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Scientific Studies and Research
Series Mathematics and Informatics
Vol. 25(2015), No. 2, 25-46

SEPARATION AXIOMS BETWEEN REGULAR SPACES AND R_0-SPACES

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Abstract: New separation axioms which lie strictly between regularity and R_0-axiom are introduced and their basic properties are studied. The interrelations and interconnections among them and the separation axioms which already exist in the mathematical literature are outlined. Their preservation under mappings is discussed. The investigation reveals several new epireflective (monoreflective) subcategories of TOP.

1. Introduction

The fundamental problem of topology is the homeomorphism / classification problem which yet remains to be resolved completely. However, a positive step in this direction is the formulation and investigation of new topological invariants. The knowledge of topological invariants is instrumental in solving the milder problem when two topological spaces are not homeomorphic by proving the presence of a topological property in one but not in the other.

Keywords and phrases: R_{D_δ}-space, R_{d_δ}-space, R_D-space, R_d-space, R_1-space, R_0-space, π_0-space, π_2-space ($\equiv P_\Sigma$-space$\equiv$ strongly s-regular space), R_δ-space, weakly Hausdorff space ($\equiv \delta T_1$-space), δT_0-space, $D_\delta T_1$-space, $D_\delta T_0$-space, initial property, epireflective (monoreflective) subcategory.
(2010) Mathematics Subject Classification: 54D10, 54B30.

It is this very spirit which constitutes the theme of the present paper as regards to lower separations axioms. Here we introduce several new separation axioms which lie strictly between regularity and R_0-axiom. The notion of regularity was introduced by Vietoris [42] (1921), while the class of R_0-spaces was introduced by Shanin [32] (1943) and rediscovered by Vaidyanathswamy [39] (1960), and by Davis [8] (1961).

Organization of the paper is as follows. Section 2 is devoted to preliminaries and basic definitions. In Section 3 we introduce certain weak regularity axioms and reflect upon their place in the hierarchy of variants of separation axioms which lie between regularity and π_0-axiom [39] and already exist in the mathematical literature. Herein examples are included to reflect upon the distinctiveness of (most of) the new axioms from the existing ones and among themselves. Section 4 is devoted to the study of basic properties of R_{D_δ}-spaces and R_{d_δ}-spaces wherein it is shown that (i) every regular space is an R_{D_δ}-space; (ii) a T_0 R_{d_δ}-space is a $D_\delta T_1$-space, a property stronger than Hausdorff axiom; (iii) the property of being an R_{D_δ}-space (R_{d_δ}-space) is preserved under disjoint topological sums and initial sources so it is hereditary, productive , sup-invariant, preimage invariant, inverse limit invariant and projective and (iv) the category of R_{D_δ}-spaces (R_{d_δ}-spaces) and continuous maps is a full isomorphism closed subcategory of TOP($\equiv$ the category of topological spaces and continuous maps) which is monoreflective (epireflective) in TOP. In Section 5 we study the properties of R_D-spaces and R_d-spaces which have properties analogous to that of R_{D_δ}-spaces and R_{d_δ}-spaces, respectively. The category of R_D-spaces (R_d-spaces) and continuous maps is a full, isomorphism closed monoreflective as well as epireflective subcategory of TOP containing the category of R_{D_δ}-spaces (R_{d_δ}-spaces). Section 6 is devoted to investigate the properties of π_2-spaces and R_δ-spaces. Some properties of R_1-spaces are discussed in Section 7.

2. Preliminaries and basic definitions

Let X be a topological space. A subset A of a space X is called a ***regular*** G_δ ***-set*** [30] if A is an intersection of a sequence of closed sets whose interiors contain A, i.e., $A = \bigcap_{n=1}^{\infty} F_n = \bigcap_{n=1}^{\infty} F_n^0$, where each F_n is a closed subset of X (here F_n^0 denotes the interior of F_n). The complement of a regular G_δ-set is called a ***regular*** F_σ***-set***. Any union of regular F_σ -sets is called $\mathbf{d}_\delta$***-open*** [22]. The complement of a d_δ-open set is referred to as a d_δ***-closed*** set.

A collection β of subsets of a space X is called an ***open complementary* system** [12] if β consists of open sets such that for every $B \in \beta$, there exist $B_1, B_2, \ldots \in \beta$ with $B = \bigcup\{X \backslash B_i : i \in N\}$. A subset A of a space X is called a ***strongly open*** F_σ ***-set*** [12] if there exists a countable open complementary system $\beta(A)$ with $A \in \beta(A)$. The complement of a strongly open F_σ-set is called ***strongly closed*** G_δ***-set.*** Brandenberg calls strongly closed G_δ-sets as D-closed in ([4] [5]). Any intersection of strongly closed G_δ -sets is called d^****-closed set*** [34].

A point $x \in X$ is called a θ***-adherent point*** [41] of a set $A \subset X$ if every closed neigbourhood of x intersects A. Let A_θ denote the set of all θ-adherent points of A. The set A is called θ***-closed*** [41] if $A = A_\theta$. The complement of a θ-closed set is referred to as a θ***-open set***.

A subset A of a space X is said to be ***regular open*** if it is the interior of its closure, i.e., $A = \overline{A}.^{\circ}$ The complement of a regular open set is referred to as a ***regular closed*** set. Any union of regular open sets is called a δ***-open set*** [41]. The complement of a δ-open set is referred to as a δ***-closed* set**. Any intersection of closed G_δ-sets is called a ***d-closed set*** [21].

