (4)$$

$$\gamma = \frac{2}{3} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{1/2} \quad (5)$$

By substituting (1) and (2) in (4) and (5), we obtain:

$$\varepsilon_1 = \frac{1}{3} \frac{\gamma}{r} \left[\sigma_1 - \frac{1}{2} (\sigma_2 + \sigma_3) \right] \quad (6)$$

$$\varepsilon_2 = \frac{1}{3} \frac{\gamma}{r} \left[\sigma_2 - \frac{1}{2} (\sigma_3 + \sigma_1) \right] \quad (7)$$

$$\varepsilon_3 = \frac{1}{3} \frac{\gamma}{r} \left[\sigma_3 - \frac{1}{2} (\sigma_1 + \sigma_2) \right] \quad (8)$$

In the plane –stress problems $\sigma_3=0$

If the cylindrical coordinates for the problems considered are used, the principal directions 1, 2 and 3 in the preceding equations become radial, circumferential and axial directions, respectively. Thus, the equations becomes:

$$\varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0 \quad (9)$$

$$r = \frac{\sqrt{2}}{3} (\sigma_r^2 - \sigma_z \sigma_\theta + \sigma_\theta^2)^{1/2} \quad (10)$$

$$\gamma = 2\sqrt{\frac{2}{3}(\varepsilon_r^2 + \varepsilon_r\varepsilon_\theta + \varepsilon_\theta^2)^{1/2}} \quad (11)$$

$$\varepsilon_r = \frac{1}{3} \frac{\gamma}{\tau} (\sigma_r - \frac{1}{2} \sigma_\theta) \quad (12)$$

$$\varepsilon_\theta = \frac{1}{3} \frac{\gamma}{\tau} (\sigma_\theta - \frac{1}{2} \sigma_r) \quad (13)$$

$$\varepsilon_z = \frac{1}{3} \frac{\gamma}{\tau} [-\frac{1}{2} (\sigma_r + \sigma_\theta)] = -(\varepsilon_r + \varepsilon_\theta) \quad (14)$$

When σ_r and σ_θ are expressed in terms of ε_r and ε_θ , we obtain:

$$\sigma_r = 2 \frac{\tau}{\gamma} (2\varepsilon_r + \varepsilon_\theta) \quad (15)$$

$$\sigma_\theta = 2 \frac{\tau}{\gamma} (2\varepsilon_\theta + \varepsilon_r) \quad (16)$$

Considering that the dimension of an element is infinitesimal compared with the original dimension is used the finite-strain concept in the evaluating of the instantaneous dimension of the element. In according with that theory, the stress is equal to the force divided by the instantaneous area and the strain are defined by the following equations:

$$\delta(\varepsilon_j) = \frac{\delta(l_j)}{l_j} \quad (17)$$

$$\varepsilon_j = \ln \frac{l_j}{(l_j)_0} \quad (18)$$

where l_j is the instantaneous length of a small element having the original length of $(l_j)_0$ and j is any principal direction.

6. THE EQUATIONS OF EQUILIBRIUM AND STRAINS FOR A CIRCULAR MEMBRANE UNDER PRESSURE

The hydro-formed sheet can be approximate with a thin membrane under pressure. The membrane considered is shown in figure 1. The membrane is clamped at the rim and subjected to a pressure and a small element defined by $\Delta\theta$ and Δs is considered at the radius $r+u$. In the un deformed state, the same element is at the radius r and defined by $\Delta\theta$ and Δs . The membrane is so thin that bending stress can be neglected.

The instantaneous thickness of the element and the stresses acting on the element are shown in figure 2.

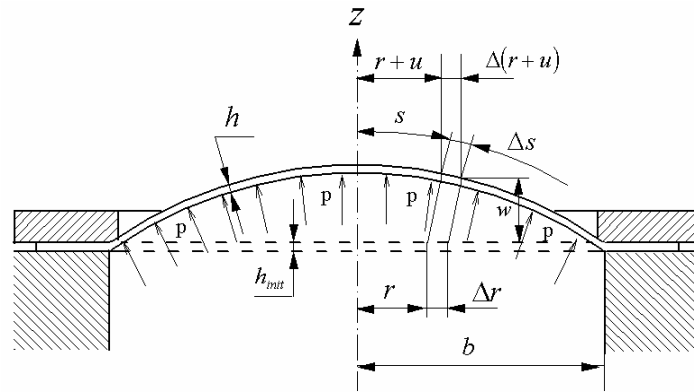


Fig.1. Thin circular membrane under pressure

6.1. THE EQUATIONS OF EQUILIBRIUM

The following equation of equilibrium is obtained when all the forces acting on the element in the direction of σ_r are summed up:

$$\begin{aligned} & \sigma_r(r+u)h\Delta\theta - (\sigma_r + \Delta\sigma_r)[r+u+\Delta(r+u)]\Delta\theta(h+\Delta h)\cos\Delta\phi + \\ & + 2\sigma_\theta\Delta s\left(h + \frac{1}{2}\Delta h\right)\sin\frac{\Delta\theta}{2}\cos\phi - p\Delta s(r+u)\Delta\theta\sin\frac{\Delta\phi}{2} = 0 \end{aligned} \quad (19)$$

When $\Delta(r+u)$ approaches zero as a limit, the differential equation of equilibrium may be obtained:

