UPON THE INFLUENCE OF SOME PARAMETERS, IN NUMERICAL ANALYSIS BY SPH

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Abstract: In some cases SPH method is more suitable than Finite Element Method (FEM) and this is more and more used also in applied mechanics. In structure numerical analysis some parameters essentially influence the results, among these the constants of the material models and some specific parameters of the method have a maximum importance. It is about the most used material models like plastic-kinematic material model, isotropic-elastic-failure material model, Johnson-Cook material model and others, which take into account the strain rate. This paper comes with some results of our experience in using of the material models together with SPH (Smoothed Particle Hydrodynamics) method. The influence of the smoothing length, distance between particles, the number of these are also numerical investigated. Our established conclusions, supported by graphics and quantitative appreciations could be useful for the researchers working in numerical analysis field.

Keywords: SPH, smoothing length, material constants, strain rate.

1. INTRODUCTION

SPH (Smoothed Particle Hydrodynamics) method is a griddles Lagrangian technique which comes from astrophysics (Lucy, 1977). The method was extended to fluid simulation, especially with free-surface (Monaghan, 1992), nowadays SPH method being also used in many scientific fields. Applied mechanics domain is perhaps the last one, but it is intensively researched and significant advances have also been made.

As the impact problems are concerned, we could emphasis the numerical analysis ability to simulate such problems, which involve large deformations and the enhancement made by using of the erosion algorithm. We could say that SPH offers an alternative Lagrangian method for approaching of large deformations, like impact problems, being an attractive method since the lack of a grid allows some calculus facilities, including the contact modeling and material erosion simulation.

Among others aspects, the material models, its appropriate constants, the specific parameters of the SPH method have a maximum importance for acceptable results having a good concordance with the experiments. This paper answers to the most important questions which appear for a user of the SPH method.

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Our numerical investigation is made upon an experimental problem represented by Taylor test. Our conclusions and observations are useful in many other problems of the applied mechanics or fluid flow field.

The numerical simulations made by us are based on the using of Ansys/Ls-Dyna program and Autodyne program, in which SPH method is implemented and which have a large material library.

2. THEORETICAL FUNDAMENTALS OF SPH METHOD

The SPH method belongs to the meshless methods, so the investigated domain is represented by a number of nodes, representing the particles of this domain, having their material and mechanical (mass, position, velocity etc.) characteristics. Each particle represents an interpolation point on which the material properties are known [1-3].

The boundary conditions have to be imposed to some of particles, according to the problem analyzed, like in the case of finite element method. The problem solution is given by the computed results, on all the particles, using an interpolation function. We can say that the fundamentals of SPH theory consist in interpolation theory; all the behavior laws are transformed into integral equations.

The kernel function, or smoothing function, often called smoothing kernel function, or simply kernel, gives a weighted approximation of the field variable (function) in a point (particle). Integral representation of a function $f(x)$, used in the SPH method starts from the following identity (1):

$$f(x) = \int_{\Omega} f(x')\delta(x - x')dx' \tag{1}$$

where f is a function of a position vector x, which can be an one-, two- or three-dimensional one; $\delta(x - x')$ is a Dirac function, having the properties (2):

$$\delta(x - x') = \begin{cases} 1 \rightarrow x = x' \\ 0 \rightarrow x \neq x' \end{cases} \tag{2}$$

In equation (1), Ω is the function domain, which can be a volume, that contains the x, and where $f(x)$ is defined and continuous. By replacing the Dirac function with a smoothing function $W(x - x', h)$ the integral representation of $f(x)$ becomes (3):

$$f(x) = \int_{\Omega} f(x')W(x - x', h)dx' \tag{3}$$

where W is the smoothing kernel function, or smoothing function, or kernel function. The parameter h, of the smoothing function W, is the smoothing length, by which the influence area of the smoothing function W is defined (Figure 1a and 1b).

