

STUDY ABOUT THE DAMPING OF THE PRESSURE WAVES

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Abstract: The study presents the numerical integration of the hyperbolic system of partial differential equations that define the pressure wave propagation into an infinite pipeline, corresponding to the barotropic fluid, in specific boundary conditions for the case of sonic transmission of energy by vibrations. It is used a systematization of Jukowski's theories, elements of Riemann and G.Constantinescu studies for the pressure wave propagation.

Key words : pressure, damping, viscous fluid, barotop.

1 INTRODUCTION

Through all ages many researchers studied the pressure wave propagation phenomena. Bernhard Riemann is the first who studies this domain and given a doctor's degree thesis in 1876 about the pressure wave. Than Jukowski, Allievi, Morozov and the other has interested by pressure propagation in a pipeline in a case of the hydraulic hammer.

On this line we must tell about romanian scientists who advanced the theory of pressure wave propagation like G Constantinescu who founded the Sonicity and M.D. Cazacu with his studies about non-permanent, one-dimensional motion of compressible fluids in the case of small pressure variations with application to the water hammer.

2. ABOUT THE WAVE PRESSURE PROPAGATION

The problem of the pressure wave propagation presents a theoretical significance being the insufficiently foundation of the non-permanent movement equations even in the case of non-compressible fluids by Leonhard Euler, and a practical significance in a case of hydraulic hammer, decrement of the noise etc. We present same conclusions about the phenomena :

1. In any relative fast variation (positive or negative) of the pressure Δp in a compressible fluid in repose or movement, exists too, a density variation (and a temperature variation in a case of gas).
2. The propagation speed c of the suppresser or depressor wave is directly, of course, from the source of the perturbation to outside.
3. If the fluid has a speed flow given by a piston or a generated wave clapped, than for the different of pressure Δp between the two parts of the front of wave exists a variation Δw of the speed, the phenomena similar of the compressible fluid flow in a pipeline.
4. If the regularity conditions of the propagation speed c , like continuos, uniform and limited function give a homothetic transformation for her definition relation, physic variable like pressure, density, temperature, flow speed can have discontinuity points where it's can use the infinitesimal relations for calculus
5. We use for to determinate the expression of the wave propagation pressure in a compressible fluid, as usually, Jukowski's relations, the low of the conserved mass, that is in fact the property of indestructibility of matter.

We consider a particular solution of the equation (2) with form:

$$f(x, t) = X(x) \cdot T(t) \quad (3)$$

and we obtain by introducing in equation (2) :

$$\frac{X''}{X} = \frac{T''}{\frac{4}{3} \nu T' + c^2 T} = \lambda = \text{const.} \quad (4)$$

were the common value of these two rapports is a strictly negative constant $\lambda < 0$, for to have non-ordinarily solutions in our case and in our conditions

We work with following limit conditions:

$$\frac{\partial p}{\partial x}(0, t) = 0 \quad \text{and} \quad p(l, t) = 0 \quad (5)$$

We considered the initial moment at the time l/c from the handle of the vane and we have the Cauchy's conditions:

$$p(x, 0) = p_{\max} = \rho c v_0 \quad (6)$$

$$\frac{\partial p}{\partial t}(x, 0) = 0$$

The first equation with partial derivatives has a general solution by form:

$$X = C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x} \quad (7)$$

that give us the system:

$$\begin{aligned} \frac{\partial X}{\partial x}(0) &= \sqrt{\lambda} (C_1 - C_2) = 0 \\ X(l) &= C_1 e^{\sqrt{\lambda} l} + C_2 e^{-\sqrt{\lambda} l} = 0 \end{aligned} \quad (8)$$

were we eliminate the ordinarily solution $\lambda=0$ for the case $C_1 = C_2 = 0$ we have:

$$C_1 = C_2 \neq 0 \quad \text{and} \quad e^{\sqrt{\lambda} l} + e^{-\sqrt{\lambda} l} = 0 \quad (9)$$

Working about the second (9) equation we have:

$$\lambda = -\frac{\pi^2 (2k+1)^2}{4l^2} < 0 \quad \text{with} \quad k \in \mathbb{Z} \quad (10)$$

and for the eigen function :

$$X_k(x) = 2C_1 \cos \frac{2\pi(2k+1)}{2l} x \quad (11)$$

With the proper values (10) the second partial equation become:

$$T'' + \frac{\nu\pi^2(2k+1)^2}{3l^2} T' + \frac{c^2\pi^2(2k+1)^2}{4l^2} T = 0 \quad (12)$$

with the general solution:

$$T = C_3 e^{r_1 t} + C_4 e^{r_2 t} \quad (13)$$

Working like usually manner we obtain the value of the all constants and the general solution of the problem is:

$$\begin{aligned} \frac{p(x,t)}{\rho c v_0} = & \sum_{k=0}^{\infty} e^{-\frac{\nu\pi^2(2k+1)^2}{6l^2} t} \cdot \cos \frac{\pi(2k+1)}{2l} x \cdot \\ & \left[\frac{4(-1)^k}{\pi(2k+1)} \cos \left(\frac{c\pi(2k+1)}{2l} t \sqrt{1 - \frac{\nu^2\pi^2(2k+1)^2}{9l^2 c^2}} \right) + \right. \\ & \left. + \frac{(-1)^k}{\sqrt{1 - \frac{\nu^2\pi^2(2k+1)^2}{9l^2 c^2}}} \frac{\nu}{lc} \sin \left(\frac{c\pi(2k+1)}{2l} t \sqrt{1 - \frac{\nu^2\pi^2(2k+1)^2}{9l^2 c^2}} \right) \right] \end{aligned} \quad (14)$$

Having the equation of $p(x,t)$ for the compressible viscous fluid we can do the graphic representation of the solutions and to compare with the ideal case.

4. CONCLUSIONS

We have the following conclusions of this mathematical study:

- The analytic solution change they form from non-periodic variation to decrement periodic variation when the report between the sonic force and viscous friction force pass through one
- The grates decrement of the particular solutions for superior frequency make that the viscosity influence of the compressible fluid given the ideal fluid to be seen by the curve of the coins of the pressure oscillation and then by the decrement of the amplitude of the oscillations. These offered a theoretical prove for the experimental observation by them the influence of the viscosity in the water hammer have always a certain delay.

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