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Scientific Studies and Research
Series Mathematics and Informatics
Vol. 34 (2024), No. 1, 17 - 32

FUZZY p-b-ALMOST COMPACT SPACES

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Abstract. This paper deals with some applications of fuzzy p-b-open sets [9]. Here we introduce fuzzy p-b-almost compactness and characterize this concept via fuzzy nets and prefilterbases. We also introduce the notion of fuzzy regularly p-b-open set, for which finite intersection property characterizes fuzzy p-b-almost compactness. It is shown that fuzzy p-b-almost compactness implies fuzzy almost compactness [11] and the converse is true only in fuzzy p-b-regular spaces [9].

1. Introduction

After the introduction of fuzzy compactness by Chang [10], many mathematicians have introduced and studied various types of fuzzy compactness. DiConcillio and Gerla [11] introduced fuzzy almost compactness. Afterwards, several weak forms of fuzzy compactness have been introduced and investigated. In this context we have to mention [1, 2, 3, 4, 5, 6, 7, 8]. In this paper we introduce the notion of fuzzy p-b-almost compactness, which is weaker than fuzzy almost compactness. We use fuzzy nets [16] and prefilterbases [13] to characterize fuzzy p-b-almost compactness. In order to characterize fuzzy p-b-almost compactness using a finite intersection property, we introduce the notion of fuzzy regularly p-b-open set. It is shown that fuzzy p-b-almost compactness implies fuzzy almost compactness [11] and the converse is true only in fuzzy p-b-regular spaces [9].

Keywords and phrases: Fuzzy p-b-open set, fuzzy p-b-regular space, fuzzy regularly p-b-closed set, fuzzy p-b-almost compact set (space), pb-adherent point of a prefilterbase, pb-cluster point of a fuzzy net.
(2020) Mathematics Subject Classification: 54A40, 03E72

2. Preliminaries

Throughout this paper, (X, τ) or simply by X we shall mean a fuzzy topological space (abbreviated, fts). In 1965, L.A. Zadeh [17] introduced the notion of fuzzy set A, which is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$ [17]. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement of a fuzzy set A in an fts X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$ [17]. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [17], while AqB means A is quasi-coincident (q-coincident, for short) with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$ [16]. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure and fuzzy interior of A respectively [10]. A fuzzy set A in X is called a fuzzy neighbourhood (fuzzy nbd, for short) of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \in G \leq A$ [16] . If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_t. A fuzzy set A is said to be a fuzzy q-nbd of a fuzzy point x_t in an fts X if there is a fuzzy open set U in X such that $x_t qU \leq A$. If, in addition, A is fuzzy open, then A is called a fuzzy open q-nbd of x_t [16] .

A fuzzy set A in an fts (X, τ) is called fuzzy preopen if $A \leq int(clA)$ [15]. The complement of a fuzzy preopen set is called fuzzy preclosed [15]. The union (intersection) of all fuzzy preopen (respectively, fuzzy preclosed) sets contained in (respectively, containing) a fuzzy set A is called fuzzy preinterior (respectively, fuzzy preclosure) of A, denoted by $pintA$ (respectively, $pclA$) [15].

Let $(D, \geq)$ be a directed set and X be an ordinary set. Let J denote the collection of all fuzzy points in X. A function $S : D \to J$ is called a fuzzy net in X [16]. It is denoted by $\{S_n : n \in (D, \geq)\}$. A nonempty family $\mathcal{F}$ of fuzzy sets in X is called a prefilterbase on X if (i) $0_X \notin \mathcal{F}$ and (ii) for any $U, V \in \mathcal{F}$, there exists $W \in \mathcal{F}$ such that $W \leq U \bigcap V$ [13].

3. Fuzzy p-b-open sets and fuzzy p-b-closed sets

In this section we recall some definitions and results from [9, 10, 11, 12, 14] for ready references. Also some properties of fuzzy p-b-open sets are discussed here.

Definition 3.1 [9]. A fuzzy set A in an fts (X, τ) is called fuzzy p-b-open if $A \leq cl(pint(clA))$.
The complement of a fuzzy p-b-open set is called fuzzy p-b-closed.
The collection of all fuzzy p-b-open (respectively, fuzzy p-b-closed) sets in an fts X is denoted by $FpbO(X)$ (respectively, $FpbC(X)$).