As long as Dirac delta function is used, the integral representation, described by equation (1), is an exact (rigorous) one, but using the smoothing function W instead of Dirac function, the integral representation can only be an approximation. This is the reason for the name of kernel approximation. Using the angle bracket, this aspect is underlined and the equation (3) can be rewritten as (4):

$$\langle f(x) \rangle = \int_{\Omega} f(x')W(x - x', h)dx' \tag{4}$$

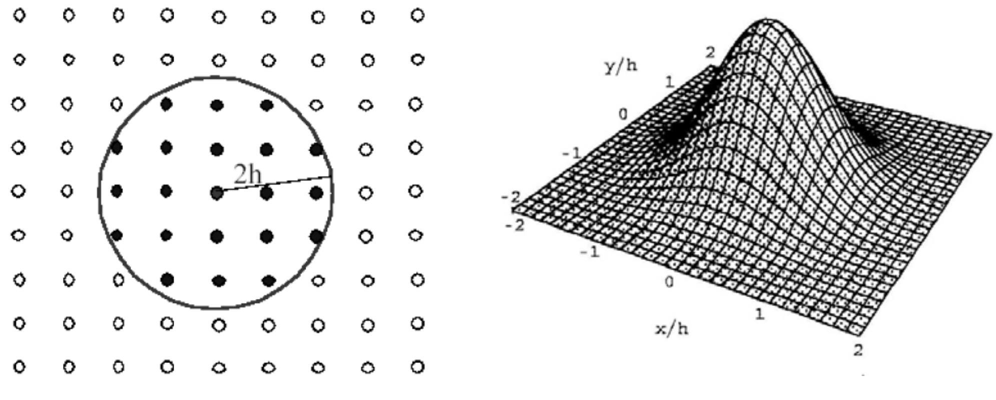


Fig. 1. Support domain of kernel function; graphical representation of 2D-Kernel function.

The smoothing function W is usually chosen to be an even one, which has to satisfy some conditions. The first condition, named normalization condition or unity condition is (5):

$$\int_{\Omega} W(x-x',h)dx' = 1 \tag{5}$$

The second condition is the Delta function property and it occures when the smoothing length approaches zero (6):

$$\lim_{h\to 0} W(x-x',h) = \delta(x-x') \tag{6}$$

The third condition is the compact condition, expressed by (7):

$$W(x-x',h) = 0 \text{ when } |x-x'| > kh \tag{7}$$

where k is a constant related to the smoothing function for point at x, defining the effective non-zero area of the smoothing function, as the Figure 2 shows.

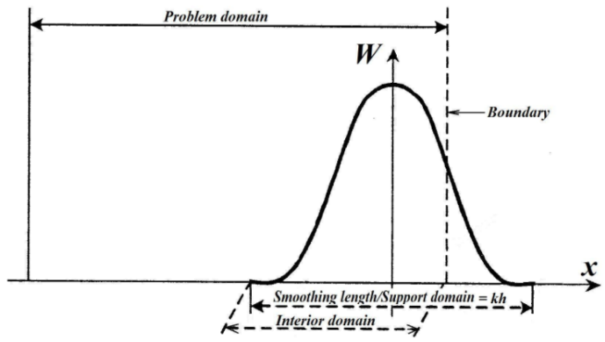


Fig. 2 Smoothing length.

As the particle approximation is concerned, the continuous integral aproximation (4) can be converted to a summation of discretized forms, over all particles belonging to the support domain. Changing the infinitesimal volume dx' with the finite volume of the particle ΔV_j, the mass of the particles m_j can be written (8):

$$m_j = \Delta V_j \rho_j \tag{8}$$

and finally, relation (3) becomes (9):

$$\langle f(x)\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W(x - x_j, h) \tag{9}$$

The particle approximation of a parameter, described by a function, for particle i, can be expressed by (10):

$$\langle f(x_i)\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij} \tag{10}$$

where,

$$W_{ij} = W(x_i - x_j, h), \tag{11}$$

being the kernel function. The most important requirements of a kernel function are presented below:

- the smoothing function has to be ***normalized*** over its support (12):

$$\int_{\Omega} W(x - x', h) dx' = 1 \tag{12}$$

- the smoothing function has to be ***compactly supported (13):***

$$W(x - x', h) = 0 \text{ for } |x - x'| > kh \tag{13}$$

- the smoothing function has to be ***positive*** for any point at x' within the support domain (14):

$$W(x - x', h) \geq 0 \tag{14}$$

- the smoothing function value has to be ***monotonically decreasing*** with the increase of the distance away from the particle.