2.1 Definitions: *A space X is said to be*

(i) $\boldsymbol{D_\delta}$***-Hausdorff*** [22] *if any two distinct points in X are contained in disjoint regular $F\sigma$ - sets.*

(ii) $\boldsymbol{D_\delta T_1}$***-space*** *if for each pair of distinct points x, y in X, there exist regular $F\sigma$ -sets U and V such that $x \in U, y \notin U$ and $y \in V, x \notin V$.*

(iii) $\boldsymbol{D_\delta T_0}$***-space*** [28] *if for each pair of distinct points x, y in X, there is a regular $F\sigma$ -set U containing one of the points x and y but not both.*

(iv) ***D-Hausdorff*** [16] *if any two distinct points in X are contained in disjoint open $F\sigma$ -sets.*

(v) ***D*-Hausdorff*** [33] *if any two distinct points in X are contained in disjoint strongly open $F\sigma$ -sets.*

(vi) θ ***-Hausdorff*** [7] *if any two distinct points in X are contained in disjoint θ- open sets.*

The following implications hold.

ultra Hausdorff
$\downarrow$
functionally Hausdorff
$\downarrow$ $\downarrow$
D^*-Hausdorff D_δ-Hausdorff $\to\theta$-Hausdorff
$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
D-Hausdorff $D_\delta T_1$-space Urysohn
$\downarrow$ $\downarrow$ $\downarrow$
Hausdorff $D_\delta T_0$-space $\to$ Hausdorff

Diagram-1

(Most of) the above implications are known to be irreversible as is shown by the following examples / observations.

2.2 Examples / Observations.

(i) A non-degenerate connected Tychonoff space is a functionally Hausdorff space which is not ultra Hausdorff.

(ii) Armentrout's [1] and Younglove's Moore spaces [47] on which every real-valued continuous functions is constant are D^*-Hausdorff spaces which are not functionally Hausdorff.

(iii) A space X is said to be D_δ-completely regular [24] if it has a base of regular F_σ - sets. The space A due to Hewitt [14] and further discussed in [24, Example 3.3] is a Hausdorff D_δ-completely regular space which is not functionally Hausdorff, so it is a D_δ-Hausdorff space which is not functionally Hausdorff.

(iv) Hewitt's example [14] (or Thomas space [38]) being a T_1-regular space is θ-Hausdorff but not D_δ-Hausdorff.

(v) For an example of a Urysohn space which is not θ-Hausdorff see [7].

(vi) The skyline space due to Heldermann [12] is a D-Hausdorff space which is not D^*-Hausdorff (see [23, Example 4.6]).

(vii) The mountain chain space due to Heldermann [12] is a Hausdorff space which is not D-Hausdorff (see [23 , Example 4.8]).

2.3 Definitions: *A space X is said to be an*

*(i) R_0-**space*** ([32] [39] [8]) *if for each open set U in X and each $x \in U$ implies that $\overline{\{x\}} \subset U$.*

*(ii) R_1-**space*** ([39] [8]) *if $x \notin \overline{\{y\}}$ implies that x and y are contained in disjoint open sets.*

(iii) π_0**-space** ([39, p 98]) *if every nonempty open set in* X *contains a nonempty closed set.*

(iv) ***weakly regular space*** [12] *if it has a base of* F_σ*-neighbourhoods.*

(v) π_2***-space*** [39] ($\equiv$ P_Σ***-space*** [44]$\equiv$ ***strongly s-regular space*** [10]) *if every open set in* X *is expressible as a union of regular closed sets.*

(vi) ***weakly Hausdorff space*** [36] *if every* x *in* X *is the intersection of regular open sets in* X*; or, equivalently, every* x *in* X *is the intersection of regular closed sets..*

(vii) δT_1***-space*** ([25] [9]) *if for each pair of distinct points* x *and* y *in* X *there exist regular open sets* U *and* V *such that* $x \in U, y \notin U$ *and* $y \in V, x \notin V$*..*

(viii) $\boldsymbol{\delta} T_0$***-space*** [26] *if for every pair of distinct points in* X *there exists a regular open set containing one of the points but not both..*

(ix) ***KC space*** [43] *if every compact set in* X *is closed in* X*..*

(x) ***US space*** [43] *if every convergent sequence in* X *has a unique limit in* X*..*

(xi) ***sober space*** [11] *if every nonempty irreducible set in* X *has a unique dense point i.e. a* T_0*-space in which every irreducible set is a point closure. A closed set in* X *is said to be* ***irreducible*** *if it can not be expressed as the union of two nonempty proper closed subsets..*

(xii) ***TD-space*** [3] *if* $\{x\}'$*,the derived set of* $\{x\}$*, is closed for every* $x \in X$*.*

2.4 Remarks: 1) Vaidyanathswamy [39] calls R_0-axiom as π_1-axiom in his text book (see [39, p 98]).

Czászár calls R_0-space as S_1-space in [6] and Worrel and Wicke call R_0-space as essentially T_1 space [45] .

2) Yang [46], while studying paracompactness, refers to R_1-space as a T_2'-space. Czászár calls R_1-spaces as S_2-spaces in his text book [6].

3) π_2-spaces were defined by Vaidyanathswamy [39] (1960) and rediscovered by Wong [44] (1981) and Ganster [10] (1980) with different terminologies.

4) Ekici [9] calls δT_1-spaces as rT_1-spaces.

The following implications are either well known or immediate from definitions.