Definition 3.2 [9]. Let (X, τ) be an fts and $A \in I^X$. The fuzzy p-b-closure of A, denoted by $pbclA$, is defined by

$$pbclA = \bigwedge\{U \in I^X : A \leq U, U \in FpbC(X)\}$$

and the fuzzy p-b-interior of A, denoted by $pbintA$, is defined by

$$pbintA = \bigvee\{G : G \leq A, G \in FpbO(X)\}.$$

Definition 3.3 [9]. A fuzzy set A in an fts (X, τ) is called fuzzy p-b-neighborhood (abbreviated, fuzzy p-b-nbd) of a fuzzy point x_t in X if there exists a fuzzy p-b-open set U in X such that $x_t \in U \leq A$. If, in addition, A is fuzzy p-b-open, then A is called fuzzy p-b-open nbd of x_t.

Definition 3.4 [9]. A fuzzy set A in an fts (X, τ) is called fuzzy p-b-neighborhood (abbreviated, fuzzy p-b-q-nbd) of a fuzzy point x_t in X if there exists a fuzzy p-b-open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy p-b-open, then A is called fuzzy p-b-open q-nbd of x_t.

Result 3.5 [9]. The union (respectively, the intersection) of any two fuzzy p-b-open (respectively, fuzzy p-b-closed) sets is also so.

Result 3.6 [9]. $x_t \in pbclA$ if and only if every fuzzy p-b-open q-nbd U of x_t, UqA.

Result 3.7. $pbcl(pbclA) = pbclA$ for any fuzzy set A in an fts (X, τ).

Proof. Let $A \in I^X$. Then $A \leq pbclA$ implies

$$pbclA \leq pbcl(pbclA) \qquad (1)$$

Conversely, let $x_t \in pbcl(pbclA)$. If possible, let $x_t \notin pbclA$. Then there exists $U \in FPbO(X)$ such that

$$x_t qU \text{ and } U \not{q} A \qquad (2)$$

But $x_t \in pbcl(pbclA)$ implies $Uq(pbclA)$. Then there exists $y \in X$ such that $U(y) + (pbclA)(y) > 1$, i.e. $U(y) + s > 1$ where $s = (pbclA)(y)$. Then $y_s \in pbclA$ and $y_s qU$ where $U \in FPbO(X)$. So by definition, UqA, which contradicts (2). So

$$pbcl(pbclA) \leq pbclA \qquad (3)$$

Combining (1) and (3), we get the result.

Result 3.8. $pbcl(A \bigvee B) = pbclA \bigvee pbclB$, for any two fuzzy sets A, B in X.

Proof. It is clear that

$$pbclA \bigvee pbclB \subseteq pbcl(A \bigvee B) \qquad (4)$$

Conversely, let $x_t \in pbcl(A \bigvee B)$. Then for any fuzzy p-b-open q-nbd U of x_t, $Uq(A \bigvee B)$. Then there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1$ and so either $U(y) + A(y) > 1$ implies that UqA or $U(y) + B(y) > 1$ implies that UqB, then either $x_t \in pbclA$ or $x_t \in pbclB$. Consequently, $x_t \in pbclA \bigvee pbclB$.

Result 3.9. For any fuzzy set A in an fts (X, τ),
(i) $pbcl(1_X \setminus A) = 1_X \setminus pbintA$,
(ii) $pbint(1_X \setminus A) = 1_X \setminus pbclA$.

Proof (i). Let $x_t \in pbcl(1_X \setminus A)$ for any $A \in I^X$. If possible, let $x_t \notin 1_X \setminus pbintA$. Then $x_t q pbintA$. Then there exists a fuzzy p-b-open set B in X with $B \leq A$ such that $x_t qB$. Then B is a fuzzy p-b-open q-nbd of x_t. By assumption, $Bq(1_X \setminus A)$ and this implies $Aq(1_X \setminus A)$, which is absurd.
Conversely, let $x_t \in 1_X \setminus pbintA$ for any $A \in I^X$. Then $x_t \not{q} pbintA$ and so $x_t \not{q} U$ for any fuzzy p-b-open set U in X with $U \leq A$, therefore $x_t \in 1_X \setminus U$. But $1_X \setminus U$ is a fuzzy p-b-closed set in X with $1_X \setminus A \leq 1_X \setminus U$. It follows that $x_t \in pbcl(1_X \setminus A)$.
(ii) Replacing A by $1_X \setminus A$ in (i) we get the result.