- the smoothing function value has to satisfy the ***Dirac delta function*** condition as the smoothing length approaches to zero:

$$\lim_{h \to 0} W(x - x', h) = \delta(x - x') \tag{15}$$

- the smoothing function value has to be an ***even function*** (symetric).

The literature presents different smoothing function (also called smoothing kernel function, smoothing kernel, or kernel). Theoretically, any function having the properties presented above, can be employed as SPH smoothing function. First time, Lucy (1977) used the following bell-shaped function as the smoothing function (16):

$$W(s,h) = \frac{\alpha}{h^n} \begin{cases} (1+3s)(1-s^2)^3 \leftarrow s \leq 1 \\ 0 \leftarrow s > 1 \end{cases} \tag{16}$$

where α is $\frac{5}{4}$, $\frac{5}{\pi}$ or $\frac{105}{16\pi}$, n is a number representing the space dimension, $s = \frac{|x - x'|}{h}$, or $s == \frac{r}{h}$, r being the distance between two points (particles). The graphical representation of this smoothing function and its derivatives (first and second) can be seen in the Figure 3. Monaghan in 1992 and Gingold and Monaghan in 1977 assumed the smoothing function to be a Gaussian (Figure 4), expressed by the relation (17).

$$W(s,h)=\frac{\alpha}{h^n}e^{-s^2} \tag{17}$$

Many notations used in relation (17) are the same, used in previous type of kernel. The notation α has the following expression: $\frac{1}{\pi^{0.5}}$, $\frac{1}{\pi}$ or $\frac{1}{\pi^{1.5}}$ in function of the space dimension (1D, 2D or 3D).

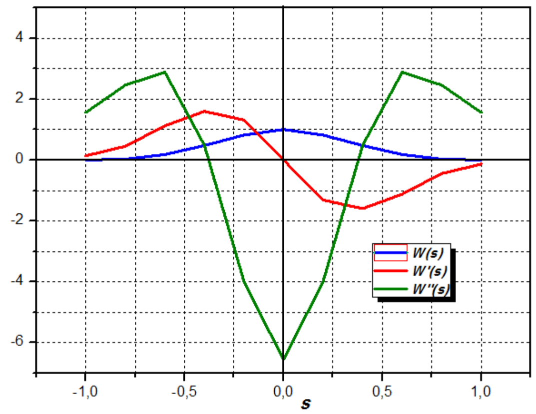


Fig. 3. Smoothing function and its derivates, used by Lucy in 1977.

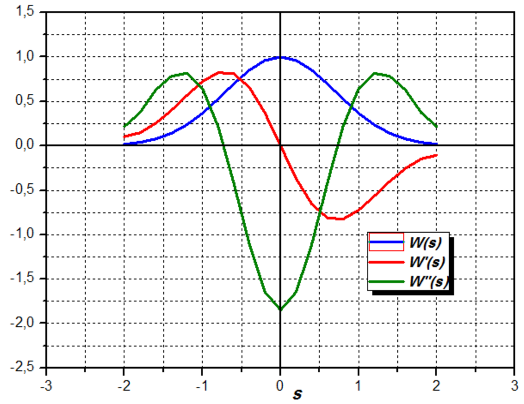


Fig. 4. Smoothing function and its derivates, used in 1977 and 1992 (Ginglod and Monaghan).

In 1996, Johnson et al. used a quadratic smoothing function to simulate the high velocity impact problem.

The graphical representation of this smoothing function and its derivatives (first and second) can be seen in the Figure 5. The expression of the Johnson smoothing function, for s being between zero and two, is (20):

$$W(s,h)=\frac{\alpha}{h^n}\left(\frac{3}{16}s^2-\frac{3}{4}s+\frac{3}{4}\right) \tag{20}$$

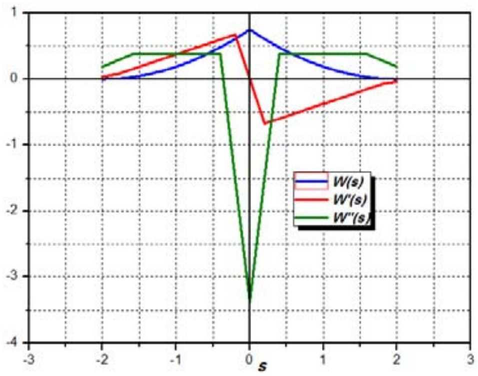


Fig. 5 Quadratic spline smoothing function and its derivates.