Hausdorff $\rightarrow$ KC $\rightarrow$ US
$\downarrow$ $\downarrow$ $\downarrow$
Sober space weakly Hausdorff ($\equiv \delta T_1$-space) $\rightarrow$ T1-space
$\downarrow$ $\downarrow$ $\downarrow$
T_0-space $\leftarrow$ δT_0-space T_0-space $\leftarrow$ TD-space

Diagram-2

However, none of the above implications is reversible as is immediate from the following assertions / examples.

2.5 (i) The implications

$$\text{Hausdorff} \rightarrow \text{KC} \rightarrow \text{US} \rightarrow \text{T1}$$

and their irreversibility are due to Wilansky [43].

(ii) The implications

$$T_1 \text{ -space} \rightarrow T_D \text{ -space} \rightarrow T_0 \text{ -space}$$

and their irreversibility are due to Aull and Thron [3].

2.6 Non-Hausdorff sober spaces are widely used in domain theory and occur in abundance in the literature. For example,

(i) Let L be a complete lattice endowed with the Scott topology $\sigma(L)$ such that sup operation in L is jointly continuous. Then the space $(L, \sigma(L))$ is a non-Hausdorff sober space (see [11, p. 106]). For the definition of Scott topology we refer the interested reader to [11].

(ii) Let L be a complete lattice and let $SpecL$ be the prime spectrum of L, i.e. the set of all prime elements of L different from endowed with the hull kernel topology is a sober space [11, p.252]. For the definition of hull kernel topology (see [11]).

(iii) Let L be a continuous lattice. Then the space $(L, \sigma(L))$ is a non-Hausdorff sober space which is not even a T_1-space space (see [11, p.106]). A lattice L is a continuous lattice if and only if it is isomorphic to a subset of cube which is closed under arbitrary infimums and directed supremums. [11, p.201].

2.7 An infinite (uncountable) set endowed with cofinite (cocountable) topology is a T_1-space which is not a δT_0-space.

2.8 Proposition: *A space X a weakly Hausdorff space if and only if it is a δT_1-space.*

3. Weak regularity axioms

3.1 Definitions: *A space X is said to be an*

*(i) R_{D_δ}**-space** if for each open set U in X and each $x \in U$ there exists a regular G_δ-set H containing x such that $H \subset U$; equivalently U is expressible as a union of regular G_δ-sets..*

*(ii) R_{d_δ}**-space** if for each open set U in X and each $x \in U$ there exists a d_δ-closed set H containing x such that $H \subset U$; equivalently U is expressible as a union of d_δ-closed sets..*

*(iii) **$\boldsymbol{R_\delta}$ -space*** [29] *if for each open set U in X and each $x \in U$ there exists a δ –closed set H containing x such that $H \subset U$; equivalently, U is expressible as a union of δ -closed sets..*

*(iv) **$\boldsymbol{R_{\mathrm{D}}}$ -space** if for each open set U in X and each $x \in U$ there exists a closed G_δ-set H containing x such that $H \subset U$; equivalently U is expressible as a union of closed G_δ- sets..*

*(v) **$\boldsymbol{R_{\mathrm{d}}}$ -space** if for each open set U in X and each $x \in U$ there exists a d-closed set H containing x such that $H \subset U$; equivalently U is expressible as a union of d-closed sets..*

3.2 Remark: The class of R_{D_δ}-spaces properly contains each of the classes of regular spaces and functionally regular spaces ([3] [40]). The class of R_{d_δ}-spaces properly contains the class of R_z-spaces ([27] [35]) which in its turn properly contains each of the classes of functionally regular spaces and functionally Hausdorff spaces.

The following diagram well illustrates the interrelations and interconnections that exist between weak variants of regularity defined in Definitions 3.1 and the separations axioms which already exist in the mathematical literature and are related to the theme of the present paper.

regular space	$\rightarrow$	π_2- space	$(\equiv P_\Sigma-$ space)			($\equiv$strongly $s-$regular space)
$\downarrow$		$\downarrow$				$\downarrow$
$\downarrow$		$R_{D_\delta}-$ space	$\rightarrow$	$R_{d_\delta}-$ space		$\equiv R_{d_\delta}-$ space
$\downarrow$		$\downarrow$		$\downarrow$		$\downarrow$
$\downarrow$		R_D- space	$\rightarrow$	R_d- space		R_1- space
$\downarrow$				$\downarrow$		$\downarrow$
weakly regular space	$\rightarrow$	$\rightarrow$	$\rightarrow$	R_0- space		$R_\delta-$ space
				$\downarrow$		
				π_0- space		

Diagram-3

Most of the above implications are known to be not reversible as is shown by the following examples / observations.

Examples / Observations.

3.3 Every expansion of a metric space is an R_{D_δ}-space which need not be regular. More generally every expansion of a T1 perfectly normal space is an R_{D_δ}-space which need not be regular. For example, Smirnov's deleted sequence topology [37, p.86] is an expansion of the Euclidean topology on the real line and is an R_{D_δ}-space which is not regular.

3.4 The real line with cofinite or cocountable topology is an R_0 –space which is not an R_δ-space.

3.5 Every weakly Hausdorff space X is an R_δ-space, since every singleton in X is δ-closed being the intersection of regular closed sets.

3.6 In [39, Ex. 23, p.99] is given an example of π_0-space which is not an R_0-space.

3.7 Example 3 of Ganster [10] gives a Hausdorff space which is not a π_2-space ($\equiv$ strongly s-regular space). So it is an R_δ-space which is not a π_2-space.