Definition 3.10. Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$, for each $x \in suppA$ [12]. If each member of $\mathcal{U}$ is fuzzy open (respectively, fuzzy p-b-open), we call $\mathcal{U}$ is fuzzy open [12] (respectively, fuzzy p-b-open) cover of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of X [10].

Definition 3.11. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to have a finite (respectively, finite proximate) subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigvee \mathcal{U}_0 \geq A$ [12] (respectively, $\bigvee\{clU : U \in \mathcal{U}_0\} \geq A$ [14]). In particular, if $A = 1_X$, we get $\bigvee \mathcal{U}_0 = 1_X$ (respectively, $\bigvee\{clU : U \in \mathcal{U}_0\} = 1_X$ [11]).

Definition 3.12 [11]. An fts (X, τ) is called fuzzy almost compact space if every fuzzy open cover has a finite proximate subcover.

4. Characterizations of fuzzy p-b-almost compact spaces

In this section fuzzy p-b-almost compactness is introduced and studied by fuzzy p-b-open and fuzzy regularly p-b-open sets and characterize this space via fuzzy net and prefilterbase.

Definition 4.1. A fuzzy set A in an fts (X, τ) is said to be a fuzzy p-b-almost compact set if every fuzzy p-b-open cover $\mathcal{U}$ of A has a finite pb-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{pbclU : U \in \mathcal{U}_0\} \geq A$. If, in addition, $A = 1_X$, we say that the fts X is fuzzy p-b-almost compact space.

Definition 4.2. A fuzzy point x_t in an fts X is said to be in the pb-closure of a fuzzy set A in X, denoted by $x_t \in pb\text{-}clA$, if for every fuzzy p-b-open q-nbd U of x_t, $pbclUqA$.

Definition 4.3. Let x_t be a fuzzy point in an fts (X, τ). A prefilterbase $\mathcal{F}$ on X is called

(a) pb-adhere at x_t, written as $x_t \in pb$-ad $\mathcal{F}$, if for each fuzzy p-b-open q-nbd U of x_t and each $F \in \mathcal{F}$, $Fq(pbclU)$, i.e., $x_t \in pb\text{-}clF$, for each $F \in \mathcal{F}$;

(b) pb-converge to x_t, written as $\mathcal{F}\overrightarrow{pb}x_t$, if to each fuzzy p-b-open q-nbd U of x_t, there corresponds some $F \in \mathcal{F}$ such that $F \leq pbclU$.

Definition 4.4. Let x_t be a fuzzy point in an fts (X, τ). A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to

(a) pb-adhere at x_t, denoted by $x_t \in pb\text{-ad}(S_n)$, if for each fuzzy p-b-open q-nbd U of x_t and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_mqpbclU$;

(b) pb-converge to x_t, denoted by $S_n\overrightarrow{pb}x_t$, if for each fuzzy p-b-open q-nbd U of x_t, there exists $m \in D$ such that $S_nqpbclU$, for all $n \geq m (n \in D)$.

Theorem 4.5. For a fuzzy set A in an fts X, the following statements are equivalent:

(a) A is a fuzzy p-b-almost compact set,

(b) for every prefilterbase $\mathcal{B}$ in X, $[\bigwedge\{pbclB : B \in \mathcal{B}\}] \bigwedge A = 0_X$ implies that there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{pbintB : B \in \mathcal{B}_0\}\not{q}A$,

(c) for any family $\mathcal{F}$ of fuzzy p-b-closed sets in X with $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{pbintF : F \in \mathcal{F}_0\}\not{q}A$,

(d) every prefilterbase on X, each member of which is q-coincident with A, pb-adheres at some fuzzy point in A.

Proof (a) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that

$[\bigwedge\{pbclB : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then for any $x \in suppA$, $[\bigwedge\{pbclB : B \in \mathcal{B}\}](x) = 0$ implies $1 - [\bigwedge\{pbclB(x) : B \in \mathcal{B}\}] = 1$, hence $\bigvee[(1_X \setminus pbclB)(x) : B \in \mathcal{B}] = 1$, therefore $sup\{pbint(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1$, which implies $\{pbint(1_X \setminus B) : B \in \mathcal{B}\}$ is a fuzzy p-b-open cover of A. By (a), there exists a finite pb-proximate subcover $\{pbint(1_X \setminus B_1), pbint(1_X \setminus B_2), ..., pbint(1_X \setminus B_n)\}$ of it for A. Thus $A \leq \bigvee_{i=1}^{n} pbcl(pbint(1_X \setminus B_i)) = \bigvee_{i=1}^{n}[1_X \setminus pbint(pbclB_i)] = 1_X \setminus \bigwedge_{i=1}^{n} pbint(pbclB_i)$, hence $\bigwedge_{i=1}^{n} pbint(pbclB_i) \leq 1_X \setminus A$, which implies $A \not{q} \bigwedge_{i=1}^{n} pbint(pbclB_i)$. Then $A \not{q} \bigwedge_{i=1}^{n} pbintB_i$.