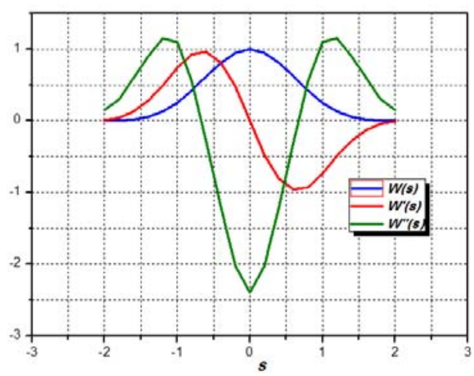


Fig. 6 The cubic B-spline kernel function used by Ls-Dyna.

The Ls-Dyna program uses a cubic B-spline kernel function, in the form given by relation (21), where s = r/h, n is the number representing the spatial dimension and α is a constant which has the value: 2/3, 10/7, depending on the space dimension (21).

$$W(s,h)=\frac{\alpha}{h^n}\begin{cases}\left(1-\frac{3}{2}s^2+\frac{3}{4}s^3\right)\leftarrow 0\le s<1\\ \frac{1}{4}(2-s)^3\leftarrow 1\le s<2\\ 0\leftarrow s\ge 2\end{cases} \tag{21}$$

The graphical representation of this smoothing function and its derivatives (first and second) can be seen in the Figure 6.

A smoothing length too small (not enough particles in the support domain) influence on the calculus efficiency and also the accuracy, this going down. A smoothing length too large all the particle properties may be smoothed out and finally the accuracy will be a low one. The best way seems to be a variable smoothing length according to calculus and accuracy efficiency. So, many ways already exist for a dinamically evolving of *h*, for geting a suitable number of the neighboring particle, which to remain relatively constant.

The simplest approaching is that the smoothing length to depend on the average density. From this point of view, the literature proposed the following relation (22):

$$h=h_0\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{d}} \tag{22}$$

where h_0 and ρ_0 are the initial smoothing length and density respectively; d is the number representing the space dimension (1D, 2D or 3D, or simply 1, 2, or 3). In 1989, Benz proposed another method, by taking into account the time derivative of the smoothing function, in terms of the continuity equation (23):

$$\frac{dh}{dt}=-\frac{1}{d}\frac{h}{\rho}\frac{d\rho}{dt} \tag{23}$$

3. SPH PARAMETERS IN ANSYS/LS-DYNA PROGRAM

The user can make a choosing regarding to the the particle approximation, having the following options, by FORM parameter (CONTROL_SPH): default formulation (0), renormalization approximation (1), symmetric formulation (2), symmetric renormalized approximation (3), tensor formulation (4), fluid particle approximation (5), or fluid particle with renormalization approximation (6).

The renormalized, symmetric or symetric renormalized approximation is reffering to the specific forms of the momentum equations, for to reduce the errors coming from the particle inconsistency problem. Which of the options is the best depends on the problem characteristics, so the right choosing is the user's task.

In conection with this subject, with the dynamic fluid flows, with SPH formulation for hydrodynamics with material strength are also many papers published and this aspect is over the target of this work. Others options can be made regarding to the computation or not of the particle approximation between two different SPH parts and regarding to the time integration type for the smoothing length h:

$$\frac{d}{dt}\big(h(t)\big)=\frac{1}{d}h(t)div(v)\,, \tag{24}$$

or,

$$\frac{d}{dt}\left(h(t)\right) = \frac{1}{d}h(t)\left(div(v)\right)^{1/3} \tag{25}$$

The smoothing length h, can be calculated by the program, just the calculus begining, if this is permited to be variable during computing simulation, or can has a defined values, established by the user (using parameters CSLH, HMIN and HMAX, of SECTION_SPH). There were some posibilities for smoothing length calculus, but the last researching lead to a formulation that considers the neighbour particles of a given particle, the particles that are included in a sphere centered in xi having a radius of $h(xi)$.

Owing to Ls-Dyna conception for implementing of SPH method, almost all the actual features of LS-DYNA can be used with this method, with its classical keywords.