3.8 Soundararajan's example in [36, Proposition 2.1] is a non-Hausdorff weakly Hausdorff space and hence an R_δ-space which is not an R_1-space.

3.9 The real line with right ray topology is not a π_0-space. More generally, a complete lattice endowed with Scott topology is not a π_0-space.

Next we give an example of an R_1-space which is not an R_{d_δ}-space. The space E_0 in the following example is due to Misra [31, Example 3.1, p.352].

3.10 Let w_1 be the first uncountable ordinal. Let the space E_o be the union of disjoint sets $\{a, b\}$, $\{a_{\alpha\beta} : 0 \leq \alpha, \beta < w_1\}$, $\{b_{\alpha\beta} : 0 \leq \alpha, \beta < w_1\}$ and $\{c_\gamma : 0 \leq \gamma < w_1\}$. The basic neighbourhoods of various points are as follows: all the points $a_{\alpha\beta}$ and $b_{\alpha\beta}$, $0 \leq \alpha$, are isolated; for each fixed γ,a typical basic neighbourhood of the point $c_\gamma$contains the points $a_{\gamma\beta}$ and $b_{\gamma\beta}$ for all but countably many indices β, $0 \leq \beta < w_1$; a typical basic neighbourhood of a (respectively b) contains, for every α greater than some ordinal $\delta < w_1$,all but countably many points $a_{\alpha\beta}$(respectively $b_{\alpha\beta}$). The space E_o is a Hausdorff, non Urysohn P-space. However, no neighbourhood of a or b is expressible as a union of d_δ-closed sets. So it is an R_1-space which is not an R_d-space and hence not an R_{d_δ}-space.

3.11 Problem: At present we do not know the following examples and leave it open for the interested reader. Example of
(i) an R_{d_δ}-space which is not an R_{D_δ}-space.
(ii) an R_d-space which is not an R_D-space.
(iii) an R_D-space which is not an R_{D_δ}-space.
(iv) an R_d-space which is not an R_{d_δ}-space.

4. Basic properties of R_{D_δ}-spaces and R_{d_δ}-spaces

4.1 Theorem: *Every regular space is an R_{D_δ}-space.*

Proof: Let X be a regular space and let U be a nonempty open subset of X. Let $x \in U$. By regularity of X there exists an open set V_1, such that $x \in V_1 \subset \bar{V}_1 \subset U$. Another application of regularity yields that there is an open set V_2 such that $x \in V_2 \subset \bar{V}_2 \subset V_1$. Continuing in this way obtain a nested sequence $\{V_n\}$ of open sets satisfying

$$... \subset V_{n+1} \subset \bar{V}_{n+1} \subset V_n \subset \bar{V}_n \subset V_{n-1} \subset ... \subset V_1 \subset \bar{V}_1 \subset U.$$

Then $\bigcap_{n=1}^{\infty} V_n = \bigcap_{n=1}^{\infty} \bar{V}_n = A_x$. So $A_x$is a regular G_δ-set containing x and is contained in U. Thus X is an R_{D_δ}-space.

4.2 Theorem: *A T_0 R_{d_δ}-space is a $D_\delta T_1$-space and so is a Hausdorff space.*

Proof: Let X be a T_0 R_{d_δ}-space. Let $x, y \in X$ with $x \neq y$. Since X is a T_0-space, there exists an open set U containing one of the point x and y but not both. To be precise assume that $x \in U$. Since X is an R_{d_δ}-space, there exists a d_δ-closed set A such that $x \in A \subset U$. Suppose that $A = \bigcap_{\alpha \in \Lambda} H_\alpha$, where each H_α is a regular G_δ-set. There exists an $\alpha_0 \in \Lambda$ such that $x \in H_{\alpha_0}$ but $y \notin H_{\alpha_0}$. Then $X \backslash H_{\alpha_0}$is a regular $F_\sigma-$set containing y but not x. So X is a $D_\delta T_0$-space. Now, since every $D_\delta T_0$-space is Hausdorff, there exist disjoint open sets G_1 and G_2 containing x and y, respectively. So there exist d_δ-closed sets B and C such that $x \in B \subset G_1$ and $y \in C \subset G_2$. Let $B = \bigcap_{\alpha \in \Lambda} B_\alpha$ and $C = \bigcap_{\beta \in \Sigma} C_\beta$, where each B_α and each C_β is a regular G_δ-set. Then there exist $\alpha_0 \in \Lambda$ and $\beta_0 \in \Sigma$ such that $x \in B_{\alpha_0}$ but $y \notin B_{\alpha_0}$ and $y \in C_{\beta_0}$ but $x \notin C_{\beta_0}$. Then $X \backslash B_{\alpha_0}$ and $X \backslash C_{\beta_0}$ are regular $F_\sigma-$sets such that $x \in X \backslash C_{\beta_0}$, $y \notin X \backslash C_{\beta_0}$ and $y \in X \backslash B_{\alpha_0}$, $x \notin X \backslash B_{\alpha_0}$. Thus X is a $D_\delta T_1$-space.

4.3 Corollary: *A T_0 R_{d_δ}-space is a $D_\delta T_0$-space and so is a Hausdorff space.*

4.4 Corollary: *A T_0 R_{D_δ}-space is a $D_\delta T_1$-space.*

4.5 Question: *Is a T_0 R_{D_δ}-space D_δ-Hausdorff ?*

4.6 Question: *Is a T_0 R_{d_δ}-space D_δ-Hausdorff ?*

4.7 Definition [22]: *A point x in a space X is said to be a $d_\delta-$**adherent point** of a set $A \subset X$ if every regular $F_\sigma-$set U containing x intersects A. Let A_{d_δ} denote the set of all $d_\delta-$adherent points of A. The set $A_{d_\delta}$is the smallest d_δ-closed set containing A.*

4.8 Lemma: *The correspondence $A \rightarrow A_{d_\delta}$is a Kuratowski closure operator.*

4.9 Theorem: *Let X be a topological space. Consider the following statements:*

(i) X is an R_{d_δ}-space

(ii) For each $x \in X$ and for each open set U containing x, we have $\{x\}_{d_\delta} \subset U$.

