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A GENERAL FIXED POINT THEOREM IN COMPLETE G - METRIC SPACES FOR WEAKLY COMPATIBLE PAIRS SATISFYING A ϕ - IMPLICIT RELATION

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Abstract. In this paper a general fixed point theorem in complete G - metric space for weakly compatible pairs satisfying a ϕ - implicit relation is proved, which generalizes and unifies the results given by Theorems 3.2 and 3.3 [20].

1. Introduction and Preliminaries

Let (X, d) be a metric space and S, $T : (X, d) \to (X, d)$ be two mappings. In 1994, Pant [13] introduced the notion of pointwise R - weakly commuting mappings. It is proved in [14] that the notion of pointwise R - weakly commutativity is equivalent to commutativity in coincidence points. Jungck [5] defined S and T to be weakly compatible if $Sx = Tx$ implies $STx = TSx$. Thus, S and T are weakly compatible if and only if S and T are pointwise R - weakly commuting.

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In [3], [4] Dhage introduced a new class of generalized metric spaces, named D - metric space. Mustafa and Sims [6], [7] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [6] - [12], [19].

Quite recently, Srivastava et al. [20] proved two fixed point theorems for weakly compatible mappings in complete G - metric spaces.

In [16], [17], Popa initiated the study of fixed points for mappings satisfying implicit relations.

In [2], Altun and Turkoglu introduced a new type of implicit relations satisfying a ϕ - map. Quite recently, Popa and Patriciu initiated in [17], [18] the study of fixed points in G - metric spaces for mappings satisfying a implicit relation. In [18], a general fixed point theorem for mappings satisfying an ϕ - implicit relation on complete G - metric spaces is proved.

The purpose of this paper is to prove a general fixed point theorem in G - metric spaces for pairs of weakly compatible mappings satisfying an ϕ - implicit relation which generalize the results from Theorems 2.3, 2.4 [20].

2. Preliminaries

Definition 2.1 ([7]). Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:

$(G_1) : G(x, y, z) = 0$ if $x = y = z$,

$(G_2) : 0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

$(G_3) : G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

$(G_4) : G(x, y, z) = G(y, z, x) = G(z, x, y) = ...$ (symmetry in all three variables),

$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Note that $G(x, y, z) = 0$, then $x = y = z$.

Definition 2.2 ([7]). Let (X, G) be a G - metric space. A sequence (x_n) in X is said to be

a) G - convergent if for $\varepsilon > 0$, there exists an $x \in X$ and $k \in \mathbb{N}$ such that for all $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$,

b) G - Cauchy if for each $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $n, m, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $m, n, p \to \infty$.

c) A G - metric space is said to be G - complete if every G - Cauchy sequence is G - convergent.

Lemma 2.3 ([7]). *Let (X, G) be a G - metric space. Then, the following properties are equivalent:*

1) (x_n) is G - convergent to x;

2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$;

3) $G(x_n, x, x) \to 0$ as $n \to \infty$;

4) $G(x_n, x_m, x) \to 0$ as $n, m \to \infty$.

Lemma 2.4 ([7]). *If (X, G) is a G - metric space and $(x_n) \in X$, then the following properties are equivalent:*

1) (x_n) is G - Cauchy.

2) For every $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $n, m \geq k$.

Lemma 2.5 ([7]). *Let (X, G) be a G - metric space, then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Lemma 2.6 ([7]). *Let (X, G) be a G - metric space. Then $G(x, y, y) \leq 2G(y, x, x)$ for all $x, y \in X$.*

The following theorems are recently proved in [20].

Theorem 2.7. *Let (X, G) be a complete G - metric space and let $S, T : X \to X$ be two mappings which satisfy the following conditions:*

(i) $T(X) \subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete, and

(iii)

$$G(Tx, Ty, Tz) \leq aG(Sx, Sy, Sz) + bG(Tx, Sx, Sx) + \\ + cG(Ty, Sy, Sy) + dG(Tz, Sz, Sz) + eG(Tx, Sy, Sy)$$

for all $x, y, z \in X$, where $a, b, c, d, e \geq 0$ and $a + 2b + 2c + 2d + 2e < 1$.

Then S and T have a unique point of coincidence in X. Moreover, if S and T are weakly compatible, then S and T have a unique common fixed point.

