CALCULATING MODEL FOR THE FRICTION DRAG OF A SHIP'S HULL

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Abstract: This paper presents the contribution of the authors regarding the friction drag calculation of a ship's hull using the quasi-plane model. The velocity calculation by a dimensional model helps to determine the friction tangential effort. Then, by integration, can be determined the friction drag of a rectangular plane panel. By extension over all the plane plates which compose the ship's side, results the friction drag for entire ship's side.

Keywords: friction drag, quasi-plane model, plane plate, curve panel

1. INTRODUCTION

Let us consider *T* the draft ship full loaded. The careen is reported to a system *Oxyz*, ensuing that:

- the longitudinal plane coincides with the plane *xOz*;
- the plane *xOy* is the waterline plane full loaded;
- the *Oz* axis is situated in the stem proximity, without crossing it (can be tangent to it, see Figure 1).

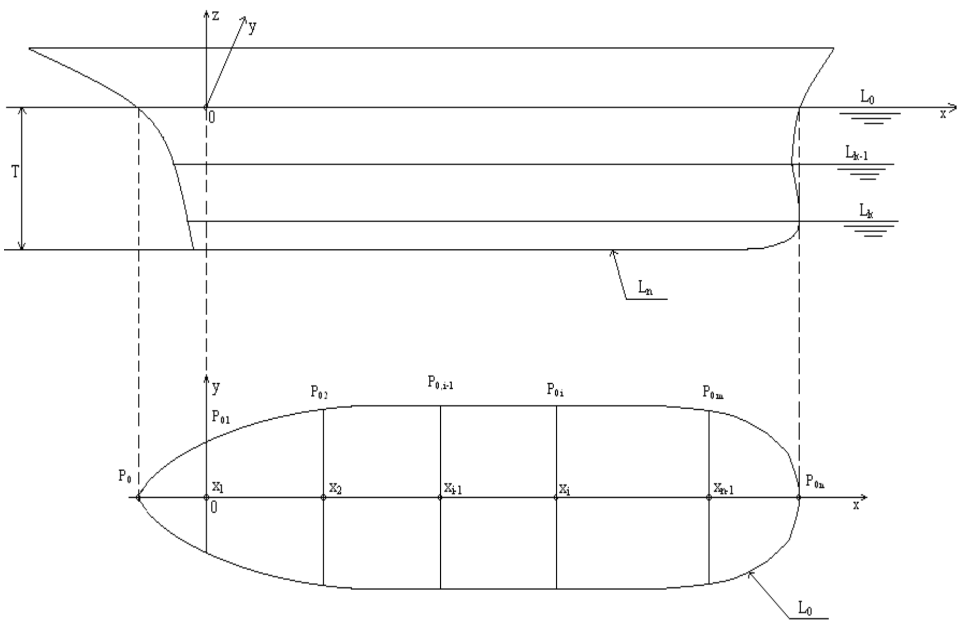

Fig. 1. The careen reported to a system *Oxyz:*
T – draft; L_k, L_{k-1} L_0 – waterlines; P_{01} P_{02} $P_{0,i-1}$ P_{0i} – curve panels.

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The waterline corresponding to the draft T_k; $(T_0 - T)$ is noted by L_k; $(k = 0,.., m)$; the planes which contain this lines are defined by equation:

$$z = z_k = -(T - T_k) = (T_k - T);\ (k=0,..., m) \tag{1}$$

Through its marks on the plane xOy, the planes $x = x_i$ $(i = 1, ..., n-1)$ has the characteristics below:

- the plane $x = x_1$ coincides with the plane yOz (so $x_1=0$);
- the plane $x = x_{n-1}$ is tangent to the stern frame (without crossing it).

The planes $z = z_k$ $(k = 0,..,m)$, $x = x_i$ $(i = 1,...,n-1)$, the stem and the stern frame establish on the side a succession of curve panels $P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}$, $(k = 0,...m-1)$, $(i=1,...n)$. P_{ki} is the notation of the half waterline point L_k , on the x_i axes (Figure 2). From Figure 2 results that the point P_{k0} belongs to the stem and the point P_{kn} belong to the stern frame.

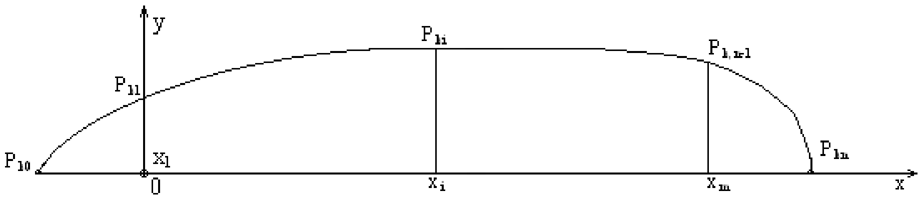

Fig. 2 Curve panels over a waterline.