(iii) There exists a subbase $\mathcal{S}$ for X such that if $x \in S$ and $S \in \mathcal{S}$, then $\{x\}_{d_\delta} \subset S$

(iv) If $x \in \{y\}_{d_\delta}$, then $y \in \{x\}_{d_\delta}$

(v) If $x \in \{y\}_{d_\delta}$, then $\{x\}_{d_\delta} = \{y\}_{d_\delta}$.

Then (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii) $\Rightarrow$ (iv) $\Leftrightarrow$ (v).

Proof: (i) $\Rightarrow$(ii). Let $x \in X$ and let U be an open set containing x. Since X is an R_{d_δ}-space, there exists a $d_\delta-$closed set A such that $x \in A \subset U$ and so, in view of Lemma 4.8, $\{x\}_{d_\delta} \subset U$.

(ii) $\Rightarrow$(i). is obvious since $\{x\}_{d_\delta}$is $d_\delta-$closed.

(ii)$\Leftrightarrow$(iii). is obvious.

(iii) $\Rightarrow$(iv). Let $x \in \{y\}_{d_\delta}$. Every subbasic open set containing y contains $\{y\}_{d_\delta}$and so it contains x. This implies $y \in \{x\}_{d_\delta}$.

(iv) $\Rightarrow$(v). $x \in \{y\}_{d_\delta}$ implies $y \in \{x\}_{d_\delta}$. So, again in view of Lemma 4.6, ($x \in \{y\}_{d_\delta}$ and $y \in \{x\}_{d_\delta}$) implies that $\{x\}_{d_\delta} \subset \{y\}_{d_\delta}$ and$\{y\}_{d_\delta} \subset \{x\}_{d_\delta}$. Hence $\{x\}_{d_\delta} = \{y\}_{d_\delta}$.

The implication $(v) \Rightarrow (iv)$ is obvious.

4.10 Theorem: *For a topological space X the following statements are equivalent.*

(i) $\{x\}_{d_\delta} \neq \{y\}_{d_\delta}$implies that x and y are contained in disjoint open sets.

(ii) $y \notin \{x\}_{d_\delta}$ implies x and y are contained in disjoint open sets.

(iii) A is compact and $\{x\}_{d_\delta} \bigcap A = \varphi$ implies x and A are contained in disjoint open sets.

(iv) If A and B are compact, and $\{a\}_{d_\delta} \bigcap B = \emptyset$ for every $a \in A$, then A and B are contained in disjoint open sets.

Proof: (i) $\Rightarrow$(ii). Suppose that $y \notin \{x\}_{d_\delta}$. Then $\{x\}_{d_\delta} \neq \{y\}_{d_\delta}$and so by (i) x and y are contained in disjoint open sets.

(ii) $\Rightarrow$(iii). Suppose A is compact and $\{x\}_{d_\delta} \bigcap A = \emptyset$. So, for each $a \in A$, by (ii) there exist disjoint open sets U_a and V_a containing a and x, respectively. Thus $\mathcal{U} = \{U_a : a \in A\}$ is an open cover of the compact set A and so there exists a finite subcollection $\{U_{a_1}, ..., U_{a_n}\}$ of $\mathcal{U}$ which covers A. Let $U = \bigcup_{i=1}^n U_{a_i}$ and $V = \bigcap_{i=1}^n V_{a_i}$. Then U and V are disjoint open sets containing A and x, respectively.

(iii) $\Rightarrow$(iv). Suppose that A and B are compact and $\{a\}_{d_\delta} \bigcap B = \emptyset$ for every$a \in A$. Then by (iii) for each $a \in A$ there exist disjoint open sets U_a and V_a containing a and B, respectively. The collection $\mathcal{U} = \{U_a : a \in A\}$ is an open cover of the compact set A and so there exists a finite subcollection $\{U_{a_1}, ..., U_{a_n}\}$ of $\mathcal{U}$ which covers A. Let $U = \bigcup_{i=1}^n U_{a_i}$ and $V = \bigcap_{i=1}^n V_{a_i}$. Then U and V are disjoint open sets containing A and B, respectively.

(iv) $\Rightarrow$(i). Suppose that $\{x\}_{d_\delta} \neq \{y\}_{d_\delta}$. Then either $x \notin \{y\}_{d_\delta}$or $y \notin \{x\}_{d_\delta}$. For definiteness assume that $y \notin \{x\}_{d_\delta}$. Then $\{x\}_{d_\delta} \bigcap \{y\} = \emptyset$ and so by (iv) there exist disjoint open sets U and V containing x and y, respectively.