Theorem 2.8. *Let (X, G) be a complete G - metric space and let $S, T : X \to X$ be two mappings which satisfy the following conditions:*

(i) $T(X) \subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete, and

(iii)

$$G(Tx, Ty, Tz) \leq \alpha \max\{G(Sx, Sy, Sz), G(Tx, Sx, Sx), G(Ty, Sy, Sy), G(Tz, Sz, Sz), (Tx, Sy, Sy)\},$$

where $\alpha \in \left(0, \frac{1}{2}\right)$.

Then S *and* T *have a unique point of coincidence. Moreover, if* S *and* T *are weakly compatible, then* S *and* T *have a unique common fixed point.*

3. Implicit relations

Definition 3.1. A function $\phi : [0, \infty) \to [0, \infty)$ is a ϕ - function if ϕ is an nondecreasing function such that $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for all $\phi(t) < t$ for $t > 0$ and $\phi(0) = 0$.

Definition 3.2 ([2]). Let $\mathfrak{F}_\phi$ be the set of all continuous functions $F(t_1, ..., t_5) : \mathbb{R}_+^5 \to \mathbb{R}$ satisfying the following conditions

1) F is nonincreasing in variables t_3 and t_4,
2) There exists a ϕ - function such that for all $u, v \geq 0$, $F(u, v, 2v, 2u, 0) \leq 0$ implies $u \leq \phi(v)$,
3) $\phi(t, t, 0, 0, t) > 0$, $\forall t > 0$.

In all the following examples, condition (F_1) is obviously.

Example 3.3. $F(t_1, ..., t_5) = t_1 - at_2 - bt_3 - (c + d)t_4 - et_5$, *where* $a, b, c, d, e \geq 0$ *and* $a + 2b + 2c + 2d + e < 1$.

(F_2) : *Let* $u, v \geq 0$ *be and* $F(u, v, 2v, 2u, 0) = u - av - 2bv - 2(c + d)u \leq 0$. *Then,* $u \leq \frac{a+2b}{1-2(c+d)}v$ *and* (F_2) *is satisfied for* $\phi(t) = \frac{a+2b}{1-2(c+d)}t$.

(F_3) : $F(t, t, 0, 0, t) = t(1 - (a + e)) > 0, \forall t > 0$.

Example 3.4. $F(t_1, ..., t_5) = t_1 - k \max\{t_2, t_3, t_4, t_5\}$, *where* $k \in \left[0, \frac{1}{2}\right)$.

(F_2) : *Let* $u, v \geq 0$ *be and* $F(u, v, 2v, 2u, 0) = u - k \max\{v, 2v, 2u\} \leq 0$. *If* $u > v$, *then* $u(1 - 2k) \leq 0$, *a contradiction, hence* $u \leq v$ *which implies* $u \leq 2kv$ *and* (F_2) *is satisfied for* $\phi(t) = 2kt$.

(F_3) : $F(t, t, 0, 0, t) = t(1 - 2k) > 0, \forall t > 0$.

Example 3.5. $F(t_1, ..., t_5) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5^2$, *where* $a, b, c \geq 0$, $a + 2b + 2c < 1$ *and* $a + d < 1$.

(F_2) : *Let* $u, v \geq 0$ *be such that* $F(u, v, 2v, 2u, 0) = u^2 - u(av + 2bv + 2cu) \leq 0$. *If* $u > v$, *then* $u \leq av - 2bv - 2cu$ *which implies* $u \leq \frac{a+2b}{1-2c}v$. *If* $u = 0$ *then* $u \leq \frac{a+2b}{1-2c}v$ *and* (F_2) *is satisfied for* $\phi(t) = \frac{a+2b}{1-2c}t$.

(F_3) : $F(t,t,0,0,t) = t^2(1-(a+d)) > 0, \forall t > 0$.

Example 3.6. *$F(t_1,...,t_5) = t_1 - a\frac{t_2+t_3}{2} - b\frac{t_4+t_5}{2}$, where $a, b \geq 0$ and $3a + 2b < 2$.*

(F_2) : Let $u, v \geq 0$ be such that $F(u,v,2v,2u,0) = u - a\frac{3v}{2} - ub \leq 0$. Hence $u \leq \frac{3a}{2-2b}v$ and (F_2) is satisfied for $\phi(t) = \frac{3a}{2-2b}t$.

(F_3) : $F(t,t,0,0,t) = t(1-\frac{a+b}{2}) > 0, \forall t > 0$.