As regarding the curve panel $P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}$, it can be approximate in a rectangular plane panel defined by two consecutive sides:

1) sides $P_{k,i-1}P_{ki}$ and $P_{k,i-1}P_{k+1,i-1}$;
2) sides $P_{k,i-1}P_{ki}$ and $P_{ki}P_{k+1,i}$;
3) sides $P_{k+1,i-1}P_{k+1,i}$ and $P_{k+1,i-1}P_{k,i-1}$;
4) sides $P_{k+1,i-1}P_{k+1,i}$ and $P_{k+1,i}P_{ki}$.

In the situations 1) and 2) the fluid moves on the vector direction and purpose-oriented $\overline{P_{k,i-1}P_{ki}}$ (so, it is parallel by the respectively plane), of constant velocity $\dfrac{\overline{v_{k,i-1}} + p' \cdot \overline{v_{ki}}}{1+p'}$, where $\overline{v_{k,i-1}}$ and $\overline{v_{ki}}$ - the velocities corresponding to the points $P_{k,i-1}$ and P_{ki} (results from the potential flow model), p' - the weight which make possible the vectors parallelism $\overline{P_{k,i-1}P_{ki}}$ and $\dfrac{\overline{v_{k,i-1}} + p \cdot \overline{v_{ki}}}{1+p'}$.

Similar, in the situations 3) and 4) the fluid moves on the vector direction $\overline{P_{k+1,i-1}P_{k+1,i}}$ (so, it is parallel by the respectively plane), of constant velocity [1]:

$$\frac{\overline{v_{k+1,i-1}} + p'' \cdot \overline{v_{k+1,i}}}{1+p''} \tag{2}$$

where: $\overline{v_{k+1,i-1}}$ and $\overline{v_{k+1,i}}$ – the velocities corresponding to the points $P_{k,i-1}$ and P_{ki} (results from the potential flow model), p'' – the weight which make possible the vectors parallelism $\overline{P_{k,i-1}P_{ki}}$ and

$$\frac{\overline{v_{k,i-1}} + p'' \cdot \overline{v_{ki}}}{1+p''} \tag{3}$$

Be F_{f_1}, F_{f_2}, F_{f_3}, F_{f_4} the friction forces developed by the fluid on the plane panels adequate to the situations 1), 2), 3), 4) and R_{f_1}, R_{f_2}, R_{f_3}, R_{f_4} the forces projections on the ship longitudinal plane. The R_{f_i}, i = (1, 2, 3, 4) projections arithmetic average, noted by $R_{f_{P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}}}$, will approximate the ship friction drag component corresponding to the curve panel $P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}$. If R_{f_c} is the keel friction drag, the ship friction drag can be written as:

$$R_f = 2 \cdot \sum_{k=0}^{m-1} \sum_{i=1}^{n} R_{f_{P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}}} + R_{f_c} \tag{4}$$

Double amount applies to all curve panels making up the port side or starboard side of the ship.

2. THE CALCULATION OF THE FRICTION DRAG COMPONENTS

Be an open polygonal segment $P_{k0}P_{k1}...P_{ki}...P_{k,n-1}P_{kn}$ which approximates the half waterline L_k from Figure 3. If this open polygonal contour is put on the straight line, it is obtained the segment $P_{k0}P_{kn}$ situated on the half axe $O\xi_k$ from Figure 3, of the origin O superimposed over a P_{k0} point.

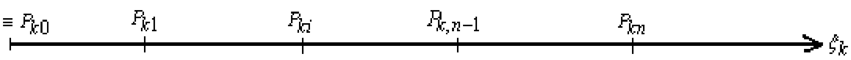

Fig. 3. Open polygonal segment.

Notations will be used below:

$$\overline{v_{P_{k,i-1}P_{ki}}} = \frac{\overline{v_{k,i-1}} + p' \cdot \overline{v_{ki}}}{1 + p'} \tag{5}$$

$$\overline{v_{P_{k+1,i-1}P_{k+1,i}}} = \frac{\overline{v_{k,i-1}} + p'' \cdot \overline{v_{ki}}}{1 + p''} \tag{6}$$

As regarding the friction tangential effort, it corresponds to the turbulent flow (characteristic to fluid flow around the ship side), defined by equation [2]:

$$\tau_{0t} = 0.028 \left(\frac{n}{n+1} \right)^{1,75} \cdot \rho v_0^2 \left(\frac{\upsilon}{v_0 \delta} \right)^{0,25} \tag{7}$$

where: ρ – fluid density; υ – kinematics viscosity of the fluid; v_0 – velocity module from relations (5) and (6); δ – boundary layer local thickness developed over the plane panel properly to one of the situations 1), 2), 3), 4) mentioned in the first paragraph; n – value from the lot (7), (9), (10) chosen depending by the size of the Reynolds number [1], defined by:

$$\mathrm{Re} = \frac{v \cdot l_k}{\upsilon} \tag{8}$$

where: v – ship rate speed; l_k – polygonal contour length $P_{k0}P_{k1}...P_{ki}...P_{k,n-1}P_{kn}$; $k = 0,...m$.