4.11 Theorem: *The disjoint topological sum of any family of R_{D_δ}-spaces (R_{d_δ}-spaces) is an R_{D_δ}-space (R_{d_δ}-space).*

4.12 Theorem: *The property of being an R_{D_δ}-space as well as the property of being an R_{d_δ}-space is an initial property.*

Proof: We shall prove the result in case of R_{D_δ}-space only. Let $\{f_\alpha : X \to Y_\alpha : \alpha \in \Lambda\}$ be a family of functions, where each Y_α is an R_{D_δ}-space and let X be endowed with the initial topology. Let U be any open set in X and let $x \in U$.Then there exist $\alpha_1, ..., \alpha_n \in \Lambda$ and open sets $V_i \in Y_{\alpha_i}$ $(i = 1, ..., n)$ such that $x \in f_{\alpha_1}^{-1}(V_1) \bigcap ... \bigcap f_{\alpha_n}^{-1}(V_n) \subset U$. Since each Y_α is an R_{D_δ}-space there exists a regular G_δ-set A_{α_i}in Y_{α_i} $(i = 1, ..., n)$ such that $f_{\alpha_i}(x) \in A_{\alpha_i} \subset V_i$. Since each f_α is continuous, it is easily verified that each $f_{\alpha_i}^{-1}(A_{\alpha_i})$ is a regular G_δ-set in X. Let $A = \bigcap_{i=1}^{n} f_{\alpha_i}^{-1}(A_{\alpha_i})$. Since finite intersection of regular G_δ-sets is a regular G_δ-set, A is a regular G_δ-set in X and $x \in A \subset U$ so X is an R_{D_δ}-space.

Th proof in other case is similar and hence omitted.

As an immediate consequence of Theorem 4.12 we have the following.

4.13 Theorem: *The property of being an R_{D_δ}-space or an R_{d_δ}-space is hereditary, productive, sup-invariant, preimage invariant, inverse limit invariant and projective.*

A topological property P is said to be projective if whenever a product space has property P every co-ordinate space possesses property P.

For the categorical terms used but not defined in the paper, we refer the reader to Herrlich and Strecker [13].

4.14 Theorem: *The category of R_{D_δ}-spaces (R_{d_δ}-spaces) and continuous maps is a full isomorphism closed subcategory of TOP which is simultaneously epireflective and monoreflective in TOP.*

4.15 Definition: *A function $f : X \to Y$ from a topological space X into a topological space Y is said to be*

(i) ***D_δ -closed*** *if $f(H)$ is a regular G_δ -set in Y for every regular G_δ -set H in X.*

(ii) ***d_δ -closed*** *if $f(A)$ is a d_δ -closed set in Y for every d_δ-closed set A in X.*

4.16 Theorem: *(a) Let $f : X \to Y$ be a continuous, D_δ -closed surjection defined on an R_{D_δ}-space X. Then Y is an R_{D_δ}-space.*

(b) Let $f : X \to Y$ be a continuous, d_δ-closed surjection defined on an R_{d_δ}-space X. Then Y is an R_{d_δ}-space.

Proof: (a) Let V be an open set in Y and let $y \in V$. Then $f^{-1}(V)$ is an open set in X containing $f^{-1}(y)$. Let $x \in X$. Since X is an R_{D_δ}-space, there exists a regular G_δ -set H in X such that $x \in H \subset f^{-1}(V)$. Since f is a D_δ -closed surjection, $f(H)$ is a regular G_δ -set in Y and $f(x) = y \in f(H) \subset V$ and so Y is an R_{D_δ}-space.

The proof of (b) is similar and hence omitted.

5. Properties of R_D-spaces and R_d-spaces

5.1 Definition [21]**:** *Let X be a topological space. A point $x \in X$ is said to be a **d-adherent point** of a set $A \subset X$ if every open F_σ-set containing x intersects A. Let A_d denote the set of all d-adherent points of A. The set $A_d$is the smallest d-closed set containing A.*

5.2 Lemma: *The correspondence $A\to A_d$is a Kuratowski closure operator.*

5.3 Theorem: *For a topological space X the following statements are equivalent:*

(i) $\{x\}_d \neq \{y\}_d$implies x and y are contained in disjoint open sets.

(ii) $y \notin \{x\}_d$ implies x and y are contained in disjoint open sets.

(iii) A is compact and $\{x\}_d \bigcap A = \emptyset$ implies x and A are contained in disjoint open sets.

(iv) If A and B are compact, and $\{a\}_d \bigcap B = \emptyset$ for every $a \in A$, then A and B are contained in disjoint open sets.

Proof: The proof of Theorem 5.3 is similar to that of the proof of Theorem 4.10 and makes use of Lemma 5.2 instead of Lemma 4.8.

5.4 Theorem: *Let X be a topological space. Consider the following statements:*

(i) X is an R_d-space.

(ii) For each $x \in X$ and for each open set U containing x, we have $\{x\}_d \subset U$.

(iii) There exists a subbase $\mathcal{S}$ for X such that $x \in S \in \mathcal{S} \Rightarrow \{x\}_d \subset S$.

(iv) If $x \in \{y\}_d$, then $y \in \{x\}_d$.

(v) If $x \in \{y\}_d$, then $\{x\}_d = \{y\}_d$

Then (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii) $\Rightarrow$ (iv) $\Leftrightarrow$ (v)

Proof: The proof of Theorem 5.4 is similar to that of the proof of Theorem 4.9 and makes use of Lemma 5.2 instead of Lemma 4.8.

5.5 Theorem: *The disjoint topological sum of any family of R_D-spaces (R_d-spaces) is an R_D-space (R_d-space).*

5.6 Theorem: *The property of being an R_D-space as well as the property of being an R_d-space is an initial property.*

We omit the proof of Theorem 5.6 which is similar to that of the proof of Theorem 4.12.

As an immediate consequence of Theorem 5.6 we have the following.