The nodal displacements, forces between the particles, pressure, energy, stresses and others are calculated by the program (using the particle approximations of the equations of mass, momentum, energy conservation) and these outcomes can be post-processed.

A connection between particles and finite elements are possible in a numerical analysis of a structure and also the contact between particles and brick and shell elements can be realised by the classical procedures.

4. NUMERICAL SIMULATION OF TAYLOR TEST

During World War II, Taylor and later Whiffen conducted tests (the Taylor test) to characterize the dynamic compressive yield strength of a variety of metals. They shot metal rods against "rigid" anvils and then measured the change in length of the rods to determine a minimum value of the dynamic compressive yield strength [4, 5, 6].

For studying the influence of characteristics parameters and material model constants, the authors used numerical simulation of Taylor test. A solid cylinder with radius of 5 mm and the length of 50 mm, made of 1018 steel was considered. Two numerical models were used for studying the impact between this metal rod with a rigid wall: FEM and SPH models.

Numerical models were built using two model types; in a first version (FEM1), the cylinder was meshed with 2993 nodes and 2560 elements (element size being 1.250x1.077x1.077 mm); in the second version (FEM2), the cylinder was meshed with 24705 nodes and 23040 elements (element size being 0.625x0.975x0.975 mm). Figure 7 presents first mesh version. SPH model consisted in 4000 particles (equal distance between particles 1.00 mm). Figure 8 (*a* and *b*) presents the SPH model.

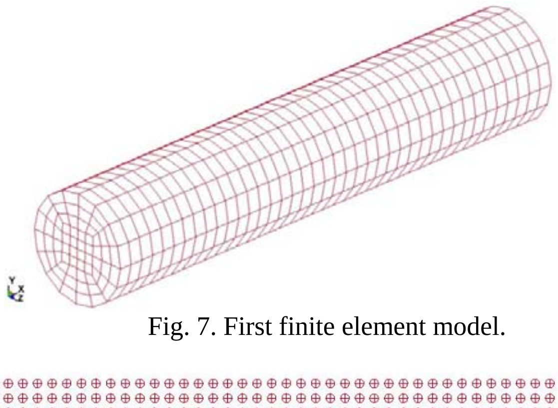
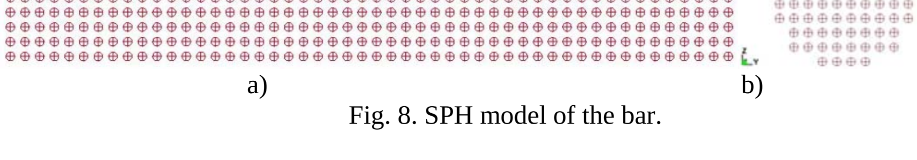


Fig. 7. First finite element model.



a) b)

Fig. 8. SPH model of the bar.

For both models, the fundamental measure units were: for length millimeter [*mm*], for time second [*s*] and for force Newton [*N*]. So, the mass measure unit will be [Ns^2/mm]. Analysis time was established at 0.003 seconds, for the stress and displacement field analysis, in a period after the impact, when the velocity changed its sign.

The study of material behavior was based on plastic-kinematic material model. In the Figure 9, deformed shape and UX-displacement field are presented, for FE and SPH modeling, for the time of 6e-5 s.

Table 1 presents some of the results for the default values of SPH using. In the Table 2, the same results are presented for different values of the parameter FORM.

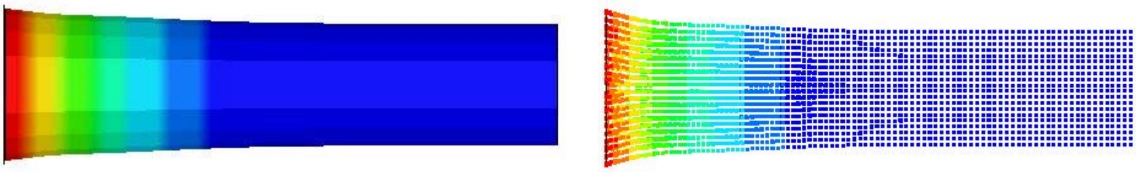


Fig. 9. UX-displacement field.

Table 1. Impact effects upon the bar.