Example 3.7. *$F(t_1,...,t_5) = t_1^2 - at_2^2 - b\dfrac{t_3^2+t_4^2}{1+t_5^2}$, where $a + 8b < 1$.*

(F_2) : Let $u, v \geq 0$ be such that $F(u,v,2v,2u,0) = u^2 - av^2 - b(4u^2+4v^2) \leq 0$ which implies $u \leq \sqrt{\frac{a+4b}{1-4b}}v$ and (F_2) is satisfied for $\phi(t) = \sqrt{\frac{a+4b}{1-4b}}t$.

(F_3) : $F(t,t,0,0,t) = t^2(1-a) > 0, \forall t > 0$.

Example 3.8. *$F(t_1,...,t_5) = t_1 - at_2 - bt_3 - c\min\{t_4,t_5\}$, where $a, b, c \geq 0$ and $a + 2b < 1$.*

(F_2) : Let $u, v \geq 0$ and $F(u,v,2v,2u,0) = u - av - 2bv \leq 0$ which implies $u \leq (a+2b)v$ and (F_2) is satisfied for $\phi(t) = (a+2b)t$.

(F_3) : $F(t,t,0,0,t) = t(1-a) > 0, \forall t > 0$.

Example 3.9. *$F(t_1,...,t_5) = t_1 - c\max\{t_2,t_3,\sqrt{t_4t_5}\}$, where $t \in \left(0,\frac{1}{2}\right)$.*

(F_2) : Let $u, v \geq 0$ and $F(u,v,2v,2u,0) = u - 2cv \leq 0$ which implies $u \leq 2cv$ and (F_2) is satisfied for $\phi(t) = 2ct$.

(F_3) : $F(t,t,0,0,t) = t(1-c) > 0, \forall t > 0$.

Example 3.10. *$F(t_1,...,t_5) = t_1 - k\max\left\{t_2,t_3,\frac{t_3+2t_4}{2},\frac{t_4+2t_5}{2}\right\}$, where $k \in \left(0,\frac{1}{3}\right)$.*

(F_2) : Let $u, v \geq 0$ and $F(u,v,2v,2u,0) = u - k\max\{2v, v+2u, u\} \leq 0$. If $u > v$, then $u(1-3k) \leq 0$, a contradiction. Hence $u \leq v$, which implies $u \leq 3kv$ and (F_2) is satisfied for $\phi(t) = 3kt$.

(F_3) : $F(t,t,0,0,t) = t(1-k) > 0, \forall t > 0$.

4. Main results

Definition 4.1. Let S and T two self mappings of a nonempty set X. If $w = Tx = Sx$ for some $x \in X$, then x is called a coincidence point of S and T and w is called a point of coincidence of T and S.

Lemma 4.2 ([1])**.** *Let T and S be weakly compatible self mappings of a nonempty set X. If T and S have a unique point of coincidence $w = Tx = Sx$, then w is the unique common fixed point of T and S.*

Theorem 4.3. *Let (X, G) be a G - metric space and T, S self mappings of X such that*

$$F(G(Tx,Ty,Ty),G(Sx,Sy,Sy),G(Tx,Sx,Sx),\\ G(Ty,Sy,Sy),G(Tx,Sy,Sy)) \leq 0 \tag{4.1}$$

for all $x, y \in X$ and F satisfying property (F_3). Then T and S have at most a point of coincidence.

Proof. Suppose that $u = Tp = Sp$ and $v = Tq = Sq$ are two distinct points of coincidence. Then, by (4.1) we have successively:

$$F(G(Tq,Tp,Tp),G(Sq,Sp,Sp),G(Tq,Sq,Sq),\\ G(Tp,Sp,Sp),G(Tq,Sp,Sp)) \leq 0,$$

$$F(G(Sq,Sp,Sp),G(Sq,Sp,Sp),0,0,G(Sq,Sp,Sp)) \leq 0,$$

a contradiction of (F_3). □

Theorem 4.4. *Let (X, G) be a G - metric space and let $T, S : (X, G) \to (X, G)$ be two mappings such that*

(i) *$T(X) \subset S(X)$,*
(ii) *$T(X)$ or $S(X)$ is G - complete,*
(iii) *T and S satisfy the inequality (4.1) for all $x, y \in X$ and $F \in \mathfrak{F}_\phi$.*

Then T and S have a unique point of coincidence. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point.