In the fluid flow case of velocity $\overline{v_{P_{k,i-1}P_{ki}}}$ along the plane panel defined in situations 1) and 2), by integrating the Kármán equation [2] on the domain $\xi_k \in \left[\xi_{P_{k,i-1}}, \xi_{P_{ki}}\right]$, it is obtained:

$$\frac{\delta^{1.25}\left(\xi_k\right)}{1.25} = 0.028 \cdot (n+2)\left(\frac{n}{n+1}\right)^{0.75} \cdot \left(\frac{\upsilon}{v_{P_{k,i-1}P_{ki}}}\right)\xi_k + C_{P_{k,i-1}P_{ki}} \tag{9}$$

$$\xi_k \in \left[\xi_{P_{k,i-1}}, \xi_{P_{ki}}\right]$$

The integration constant $C_{P_{k,i-1}P_{ki}}$ is determined from condition: $\xi_{P_{ki}} = \xi_{P_{k,i-1}}$, $\delta = \delta_{P_{k,i-1}}$ (k = 0,...m–1; n = 1,..., n), and $\delta_{P_0} = 0$.

In the fluid flow case of velocity $\overline{v_{P_{k+1,i-1}P_{k+1,i}}}$ along the plane panel defined in situations 3) and 4), by integrating the Kármán equation [2] on the domain $\xi_{k+1} \in \left[\xi_{P_{k+1,i-1}}, \xi_{P_{k+1,i}}\right]$, it is obtained:

$$\frac{\delta^{1.25}\left(\xi_{k+1}\right)}{1.25} = 0.028 \cdot (n+2)\left(\frac{n}{n+1}\right)^{0.75} \cdot \left(\frac{\upsilon}{v_{P_{k+1,i-1}P_{k+1,i}}}\right)\xi_{k+1} + C_{P_{k+1,i-1}P_{k+1,i}} \tag{10}$$

$$\xi_{k+1} \in \left[\xi_{P_{k+1,i-1}}, \xi_{P_{k+1,i}}\right]$$

The integration constant $C_{P_{k+1,i-1}P_{k+1,i}}$ is determined from condition:

- for $\xi_{P_{k+1,i}} = \xi_{P_{k+1,i-1}}$, $\delta = \delta_{P_{k+1,i-1}}$ (k = 0,...m–1; n = 1,..., n), and $\delta_{P_{k+1,0}} = 0$.

Using relations (7), (9) and (10), the turbulent tangential effort developed over the plane panel along the segments $P_{k,i-1}P_{ki}$ (in situations 1) and 2)), respectively $P_{k+1,i-1}P_{k+1,i}$ (in situations 3) and 4)), noted by $\tau_{P_{k,i-1}P_{ki}}$ and $\tau_{P_{k+1,i-1}P_{k+1,i}}$.

The average values of the efforts $\tau_{P_{k,i-1}P_{ki}}$ and $\tau_{P_{k+1,i-1}P_{k+1,i}}$ are obtained from relations:

$$\tau^*_{P_{k,i-1}P_{ki}} = \frac{1}{\xi_{P_{ki}} - \xi_{P_{k,i-1}}} \int_{\xi_{P_{k,i-1}}}^{\xi_{P_{ki}}} \tau_{P_{k,i-1}P_{ki}} \cdot d\xi_k \tag{11}$$

$$\tau^*_{P_{k+1,i-1}P_{k+1,i}} = \frac{1}{\xi_{P_{k+1,i}} - \xi_{P_{k+1,i-1}}} \int_{\xi_{P_{k+1,i-1}}}^{\xi_{P_{k+1,i}}} \tau_{P_{k+1,i-1}P_{ki}} \cdot d\xi_{k+1} \tag{12}$$

In the fluid flow case along a rectangular plane panel, of constant velocity parallel with one of the panel segments (for example MQ, Figure 4), the friction tangential effort along the segment MQ has the same distribution along any segment RS parallel to the segment MQ from the parallelogram. I.e. the average value of the friction tangential effort calculated along the segment MQ (or NP) is the same with the average value of the same effort for entire parallelogram surface.

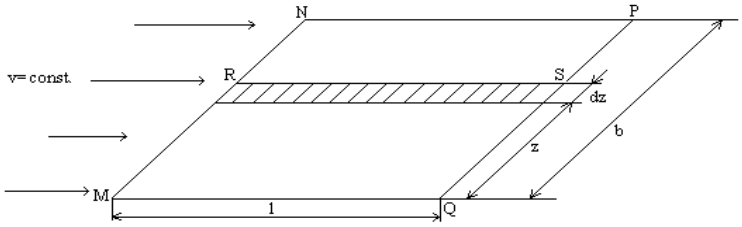

Fig. 4. Rectangular plane plate MNPQ:
l – plane plate length; b – plane plate width; ldz – plane plate elementary surface; z – current surface height; v = const. – current velocity.

