EFFECTS OF NATURAL CONVECTION IN THE HORIZONTAL CYLINDER

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Abstract: The magnitude of convection is proportional to the Rayleigh number based on the width of the melted region. The importance of the natural convection increase with time. For a short period after the beginning of melting, heat transfer is dominated by conduction. A regular solution is presented to demonstrate the increasing effect of natural convection on the melting process.

Keywords: physical model, Rayleigh number, melting

1. ANALYSIS

The physical model considered is infinity long circular cylinder of radius horizontally embedded solid material (figure 1). The temperature of the solid is held at the melting temperature, Tm, originally. The surface temperature of the hot cylinder is kept at a constant temperature, $T_i > T_m$ when the melting begins. The position of the melting front is denoted by $R\left(\psi, t\right)$ which not asymmetric due to the natural convection is. The variation of density through phase change which is the most commonly adopted assumption in the theoretical study of phase change problems is not considered here.

The equations governing the stream function and the temperature in cylinder polar coordinates are:

$$\frac{\partial}{\partial t}\left(\nabla^2 \overline{f}\right) + \frac{1}{\overline{r}} \overline{J}\left(\overline{f}, \nabla^2 \overline{f}\right) = \beta g \left[\sin\psi \frac{\partial T}{\partial \overline{r}} + \frac{\cos\psi}{\overline{r}} \frac{\partial T}{\partial \psi}\right] + \upsilon \nabla^2 \overline{f} \tag{1}$$

$$\frac{\partial T}{\partial \overline{t}} + \frac{1}{\overline{r}} \overline{J}\left(\partial f, T\right) = \alpha \nabla^2 T$$

where:

R = contour of melted region;

$\overline{r}, \psi$ denote the radial and the azimuthally coordinates;

t is the time;

υ is the kinematics' viscosity;

β is the thermal expansion coefficient;

g gravitational acceleration;

T represents temperature;

$\overline{f}$ is the stream function which is related to the velocities by

$$\overline{u} = -\frac{1}{\overline{r}}\frac{\partial \overline{f}}{\partial \psi}; \quad \overline{v} = \frac{\partial \overline{f}}{\partial \overline{r}};$$

where $\overline{u}, \overline{v}$ is the radial and azimuthally velocity.

α thermal diffusivity;

$$\nabla^2 = \frac{\partial^2}{\partial \overline{r}^2} + \frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}} + \frac{1}{\overline{r}^2}\frac{\partial^2}{\partial \psi^2}$$

$$\overline{J}\left(\overline{f}, \nabla^2 \overline{f}\right) = \frac{\partial \overline{f}}{\partial \overline{r}}\frac{\partial}{\partial \psi}\nabla^2 \overline{f} - \frac{\partial \overline{f}}{\partial \psi}\frac{\partial}{\partial \overline{r}}\nabla^2 \overline{f}$$

∇^2 is the Laplace operator in cylindrical polar coordinates and $\overline{J}$ is the Jacobian.

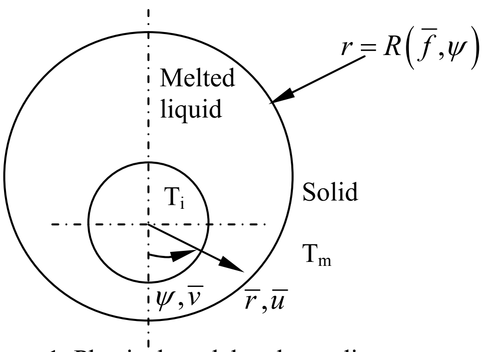

Figure 1. Physical model and coordinates

The moving melting front presents some difficulties in the analysis of the melting problem. A dimension less gap width of the melted liquid $B\left(\beta, t\right) = \frac{\left[R\left(\psi, t\right) - a\right]}{a}$ is introduced (a is radius of the hot cylinder). $f = \frac{\overline{f}}{\alpha}$ is stream function. $r = \frac{\overline{r} - a}{aB}$ is radial – coordinate transformation. $t = \frac{\overline{t}}{a^2 \alpha \varepsilon}; \varepsilon = \frac{C\left(T_i - T_m\right)}{L\left(\frac{\rho_l}{\rho_s}\right)}$ are time and Stefan number where C is the specific heat and L is the latent heat.

Substitution of anterior equations in first equation (1) results:

$$\Pr \cdot \left[\varepsilon B\left(\frac{\partial}{\partial t} - \frac{r\dot{B}}{B}\frac{\partial}{\partial t}\right)\nabla_1^2 f + \frac{1}{r + \frac{1}{B}} J\left(f, \nabla_1^2 f\right)\right] = Ra\left[\sin\psi \frac{\partial \theta}{\partial r} + \frac{\cos\psi}{r + \frac{1}{B}}\left(\frac{\partial \theta}{\partial \psi} - \frac{rB'}{B}\frac{\partial \theta}{\partial r}\right)\right] + B\nabla_1^4 f$$

$$\varepsilon B\left(\frac{\partial \theta}{\partial r} - \frac{r\dot{B}}{B}\frac{\partial \theta}{\partial r}\right) + \frac{1}{r + \frac{1}{B}} J\left(f, \theta\right) = B\nabla_1^2 f \tag{2}$$

where

$\Pr = \frac{\upsilon}{\alpha}$ is Prandtl number;

$Ra = \frac{\beta g\left(T_i - T_m\right)a^3}{\upsilon \alpha}$ is Rayleigh number;

$\dot{B}$ is B derivative with respect to t;

B' is B derivative with respect to ψ .