Proof. First suppose that $S(X)$ is G - complete. Let $x_0 \in X$ be an arbitrary point. Then, by (i) there exists $x_1 \in X$ such that $Tx_0 = Sx_1$. In this way, we define a sequence (Sx_n) with $Tx_{n-1} = Sx_n$ for $n = 1, 2, ...$ Then by (4.1) we have successively:

$$F(G(Tx_{n-1},Tx_n,Tx_n),G(Sx_{n-1},Sx_n,Sx_n),G(Tx_{n-1},Sx_{n-1},Sx_{n-1}),\\ G(Tx_n,Sx_n,Sx_n),G(Tx_{n-1},Sx_n,Sx_n)) \leq 0,$$

$$F(G(Sx_n,Sx_{n+1},Sx_{n+1}),G(Sx_{n-1},Sx_n,Sx_n),\\ G(Sx_n,Sx_{n-1},Sx_{n-1}),G(Sx_{n+1},Sx_n,Sx_n),0) \leq 0.$$

By Lemma 2.6

$$G(Sx_n,Sx_{n-1},Sx_{n-1}) \leq 2G(Sx_{n-1},Sx_n,Sx_n)$$

and

$$G(Sx_{n+1},Sx_n,Sx_n) \leq 2G(Sx_n,Sx_{n+1},Sx_{n+1}).$$

By (F_1) we obtain:

$$F(G(Sx_n, Sx_{n+1}, Sx_{n+1}), G(Sx_{n-1}, Sx_n, Sx_n), \\ 2G(Sx_{n-1}, Sx_n, Sx_n), 2G(Sx_n, Sx_{n+1}, Sx_{n+1}), 0) \leq 0$$

which implies by (F_2) that

$$G(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq \phi(G(Sx_{n-1}, Sx_n, Sx_n)).$$

Then, we obtain that

$$G(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq \phi(G(Sx_{n-1}, Sx_n, Sx_n)) \leq \\ \leq ... \leq \phi^n(G(Sx_0, Sx_1, Sx_1)).$$

For any $p > m > n$, by rectangle inequality we obtain

$$G(Sx_n, Sx_n, Sx_p) \leq G(Sx_n, Sx_{n+1}, Sx_{n+1}) + ... + G(Sx_{p-1}, Sx_p, Sx_p) \\ \leq \phi^n(G(Sx_0, Sx_1, Sx_1)) + \phi^{n+1}(G(Sx_0, Sx_1, Sx_1)) + \\ +... + \phi^{p-2}(G(Sx_0, Sx_1, Sx_1)) \\ \leq \sum_{k=n}^{\infty} \phi^k(G(Sx_0, Sx_1, Sx_1)).$$

Since

$$\sum_{k=0}^{\infty} \phi^k(G(Sx_0, Sx_1, Sx_1)) < +\infty$$

it follows that $G(Sx_n, Sx_m, Sx_p) \to 0$ as $n, m, p \to \infty$. Hence (Sx_n) is a G - Cauchy sequence. Now, since $S(X)$ is G - complete, there exists a point $q \in X$ such that $Sx_n \to q$ as $n \to \infty$. Consequently, we can find a point $p \in X$ such that $Sp = q$.

If $T(X)$ is complete, there exists $q \in T(X)$ such that $Sx_n \to q$ as $T(X) \subset S(X)$ we have $q \in Sx$. Then, there exists $p \in X$ such that $Sp = q$. We prove that p is a coincidence point for T and S. By (4.1) we have successively:

$$F(G(Tx_{n-1}, Tp, Tp), G(Sx_{n-1}, Sp, Sp), G(Tx_{n-1}, Sx_{n-1}, Sx_{n-1}), \\ G(Tp, Sp, Sp), G(Tx_{n-1}, Sp, Sp)) \leq 0,$$

$$F(G(Sx_n, Sp, Sp), G(Sx_{n-1}, Sp, Sp), \\ G(Sx_n, Sx_{n-1}, Sx_{n-1}), G(Sp, Sp, Sp), G(Sx_n, Sp, Sp)) \leq 0.$$

Letting n tends to infinity, by Lemma 2.5 we obtain

$$F(G(Sp, Tp, Tp), 0, 0, G(Tp, Sp, Sp), 0) \leq 0.$$

By Lemma 2.6, $G(Tp, Sp, Sp) \leq 2G(Sp, Tp, Tp)$. By (F_1) we obtain

$$F(G(Sp, Tp, Tp), 0, 0, 2G(Sp, Tp, Tp), 0) \leq 0.$$

By (F_2), $G(Sp, Tp, Tp) = 0$ which implies $w = Tp = Sp$. By Theorem 4.3 w is the unique point of coincidence of T and S. Moreover, if