(b) $\Rightarrow$ (a). Let the condition (b) hold. Suppose that there exists a fuzzy p-b-open cover $\mathcal{U}$ of A having no finite pb-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $sup\{pbclU(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - sup\{(pbclU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0$, which implies $inf\{(1_X \setminus pbclU)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0} (1_X \setminus pbclU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ ($=\mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^{n} pbint(1_X \setminus pbclU_i) \not{q} A$, then $A \leq 1_X \setminus \bigwedge_{i=1}^{n} pbint(1_X \setminus pbclU_i) = \bigvee_{i=1}^{n}[1_X \setminus pbint(1_X \setminus pbclU_i)] = \bigvee_{i=1}^{n} pbcl(pbclU_i) = \bigvee_{i=1}^{n} pbclU_i$ (by Result 3.7). Thus $\mathcal{U}$ has a finite pb-proximate subcover for A, which contradicts our hypothesis. Hence for every finite subcollection $\{\bigwedge_{U \in \mathcal{U}_1} (1_X \setminus pbclU), ..., \bigwedge_{U \in \mathcal{U}_k} (1_X \setminus pbclU)\}$ of $\mathcal{B}$, where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subset of $\mathcal{U}$, we have $[\bigwedge_{U \in \mathcal{U}_1 \bigvee ... \bigvee \mathcal{U}_k} pbint(1_X \setminus pbclU)] q A$. By(b), $[\bigwedge_{U \in \mathcal{U}} pbcl(1_X \setminus pbclU)] \bigwedge A \neq 0_X$. Then there exists $x \in suppA$, such that $\inf_{U \in \mathcal{U}}[pbcl(1_X \setminus pbclU)](x) > 0$. Then $1 - \inf_{U \in \mathcal{U}}[pbcl(1_X \setminus pbclU)](x) < 1$, hence $\sup_{U \in \mathcal{U}}[1_X \setminus pbcl(1_X \setminus pbclU)](x) < 1$, that implies $\sup_{U \in \mathcal{U}} U(x) \leq \sup_{U \in \mathcal{U}} pbint(pbclU)(x) < 1$, which contradicts that $\mathcal{U}$ is a fuzzy p-b-open

cover of A.
(a) $\Rightarrow$ (c). Let $\mathcal{F}$ be a family of fuzzy p-b-closed sets in X such that $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$. Then for each $x \in suppA$ and for each positive integer n, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n$, hence $1 - F_n(x) > 1 - 1/n$. It follows that $\sup\limits_{F\in\mathcal{F}}[(1_X \setminus F)(x)] = 1$ and so $\{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy p-b-open cover of A. By (a), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigvee\limits_{F\in\mathcal{F}_0} pbcl(1_X \setminus F)$, therefore

$$\begin{aligned} 1_X \setminus A &\geq 1_X \setminus \bigvee_{F\in\mathcal{F}_0} pbcl(1_X \setminus F) \\ &= \bigwedge_{F\in\mathcal{F}_0} (1_X \setminus pbcl(1_X \setminus F)) = \bigwedge_{F\in\mathcal{F}_0} pbintF. \end{aligned}$$