5.7 Theorem: *The property of being an R_D-space as well as the property of being an R_d-space is hereditary, productive, sup-invariant, preimage invariant, inverse limit invariant and projective.*

5.8 Theorem: *The category of R_D-spaces (R_d-spaces) and continuous maps is a full isomorphism closed subcategory of TOP which is simultaneously monoreflective and epireflective subcategory of TOP.*

5.9 Definition: *A function $f : X \to Y$ from a topological space X into a topological space Y is said to be*

(i) ***D -closed*** *if $f(A)$ is a closed G_δ -set in Y for every closed G_δ-set A in X.*

(ii) ***d -closed*** *if $f(A)$ is a d -closed set in Y for every d -closed set A in X.*

5.10 Theorem: *(a) Let $f : X \to Y$ be a continuous, D-closed function from X onto Y. If X is an R_D-space, then so is Y.*

(b) Let $f : X \to Y$ be a continuous, d-closed surjection from X to Y. If X is an R_d-space, then so is Y.

6. Properties of π_2-spaces and R_δ-spaces

6.1 Definition [41]**:** *Let X be a topological space. A point $x \in X$ is said to be a δ-**adherent** point of a set $A \subset X$ if every regular open set containing x intersects A. Let A_δ denote the set of all δ-adherent points of the set A. Then A_δ is the smallest δ-closed set containing A.*

The following lemma is essentially due to Veličko [41].

6.2 Lemma: *The correspondence $A \to A_\delta$ is a Kuratowski closure operator.*

6.3 Theorem: *For a topological space X,.consider the following statements*

(i) X is an R_δ-space

(ii) For each $x \in X$ and each open set U containing x, we have $\{x\}_\delta \subset U$.

(iii) There exists a subbase $\mathcal{S}$ for X such that $x \in S \in \mathcal{S} \Rightarrow \{x\}_\delta \subset S$.

(iv) For any $x, y \in X$, $x \in \{y\}_\delta$ implies $y \in \{x\}_\delta$.

(v) For any $x, y \in X$, $x \in \{y\}_\delta$ implies $\{x\}_\delta = \{y\}_\delta$.

Then (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii) $\Rightarrow$ (iv) $\Leftrightarrow$ (v).

The proof of the Theorem 6.3 is similar to that of the proof of Theorem 4.9 and makes use of Lemma 6.2 instead of Lemma 4.8.

6.4 Theorem: *For a topological space X, the following statements are equivalent:*

(i) For any $x, y \in X$, $\{x\}_\delta \neq \{y\}_\delta$implies x and y are contained in disjoint open sets.

(ii) For any $x, y \in X$, $y \notin \{x\}_\delta$ implies x and y are contained in disjoint open sets.

(iii) A is compact and $\{x\}_\delta \bigcap A = \emptyset$ implies x and A are contained in disjoint open sets.

(iv) If A and B are compact, and $\{a\}_\delta \bigcap B = \emptyset$ for every $a \in A$, then A and B are contained in disjoint open sets.

The proof of the Theorem 6.4 is similar to that of the proof of Theorem 4.10 and makes use of Lemma 6.2 instead of Lemma 4.8.

6.5 Theorem: *The disjoint topological sum of any family of R_δ-spaces (π_2-spaces) is an R_δ-space (π_2-space).*

6.6 Definition: *A function $f : X \to Y$ from a topological space X into a topological space Y is said to be:*

(i) ***regular closed*** *if $f(A)$is a regular closed in Y for every regular closed set A in X.*

(ii) δ ***-closed*** *if $f(A)$ is a δ -closed set in Y for every δ -closed set A in X.*

6.7 Theorem: *(a) Let $f : X \to Y$ be a continuous regular closed surjection from X to Y. If X is a π_2-space, then so is Y.*

(b) Let $f : X \to Y$ be a continuous δ-closed surjection from X onto Y. If X is an R_δ-space, then so is Y.

The following result yields a factorization of the property of being a weakly Hausdorff space and in turn improves a result of Soundararajan [36].

6.8 Theorem: *A space X is a weakly Hausdorff ($\equiv \delta T_1$-space) if and only if it is a T_1-space and an R_δ-space.*

Proof: Clearly every weakly Hausdorff space is a T_1-space and an R_δ-space, since each point is the intersection of regular closed sets i.e., a δ-closed set. Conversely, let X be a T_1-space and an R_δ-space. Let $x, y \in X$ and $x \neq y$. By T_1-property there exists an open set U containing x but not y. Since X is an R_δ-space, there exists a δ-closed set A such that $x \in A \subset U$. Then $y \in X \backslash A$ which is a δ-open set and so there exists a regular open set V such that $y \in V \subset X \backslash A$. Thus $X \backslash V$ is a regular closed set containing x but not y and so $\{x\}$ is the intersection of regular closed sets. Hence X is a weakly Hausdorff space.

6.9 Corollary [36, Proposition 2.7]**:** *Every π_2, T_1-space is a weakly Hausdorff space.*

Proof: Since every π_2-space is an R_δ-space, it is immediate in view of Theorem 6.8.

6.10 Corollary [36]**:** *EveryT_1 semiregular space is a weakly Hausdorff space.*

Proof: Let X be a T_1 semiregular space. It is easily verified that every singleton in X is a δ-closed set and so it is an R_δ-space. By Theorem 6.9 X is weakly Hausdorff.