Equation (2) shows that the buoyancy forces are proportional to *Ra* which is evaluated at the radius of the inner cylinder.

In this paper, we concentrate on the short-time solution where natural convection is weak and can be treated as a perturbed quantity and condition is the dominant head transfer mode. Also, we assume that the Stefan number is small. This is a desirable characteristic in selecting thermal storage materials. For a small number, the melting front moves slowly and a quasi - steady approximation can be justified. With this assumptions the regular perturbation series can be expressed

$$\begin{aligned} \theta &= \left[\theta_{00} + \varepsilon\theta_{01} + \ldots\right] + Ra\cos\psi\left[\theta_{10} + \varepsilon\theta_{11} + \ldots\right] + Ra^2\cos 2\psi\left[\theta_{20} + \varepsilon\theta_{21} + \ldots\right] + \ldots \\ f &= Ra\sin\psi\left[f_{10} + \varepsilon f_{11} + \ldots\right] + \ldots \end{aligned} \tag{3}$$

where all the expression depends on *r*.

The governing equations of θ *'s* and *f's* can be obtained by substituting equations (3) into equations (2) and collecting the terms of equal order ε and *Ra*.The equations governing the gap functions can be obtained from the principle of energy conservation along the melting front, the boundary between two phases. It is:

$$-k_1\left(\frac{\partial T}{\partial \overline{r}}\right)_{\overline{r}=R} = \rho_s L\left(\frac{dR}{d\overline{t}}\right)$$

where *k* is the thermal conductivity and ρ is the density.

The gap width can be expanded into a power series of *Ra* and ε such as,

$$B = \left[B_{00} + \varepsilon B_{01} + \ldots\right] + Ra\cos\psi\left[B_{10} + \varepsilon B_{11} + \ldots\right] + Ra^2\cos 2\psi\left[B_{20} + \varepsilon B_{21} + \ldots\right] + \ldots \tag{4}$$

2. BOUNDARY CONDITIONS

Boundary conditions required to solve the above equations are:

a. $r = 0; \theta = 1$ constant temperature;

$f = \frac{\partial f}{\partial r} = 0$ no slip condition;

b. $r = 1; \theta = 0$ melting temperature

The initial condition for the melting front is:

$t = 0; B = 0.$

The solutions of equations which satisfy the boundary conditions are

$$\theta_{00} = 1 - A_1 \ln\left(1 + B_{00} r\right)$$

$$A_1 = \frac{1}{\ln\left(1 + B_{00}\right)}$$

$$\theta_{01} = A_2 \ln\left(r + \frac{1}{B_{00}}\right) + A_3 + C_{01}(r)$$
$$A_2 = \frac{C_{01}(0) - C_{01}(1)}{\ln(1 + B_{00})};$$
$$A_3 = A_2 \ln B_{00} - C_{01}(0); \tag{5}$$
$$C_{01}(r) = \frac{A_1 B_{01}}{B_{00}^2\left(r + \frac{1}{B_{00}}\right)} + ...$$

Equations (5) show that the solution is independent of the Prandtl number, at least up to the first order of *Ra* and ε.

3. RESULTS AND DISCUSSION

The propagation of the melting front can be described from equation (4) after the values of $B_{00}, B_{10}, B_{10}, ...$ are calculated. Since the regular perturbation solutions with the quasi-steady approximation are valid only for small RaB^3 and ε, we present the solutions up to the first order of *Ra* and ε. The value of $B_{00}, B_{10}, B_{10}, ...$ are show in figure 2. B_{00} represents the location of the melting front due to the heat conduction. B_{01} is the first order unsteady effect. Its values are negative. B_{10} is the first order effect due to the natural convection. Its values are also negative. Physically, B_{10} show that the natural convection sends a hot fluid upward along the surface of the hot cylinder. The liquid is cooled along the melting front and flows toward the bottom of the annulus region.

At the beginning of the melting process, the magnitude of natural convection is too small to be measured accurately. Since the gradient of B_{10} becomes steeper and that of B_{00} less steep as the size of the melted region grows, the natural convection eventually becomes the dominant heat transfer mode in the melting process.

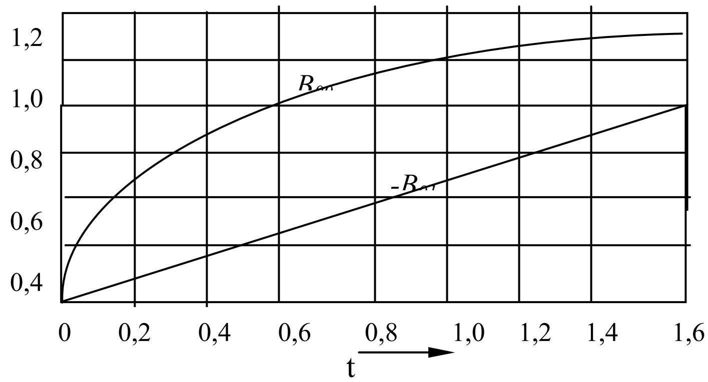

Figure 2. Melting front

REFERENCES

[1] Zan, La, Chen, F. – *Effects of Natural Convection*, Journal of Heat Transfer, Vol. 102, 1980.
[2] Sparrow, E., Schmidt, R. – *Experiments on the Role of Natural convection in the Melting of Solids*, ASME Journal of Heat Transfer, Vol. 100, 1978.
[3] Florescu, D., Florescu, I. – *Mecanica. Statica*. Vol. I, Editura Tehnica – Info, Chişinău, 2004.