Models	Bar Head		Bar Tail	
	UX_{max}	VX_{max}	UX_{max}	VX_{max}
	mm	mm/s	mm	mm/s
FEM	43.390	26557	38.404	28720
SPH	43.475	27295	38.172	26891
	Er. 0,2%	*Er. 2,77%*	*Er. -0.6%*	*Er. - 6,4%*

Table 2. The influence of the kernel type.

FORM parameter	Bar Head		Bar Tail	
	UX_{max}	VX_{max}	UX_{max}	VX_{max}
	mm	mm/s	mm	mm/s
FORM=1	42.592	25176	37.866	27575
Er. [%]	*-1.84*	*-5.20*	*-1.40*	*-3.98*
FORM=2	43.556	25371	38.245	27010
Er. [%]	*0.38*	*-4.46*	*-0.41*	*-5.95*
FORM=3	42.055	24571	37.302	26559
Er. [%]	*-3.10*	*-7.47*	*-2.87*	*-7.52*
FORM=5	43.545	25384	38.235	26997
Er. [%]	*0.36*	*-4.41*	*-0.44*	*-5.99*
FORM=6	42.272	25055	37.546	27687
Er. [%]	*-2.57*	*-5.65*	*-2.23*	*-3.59*

Another researched aspect was the influence of the ratio *h/d* upon results, especially in the contact zone. The quantitative results can be watched looking at Table 3. The best results are obtained for automatic calculus of the smoothing length (by the program), or for a ratio *h/d* = 1 established by the user.

Table 3. The Influence of ratio *h/d* upon results.

Models	Ratio h/d	Bar Head	
		UX_{max}	VX_{max}
		mm	mm/s
FEM2	-	**43.390**	**26557**
SPH1	automatic	**43.475**	**27295**
	h/d = 1.50	51.825	25310
	h/d = 1.25	41.166	23.444
	h/d = 1.00	**43.585** *Er. 0.45%*	**27535** *Er. 3.68%*
	h/d = 0.75	43.182	27936
	h/d = 0.50	25.333	18885
	h/d = 0.25	0.000	0.000

The results of the numerical simulation are strongly determined by the material coefficients used by the material model. One of the most used material model, adopted for dynamic analysis, is the Elastic Plastic with Kinematic

Hardening Model, being strain rate dependent plasticity for isotropic materials.

The strain rate is taken into account by Cowper-Symonds model using the coefficients *C* and *P*, having the same name. The yield function σ_y is:

$$\sigma_y = \left[1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}}\right]\left(\sigma_0 + \beta E_p \varepsilon_p^{ef}\right) \tag{24}$$

where σ_0 is the initial yield stress, ε_p^{ef} is the effective plastic strain, E_p is the plastic hardening modulus which is given by:

$$E_p = \frac{E_T E}{E - E_T} \tag{25}$$

where β is the hardening parameter that can vary between 0 and 1 depending on plasticity type (0 for kinematic and 1 for isotropic, respectively) and E_T is the tangent modulus. For this model, the user has to specify the failure strain, for which, elements will be eliminated.

The results of our numerical simulation are strongly determined by the material coefficients used by the material model. The Figures 10…15 show such a dependence, when each coefficient, alternatively, was kept constant, around its common value, in the case of SPH1 model.

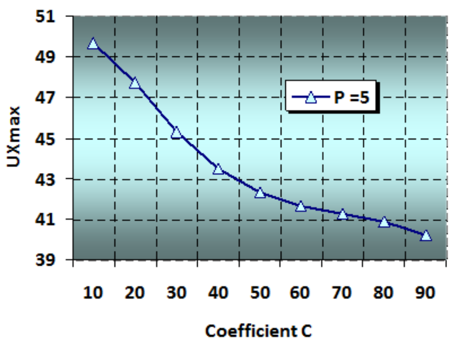


Fig. 10. UX_{max} versus C coefficient.