Hence $A\not{q}(\bigwedge\limits_{F\in\mathcal{F}_0} pbintF)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.
(c) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{pbclB : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then the family $\mathcal{F} = \{pbclB : B \in \mathcal{B}\}$ is a family of fuzzy p-b-closed sets in X with $(\bigwedge F) \bigwedge A = 0_X$. By (c), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigwedge\{pbint(pbclB) : B \in \mathcal{B}_0\}]\not{q}A$. Consequently, $(\bigwedge\limits_{B\in\mathcal{B}_0} pbintB)\not{q}A$.
(a) $\Rightarrow$ (d). Let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. If possible, let $\mathcal{F}$ do not pb-adhere at any fuzzy point in A. Then for each $x \in suppA$, there exists $n_x \in \mathbb{N}^*$ such that $x_{1/n_x} \in A$. Then there are a fuzzy p-b-open set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x} q U^x_{n_x}$ and $pbclU^x_{n_x}\not{q}F^x_{n_x}$. Thus $U^x_{n_x}(x) > 1 - 1/n_x$ so that $sup\{U^x_n(x) : n \in \mathbb{N}^*, n \geq n_x\} = 1$. Thus $\{U^x_n : n \in \mathbb{N}^*, n \geq n_x, x \in suppA\}$ forms a fuzzy p-b-open cover of A. By (a), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in \mathbb{N}^*$ such that $A \leq \bigvee\limits_{i=1}^{k} pbclU^{x_i}_{n_{x_i}}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigwedge\limits_{i=1}^{k} F^{x_i}_{n_i}$. Then $F\not{q}[\bigvee\limits_{i=1}^{k} pbclU^{x_i}_{n_{x_i}}]$, i.e., $F\not{q}A$, a contradiction.
(d) $\Rightarrow$ (a). If possible, let there exist a fuzzy p-b-open cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\bigvee\{pbclU : U \in \mathcal{U}_0\} \ngeq A$. Then $\mathcal{F} = \{1_X \setminus \bigvee\limits_{U\in\mathcal{U}_0} pbclU : \mathcal{U}_0$ is a finite subset of $\mathcal{U}\}$ is a prefilterbase on X such that FqA, for each $F \in \mathcal{F}$. By (d), $\mathcal{F}$ pb-adheres at

some fuzzy point $x_t \in A$. As $\mathcal{U}$ is a fuzzy cover of A, $\sup\limits_{U \in \mathcal{U}} U(x) = 1$, therefore there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1 - t$, hence $x_t q U_0$. As $x_t \in pb\text{-ad}\mathcal{F}$ and $1_X \setminus pbclU_0 \in \mathcal{F}$, we have $pbclU_0 q(1_X \setminus pbclU_0)$, a contradiction.

Theorem 4.6. For a fuzzy set A in an fts X, the following implications hold :
(a)$\Leftrightarrow$(b)$\Leftrightarrow$(c)$\Rightarrow$ (d)$\Rightarrow$(e), where
(a) every fuzzy net in A *pb*-adheres at some fuzzy point in A,
(b) every fuzzy net in A has a *pb*-convergent fuzzy subnet,
(c) every prefilterbase in A *pb*-adheres at some fuzzy point in A,
(d) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge\limits_{\alpha \in \Lambda} pb\text{-}clB_\alpha] \bigwedge A = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha) \bigwedge A = 0_X$,
(e) A is fuzzy *p*-*b*-almost compact set.

Proof (a) $\Rightarrow$ (b). Let a fuzzy net $\{S_n : n \in (D, \geq)\}$ in A where $(D, \geq)$ is a directed set, *pb*-adhere at a fuzzy point $x_\alpha \in A$. Let Q_{x_α} denote the set of the fuzzy *p*-*b*-closures of all fuzzy *p*-*b*-open *q*-nbds of x_α. For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n q B$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n q B$. Then $(E, \gg)$ is a directed set where $(m, C) \gg (n, B)$ if and only if $m \geq n$ in D and $C \leq B$. Then $T : (E, \gg) \to (X, \tau)$ given by $T(n, B) = S_n$, is a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$. Let V be any fuzzy *p*-*b*-open *q*-nbd of x_α. Then there is $n \in D$ such that that $(n, pbclV) \in E$ and hence $S_n q(pbclV)$. Now, for any $(m, U) \gg (n, pbclV)$, $T(m, U) = S_m q U \leq pbclV$, which implies $T(m, U) q(pbclV)$. Hence $T \xrightarrow{pb} x_\alpha$.

(b) $\Rightarrow$ (a). If a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not *pb*-adhere at a fuzzy point x_α, then there is a fuzzy *p*-*b*-open *q*-nbd U of x_α and an $n \in D$ such that $S_m \not{q}(pbclU)$, for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can *pb*-converge to x_α.