6.11 Theorem: *Every $T_0 R_\delta$-space is a δT_0-space.*

Proof: Let X be a $T_0 R_\delta$-space and let $x, y \in X$,$x \neq y$. Since X is T_0-space, there exists an open set U containing one of the points x and y but not both. To be precise, assume that $x \in U$. Since X is an R_δ-space, there exists a δ-closed set C_x such that $x \in C_x \subset U$. Let $C_x = \bigcap_{\alpha \in \Lambda} F_{\alpha x}$,where each $F_{\alpha x}$ is a regular closed set. So there exists an $\alpha_0 \in \Lambda$ such that $y \notin F_{\alpha_0 x}$.Then $X \backslash F_{\alpha_0 x}$ is a regular open set containing y but not x. Hence X is a δT_0-space.

7. R_1-SPACES

7.1 Definition: *A topological space X is said to be an R_θ**-space** if for each $x \in X$ and each open set U containing x there exists a θ-closed set F in X such that $x \in F \subset U$.*

7.2 Definition: *Let X be a topological space.* A point $x \in X$ is said to be a ***u_θ-adherent point*** ([18] [19]) of a set $A \subset X$ *if every θ-open set containing x intersects A. Let A_{u_θ} denote the set of all u_θ-adherent points of A.*

7.3 Lemma ([18] [19])**:** *A_{u_θ} is the smallest θ-closed set containing A and the correspondence $A \to A_{u_\theta}$is a Kuratowski closure operator.*

7.4 Lemma ([17] [20])**:** *A subset A of a topological space X is θ-open if and only if for each $x \in A$, there exists an open set U such that $x \in U \subset \overline{U} \subset A$.*

The following result is essentially due to Janković [15].

7.5 Theorem: *A topological space X is an R_1-space if and only if it is an R_θ-space.*

Proof: Suppose X is an R_1-space. Let U be an open set in X and let $x \in U$.Since every R_1-space is an R_0-space, $\{\overline{x}\} \subset U$. By a result of Janković [15], $\{\overline{x}\}$is $\theta-$closed and so X is an R_θ-space. Conversely, suppose X is an R_θ-space. Let $x, y \in X$, $x \notin \{\overline{y}\}$.Then $X\backslash\{\overline{y}\}$is an open set containing x but not y. Since X is an R_θ-space, there is a $\theta-$closed set A such that $x \in A \subset X\backslash\{\overline{y}\}$. Then $X\backslash A$ is a $\theta-$open set containing y but not x. By Lemma 7.4 there is an open set V such that $y \in V \subset \overline{V} \subset X\backslash A$. So $X\backslash\overline{V}$ and V are disjoint open sets containing x and y, respectively and hence X is an R_1-space.

7.6 Theorem: *For a topological space X consider the following statements:*

(i) X is an R_θ-space

(ii) For each $x \in X$ and for each open set U containing x, we have $\{x\}_{u_\theta} \subset U$.

(iii) There exists a subbase $\mathcal{S}$ for X such that $x \in S \in \mathcal{S} \Rightarrow \{x\}_{u_\theta} \subset S$.

(iv) If $x \in \{y\}_{u_\theta}$, then $y \in \{x\}_{u_\theta}$.

(v) If $x \in \{y\}_{u_\theta}$, then $\{x\}_{u_\theta} = \{y\}_{u_\theta}$.

Then (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii) $\Rightarrow$ (iv) $\Leftrightarrow$ (v).

Proof: The proof of Theorem 7.6 is similar the proof of Theorem 4.9 and makes use of Lemma 7.3 instead of Lemma 4.8.

7.7 Theorem: *For a topological space X, the following statements are equivalent:*

(i) For any $x, y \in X$, $\{x\}_{u_\theta} \neq \{y\}_{u_\theta}$implies x and y are contained in disjoint open sets.

(ii) For any$x, y \in X$, $y \notin \{x\}_{u_\theta}$ implies x and y are contained in disjoint open sets.

(iii) A is compact and $\{x\}_{u_\theta} \bigcap A = \emptyset$ implies x and A are contained in disjoint open sets.

(iv) If A and B are compact, and $\{a\}_{u_\theta} \bigcap B = \emptyset$ for every $a \in A$, then A and B are contained in disjoint open sets.

It is well known that the property of being an R_1-space is an initial property and preserved under disjoint topological sums. So it is hereditary, productive, sup invariant, pre image invariant, inverse limit invariant and projective.

7.8 Theorem: *The category of R_1-spaces and continuous maps is a full, isomorphism closed subcategory of TOP which is simultaneously a monoreflective and epireflective subcategory of TOP.*

7.9 Definition: *A function $f : X \to Y$ from a topological space X into a topological space Y is said to be θ* ***-closed*** *if $f(A)$ is a θ -closed set in Y for every θ -closed set A in X.*

7.10 Theorem: *Let $f : X \to Y$ be a continuous, θ-closed function from a space X onto Y. If X is an R_1-space, then so is Y.*

Proof: This is immediate in view of Theorem 7.5.

7.11 Theorem (Davis [8], Czászár [6])**:** *A T_0 R_1-space X is Hausdorff.*

Proof: Let $x, y \in X$ with $x \neq y$. By T_0-property of X, there exists an open set U containing one of the point x and y not both. To make a choice assume that $x \in U$. By Theorem 7.5 there exists a θ-closed set A such that $x \in A \subset U$. Then $X \backslash A$ is a θ-open set containing y. By Lemma 7.4 there is an open set V such that $y \in V \subset \overline{V} \subset X \backslash A$. So $X \backslash \overline{V}$ and V are disjoint open sets containing x and y, respectively.

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