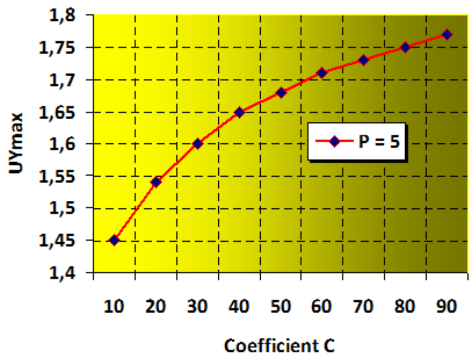


Fig. 11. UY_{max} versus C coefficient

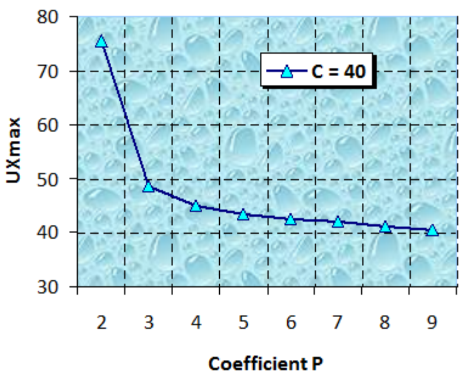


Fig. 12. UX_{max} versus P coefficient

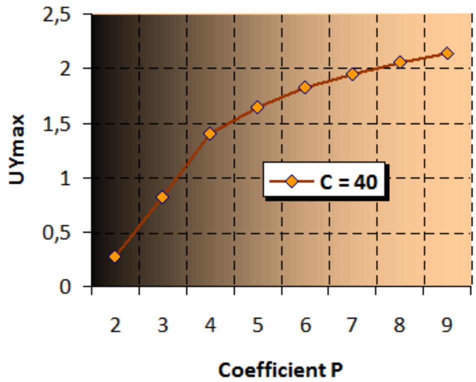


Fig. 13. UY_{max} versus P coefficient

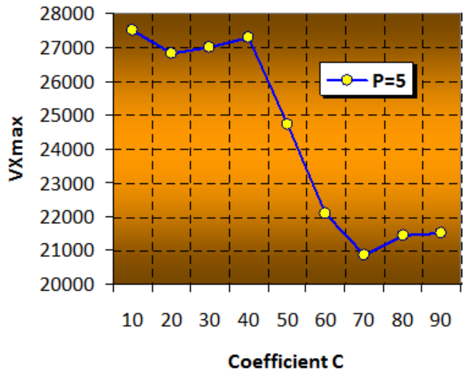


Fig. 14. VX_{max} versus C coefficient.

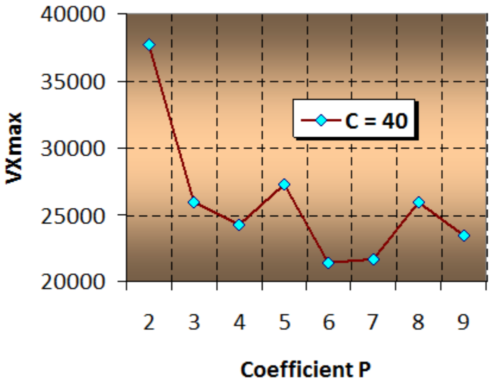


Fig. 15. VX_{max} versus P coefficient.

7. CONCLUSIONS

For problems like high velocity impact, impact with special materials (ceramics, glass etc), direct interaction between solid-fluid and others, the SPH method is a powerful numerical tool.

The best appropriate method, for studying of debris cloud developing and their effects, is only the smoothed particle hydrodynamics. There are many aspects when the SPH method is better than FEM and conversely.

Many problems belonging to applied mechanics field, especially those involving large deformations, can be solved by SPH method.

Next to the aspects presented in this paper, saving the computer time has to be added. In the SPH modeling, a condition, very important for good results, is that the distance between particles to be uniform one. This condition seems to be more important than the distance length.

The ratio *h/d* has the greatest important; this ratio, for some values, can lead to wrong (unexpected, unrealistic) results. For a right choosing of the ratio *h/d,* the most suitable way is the using of special criterions for it, or to use the facilities offered by the program.

Our research shows that a value around 1 (0.9…1.1) for *h/d* ratio, could be a right choosing. When a problem involves the using of the material models, the model constants have a maximum importance. The results a deeply affected by the material constants (like Cowper-Symonds, but all the constants used by different material models).

The curves presented in Figures 10…15 show us that these parameters are strongly influenced and surely, the errors would be unacceptable. For 1018 steel, the right values for C and P constants are 40 and 5, respectively.

The research of the influence of different parameters upon SPH models used in applied mechanics has to be continued. Many others parameters could be studied from this point of view.

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