(a) $\Rightarrow$ (c). Let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A. For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \in F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ if and only if $F_\alpha \leq F_\beta$. By (a), the fuzzy net S *pb*-adheres at some fuzzy point x_t $(0 < t \leq 1) \in A$. Then for any fuzzy *p*-*b*-open *q*-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} q(pbclU)$. Then $F_\beta q(pbclU)$ and

hence $F_\alpha q(pbclU)$. Thus $\mathcal{F}$ *pb*-adheres at x_t.
(c) $\Rightarrow$ (a). Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. By (c), there exists a fuzzy point $a_\alpha \in A$ such that $\mathcal{F}$ *pb*-adheres at a_α. Then for each fuzzy p-b-open q-nbd U of a_α and each $F \in \mathcal{F}$, $Fq(pbclU)$, i.e., $(pbclU)qT_n$, for all $n \in D$. Hence the given fuzzy net *pb*-adheres at a_α.
(c) $\Rightarrow$ (d). Let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ, $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) \bigwedge A \neq 0_X$. Then $\mathcal{F} = \{(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) \bigwedge A : \Lambda_0$ is a finite subset of $\Lambda\}$ is a prefilterbase in A. By (c), $\mathcal{F}$ *pb*-adheres at some fuzzy point $a_t \in A$ $(0 < t \leq 1)$. Then for each $\alpha \in \Lambda$ and each fuzzy p-b-open q-nbd U of a_t, $B_\alpha q(pbclU)$, i.e., $a_t \in pb\text{-}clB_\alpha$, for each $\alpha \in \Lambda$. Consequently, $(\bigwedge_{\alpha \in \Lambda} pb\text{-}clB_\alpha) \bigwedge A \neq 0_X$.
(d) $\Rightarrow$ (e). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy p-b-open cover of a fuzzy set A. Then by (d), $A \bigwedge [\bigwedge_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = A \bigwedge [1_X \setminus \bigvee_{\alpha \in \Lambda} U_\alpha] = 0_X$. If for some $\alpha \in \Lambda$, $1_X \setminus pbclU_\alpha = 0_X$, then we are done. If $1_X \setminus pbclU_\alpha$ ($=B_\alpha$, say) $\neq 0_X$, then for each $\alpha \in \Lambda$, $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigwedge_{\alpha \in \Lambda} pb\text{-}clB_\alpha \leq \bigwedge_{\alpha \in \Lambda} (1_X \setminus U_\alpha)$. In fact, let x_t $(0 < t \leq 1)$ be a fuzzy point such that $x_t \in pb\text{-}clB_\alpha = pb\text{-}cl(1_X \setminus pbclU_\alpha)$. If $x_t q U_\alpha$, then $pbclU_\alpha q(1_X \setminus pbclU_\alpha)$, which is absurd. Hence $x_t \not{q} U_\alpha$, therefore $x_t \in 1_X \setminus U_\alpha$. Then $[\bigwedge_{\alpha \in \Lambda} pb\text{-}clB_\alpha] \bigwedge A \leq A \bigwedge [\bigwedge_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = 0_X$. By (d), there exists a finite subset Λ_0 of Λ such that $[\bigwedge_{\alpha \in \Lambda_0} B_\alpha] \bigwedge A = 0_X$, i.e., $A \leq 1_X \setminus \bigwedge_{\alpha \in \Lambda_0} B_\alpha = \bigvee_{\alpha \in \Lambda_0} (1_X \setminus B_\alpha) = \bigvee_{\alpha \in \Lambda_0} pbclU_\alpha$ and (e) follows.

Definition 4.7. A fuzzy set A in an fts (X, τ) is said to be fuzzy regularly p-b-open if $A = pbint(pbclA)$. The complement of such a set is called fuzzy regularly p-b-closed.

Definition 4.8. A fuzzy point x_α in X is said to be a fuzzy *pb*-cluster point of a prefilterbase $\mathcal{B}$ if $x_\alpha \in pbclB$, for all $B \in \mathcal{B}$. If, in addition, $x_\alpha \in A$, for a fuzzy set A, then $\mathcal{B}$ is said to have a fuzzy *pb*-cluster point in A.