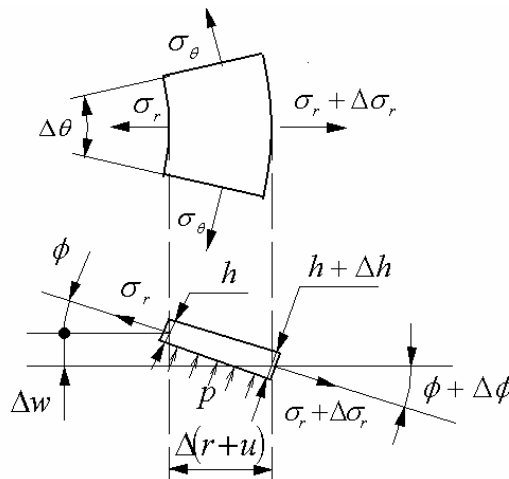


Fig.2. The element of the membrane in the deformed state

$$(r+u)\frac{d(\sigma_r h)}{d(r+u)} = h(\sigma_\theta - \sigma_r) \quad (20)$$

Summing up the forces in the z-direction yields:

$$p\pi(r+u)^2 = \sigma_r \frac{dw}{ds} 2\pi h(r+u) \quad (21)$$

or

$$\left[\frac{dw}{d(r+u)} \right]^2 = \frac{1}{\left[\frac{2h\sigma_r}{p(r+u)} \right]^2 - 1} \quad (22)$$

6.2. THE EQUATIONS OF STRAINS

As the element at radius r , defined by $\Delta\theta$ and Δr in the undeformed state is moved by the application of the pressure p to radius $r+u$ and defined by $\Delta\theta$ and Δs , the resulting strains are following:

$$\varepsilon_r = \ln \frac{ds}{dr} \quad (23)$$

$$\varepsilon_\theta = \ln \frac{r+u}{r} \quad (24)$$

$$\varepsilon_z = \ln \frac{h}{h_{init}} \quad (25)$$

Then:

$$e^{\varepsilon_r} = \frac{d(r+u)}{dr} \left\{ 1 + \left[\frac{dw}{d(r+u)} \right]^2 \right\}^{1/2} \quad (26)$$

$$e^{\varepsilon_\theta} = \frac{r+u}{r} \quad (27)$$

$$e^{\varepsilon_z} = \frac{h}{h_{init}} \quad (28)$$

The set of the independent equations (9), (2), (5), (12), (13), (20), (22), (26), (27) and (28) involving the ten unknowns σ_r , σ_θ , ε_r , ε_θ , ε_z , γ , r , h , u , w resolve the problem of stresses and strains for the circular membrane under pressure. If differentiate equation (27) with respect to r and combined with equation (26) results:

$$r \frac{d\varepsilon_\theta}{dr} = \frac{e^{(\varepsilon_r - \varepsilon_\theta)}}{\left\{ 1 + \left[\frac{dw}{d(r+u)} \right]^2 \right\}^{1/2}} - 1 \quad (29)$$

Substituting equation (22) in equation (29) to eliminate w , yields the following equation of compatibility:

$$r \frac{d\varepsilon_\theta}{dr} = e^{(\varepsilon_r - \varepsilon_\theta)} \left\{ 1 - \left[\frac{p(r+u)}{2h\sigma_r} \right]^2 \right\}^{1/2} - 1 \quad (30)$$

By eliminate u and h from equations (20), (23), (24), (25) and (30) results:

$$r \frac{d\sigma_r}{dr} + \sigma_r r \frac{d\varepsilon_z}{dr} = (\sigma_\theta - \sigma_r) e^{(\varepsilon_r - \varepsilon_\theta)} \left\{ 1 - \left[\frac{rpe^{(\varepsilon_\theta - \varepsilon_z)}}{2h_{init}\sigma_r} \right]^2 \right\}^{1/2} \quad (31)$$

$$r \frac{d\varepsilon_\theta}{dr} = e^{(\varepsilon_r - \varepsilon_\theta)} \left\{ 1 - \left[\frac{rpe^{\varepsilon_\theta - \varepsilon_z}}{2h_{init}\sigma_r} \right]^2 \right\}^{1/2} - 1 \quad (32)$$

By introducing an arbitrary constant k into equation (31) and (32) results a system of two equations with the parameters r/k and pk/h_{init} (33), that can be solved in a simple, direct way without the use of the iteration.

$$\left. \begin{aligned} \frac{r}{k} \frac{d\sigma_r}{d\left(\frac{r}{k}\right)} + \sigma_r \frac{r}{k} \frac{d\varepsilon_z}{d\left(\frac{r}{k}\right)} &= (\sigma_\theta - \sigma_r) e^{(\varepsilon_r - \varepsilon_\theta)} \left\{ 1 - \left[\frac{\frac{pk}{h_{init}} \frac{r}{k} e^{(\varepsilon_\theta - \varepsilon_z)}}{2\sigma_r} \right]^2 \right\}^{1/2} \\ \frac{r}{k} \frac{d\varepsilon_\theta}{d\left(\frac{r}{k}\right)} &= e^{(\varepsilon_r - \varepsilon_\theta)} \left\{ 1 - \left[\frac{\frac{pk}{h_{init}} \frac{r}{k} e^{(\varepsilon_\theta - \varepsilon_z)}}{2\sigma_r} \right]^2 \right\}^{1/2} - 1 \end{aligned} \right\} \quad (33)$$

7. CONCLUSIONS

The method developed solves the plane-plastic-stresses problem with axial symmetry in a comfortable manner and shows the octahedral shear strains distribution and the ratio of principal stresses during loading. This method can be applied at the necessary pressure estimation from large deflection of a thin circular plates clamped to rim under pressure. The method offers the possibility to estimate the technological conditions for hydroforming various dimensions sheets.

8. REFERENCES

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