If τ^* is the average value of the friction tangential effort along MQ and RS, then on an elementary surface the friction drag is developed (see the Fig. 4),

$$F_f = \int_A dF_f = \int_0^b \tau^* l dz = \tau^* lb \tag{13}$$

where τ^* does not depend by z.

The friction drag for the entire parallelogram MNPQ is obtained by integration:

$$F_f = \int_A dF_f = \int_0^b \tau^* l dz = \tau^* lb \tag{14}$$

The average friction tangential effort τ' for entire parallelogram MNPQ surface can be writing as:

$$\tau' = \frac{F_f}{A} = \frac{F_f}{lb} = \tau^* \tag{15}$$

From these considerations, the friction forces developed by the fluid over the plane panel determined in the situations 1), 2), 3), 4) are:

$$F_{f1} = \tau^*_{P_{k,i-1}P_{ki}} \cdot \left| \overline{P_{k,i-1}P_{ki}} \times \overline{P_{k,i-1}P_{k+1,i}} \right| \tag{16}$$

$$F_{f2} = \tau^*_{P_{k,i-1}P_{ki}} \cdot \left| \overline{P_{k,i-1}P_{ki}} \times \overline{P_{ki}P_{k+1,i}} \right| \tag{17}$$

$$F_{f3} = \tau^*_{P_{k+1,i-1}P_{k+1,i}} \cdot \left| \overline{P_{k+1,i-1}P_{k+1,i}} \times \overline{P_{k+1,i-1}P_{k,i-1}} \right| \tag{18}$$

$$F_{f4} = \tau^*_{P_{k+1,i-1}P_{k+1,i}} \cdot \left| \overline{P_{k+1,i-1}P_{k+1,i}} \times \overline{P_{k+1,i}P_{ki}} \right| \tag{19}$$

Be $\alpha_{P_{k,i-1}P_{ki}}$ and $\alpha_{P_{k+1,i-1}P_{k+1,i}}$ the angles formed by the horizontal segments $P_{k,i-1}P_{ki}$ and $P_{k+1,i-1}P_{k+1,i}$ with the longitudinal plane, resulting that:

$$\cos\alpha_{P_{k,i-1}P_{ki}} = \frac{x_i - x_{i-1}}{P_{k,i-1}P_{ki}} \tag{20}$$

$$\cos\alpha_{P_{k+1,i-1}P_{k+1,i}} = \frac{x_i - x_{i-1}}{P_{k+1,i-1}P_{k+1,i}} \tag{21}$$

The forces projections F_{f_i} (i = 1, 2, 3, 4) over the longitudinal plane can write as:

$$R_{f_1} = F_{f_1} \cdot \cos\alpha_{P_{k,i-1}P_{ki}} \tag{22}$$

$$R_{f_2} = F_{f_2} \cdot \cos\alpha_{P_{k,i-1}P_{ki}} \tag{23}$$

$$R_{f_3} = F_{f_3} \cdot \cos\alpha_{P_{k+1,i-1}P_{k+1,i}} \tag{24}$$

$$R_{f_4} = F_{f_4} \cdot \cos\alpha_{P_{k+1,i-1}P_{k+1,i}} \tag{25}$$

The arithmetic average of the forces R_{f_i} (i = 1, 2, 3, 4) represent the ship friction drag component corresponding to the curve panel $P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}$:

$$R_{f_{P_{k,i-1}P_{k+1,i-1}P_{k+1,i}P_{ki}}} = \frac{\sum_{i=1}^{4} R_{f_i}}{4} \tag{26}$$

As regarding the R_{f_c} term from relation above it is calculated simply: is the plane plate case in a parallel current for constant velocity (the ship regime speed).

3. CONCLUSIONS

The model presented in this paper is a quasi-plane model, based over a velocity field with two scalar components, v_x and v_y, defined by equation:

$$\begin{cases} v_x = v_x(x, y, z) \\ v_y = v_y(x, y, z) \end{cases} \tag{27}$$

It are obtained through the Kármán method of sources [3] arranged in the ship longitudinal plane of variable intensity z; The curve panel approximation of a side in rectangular plane plates (situations 1), 2), 3), 4) from paragraph 1), the consideration that the fluid flow is parallel to the velocities defined by relations (5) and (6) (dependents by z), and the forces average R_{f_i} (i = 1, 2 , 3, 4) represents the deviation improve ways from the curve side panels and from velocity variation on the z height. Obviously, the model precision will be better if the calculation is achieved with large number of side panels.

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