T and S are weakly compatible, by Lemma 4.2 w is the unique fixed point of T and S. □

Corollary 4.5. *Let (X, G) be a G - metric space and let $T, S : (X, G) \to (X, G)$ be two mappings such that:*

(i) $T(X) \subset S(X)$,

(ii) $S(X)$ *or* $T(X)$ *is* G *- complete,*

(iii) *one of the following inequalities hold for all* $x, y \in X$

(1)

$$G(Tx, Ty, Ty) \leq aG(Sx, Sy, Sy) + bG(Tx, Sx, Sx)+ \\ +(c + d)G(Ty, Sy, Sy) + eG(Tx, Sy, Sy),$$

where $a, b, c, d, e \geq 0$ *and* $a + 2b + 2c + 2d + e < 1$.

(2)

$$G(Tx, Ty, Ty) \leq k \max\{G(Sx, Sy, Sy), G(Tx, Sx, Sx), \\ G(Ty, Sy, Sy), G(Tx, Sy, Sy)\},$$

where $k \in \left(0, \frac{1}{2}\right)$.

(3)

$$G^2(Tx, Ty, Ty) \leq G(Tx, Ty, Ty)[aG(Sx, Sy, Sy)+ \\ +bG(Tx, Sx, Sx) + cG(Ty, Sy, Sy)] + dG^2(Tx, Sy, Sy),$$

where $a, b, c \geq 0$, $a + 2b + 2c < 1$ *and* $a + d < 1$.

(4)

$$G(Tx, Ty, Ty) \leq a\tfrac{G(Sx,Sy,Sy)+G(Tx,Sx,Sx)}{2}+ \\ +b\tfrac{G(Ty,Sy,Sy)+G(Tx,Sy,Sy)}{2},$$

where $3a + 2b < 2$.

(5)

$$G^2(Tx, Ty, Ty) \leq aG^2(Sx, Sy, Sy)+ \\ +b\tfrac{G^2(Tx,Sx,Sx)+G^2(Ty,Sy,Sy)}{1+G^2(Tx,Sy,Sy)},$$

where $a, b \geq 0$ *and* $a + 2b < 1$.

(6)

$$G(Tx, Ty, Ty) \leq aG(Sx, Sy, Sy) + bG(Tx, Sx, Sx)+ \\ +c \min\{G(Ty, Sy, Sy), G(Tx, Sy, Sy)\},$$

where $a, b, c \geq 0$ *and* $a + 2b < 1$.

(7)

$$G(Tx, Ty, Ty) \leq c \max\{G(Sx, Sy, Sy), G(Tx, Sx, Sx), \\ [G(Ty, Sy, Sy) \cdot G(Tx, Sy, Sy)]^{1/2}\},$$

where $c \in \left(0, \frac{1}{2}\right)$.

(8)

$$G(Tx,Ty,Ty) \le k\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), \frac{1}{2}[G(Tx,Sx,Sx)+2G(Ty,Sy,Sy)], \frac{1}{2}[G(Ty,Sy,Sy)+2G(Tx,Sy,Sy)]\},$$

where $k \in \left(0, \frac{1}{3}\right)$.

If S and T are weakly compatible, then S and T have a unique common fixed point.

Proof. The proof follows by Theorem 4.4 and Examples 3.3 - 3.10. ☐

Remark 4.6. *By Theorem 2.7 and* $a+2b+2c+2d+2e<1$, *for* $y=z$ *we obtain*

$$G(Tx,Ty,Ty) \le aG(Sx,Sy,Sy)+bG(Tx,Sx,Sx)+ +(c+d)G(Ty,Sy,Sy)+eG(Tx,Sy,Sy)$$

and $a+2b+2c+2d+e<1$, *Theorem 2.7 it follows from Corollary 4.5 (1).*

Remark 4.7. *By Theorem 2.8 for* $y=z$ *we obtain*

$$G(Tx,Ty,Ty) \le k\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), G(Ty,Sy,Sy), G(Tx,Sy,Sy)\},$$

and Theorem 2.8 follows from Corollary 4.5 (2).

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