Theorem 4.9. A fuzzy set A in an fts (X,τ) is fuzzy p-b-almost compact if and only if for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2, ..., F_n$ from $\mathcal{F}$ and for any fuzzy regularly p-b-closed set C containing A, one has $(F_1 \bigwedge ... \bigwedge F_n) q C$, $\mathcal{F}$ has a fuzzy pb-cluster point in A.
Proof. Let A be fuzzy p-b-almost compact set and suppose $\mathcal{F}$ be a prefilterbase in X such that

$$[\bigwedge\{pbclF : F \in \mathcal{F}\}] \bigwedge A = 0_X \qquad (5)$$

Let $x \in suppA$. Consider any $n \in \mathbb{N}^*$ such that $1/n < A(x)$, i.e., $x_{1/n} \in A$. By (5), $x_{1/n} \notin pbclF^n_x$, for some $F^n_x \in \mathcal{F}$. Then there exists a fuzzy p-b-open q-nbd U^n_x of $x_{1/n}$ such that $U^n_x \not{q} F^n_x$. Now $U^n_x(x) > 1 - 1/n$ for every $n \in \mathbb{N}^*$, therefore $\sup\{U^n_x(x) : 1/n < A(x), n \in \mathbb{N}^*\} = 1$, hence $\mathcal{U} = \{U^n_x : x \in suppA, n \in \mathbb{N}^*\}$ forms a fuzzy p-b-open cover of A such that for U^n_x, there exists $F^n_x \in \mathcal{F}$ with $U^n_x \not{q} F^n_x$. Since A is fuzzy p-b-almost compact, there exist finitely many members $U^{n_1}_{x_1}, , , , , U^{n_k}_{x_k}$ of $\mathcal{U}$ such that $A \leq \bigvee\limits_{i=1}^{k} pbclU^{n_i}_{x_i} = pbcl(\bigvee\limits_{i=1}^{k} U^{n_i}_{x_i})(=U$, say) (by Result 3.8) . Now $F^{n_1}_{x_1}, ..., F^{n_k}_{x_k} \in \mathcal{F}$ such that $U^{n_i}_{x_i} \not{q} F^{n_i}_{x_i}$ for $i = 1, 2, ..., k$. Now U is a fuzzy regularly p-b-closed set containing A such that $U \not{q} (F^{n_1}_{x_1} \bigwedge .. \bigwedge F^{n_k}_{x_k})$.
Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy pb-cluster point in A. Then by hypothesis, there is a fuzzy regularly p-b-closed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigwedge \mathcal{B}_0) \not{q} C$. Then $(\bigwedge \mathcal{B}_0) \not{q} A$. By Theorem 4.5 (b) $\Rightarrow$ (a), A is fuzzy p-b-almost compact set.

From Theorem 4.5, Theorem 4.6 and Theorem 4.9, we obtain characterizations of fuzzy p-b-almost compact space, as follows.

Theorem 4.10. For an fts X, the following statements are equivalent :
(a) X is fuzzy p-b-almost compact,
(b) every fuzzy net in X pb-adheres at some fuzzy point in X,
(c) every fuzzy net in X has a pb-convergent fuzzy subnet,
(d) every prefilterbase in X pb-adheres at some fuzzy point in X,
(e) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge\limits_{\alpha \in \Lambda} pbclB_\alpha] = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha) = 0_X$,
(f) for every prefilterbase $\mathcal{B}$ in X with $\bigwedge\{pbclB : B \in \mathcal{B}\} = 0_X$, there

is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{pbintB : B \in \mathcal{B}_0\} = 0_X$,
(g) for any family $\mathcal{F}$ of fuzzy p-b-closed sets in X with $\bigwedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{pbintF : F \in \mathcal{F}_0\} = 0_X$.

Theorem 4.11. An fts X is fuzzy p-b-almost compact if and only if for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy p-b-open sets in X having finite intersection property $\bigwedge\{pbclF_\alpha : \alpha \in \Lambda\} \neq 0_X$.

Proof. Let X be fuzzy p-b-almost compact space and $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy p-b-open sets in X with finite intersection property. Suppose $\bigwedge\{pbclF_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\{1_X \setminus pbclF_\alpha : \alpha \in \Lambda\}$ is a fuzzy p-b-open cover of X. By hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \bigvee\{pbcl(1_X \setminus pbclF_\alpha) : \alpha \in \Lambda_0\} = \bigvee\{1_X \setminus pbint(pbclF_\alpha) : \alpha \in \Lambda_0\} \leq \bigvee\{1_X \setminus F_\alpha : \alpha \in \Lambda_0\} = 1_X \setminus \bigwedge\limits_{\alpha\in\Lambda_0} F_\alpha \Rightarrow \bigwedge\limits_{\alpha\in\Lambda_0} F_\alpha = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.
Conversely, suppose that X is not fuzzy p-b-almost compact space. Then there is a fuzzy p-b-open cover $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of X such that for every finite subset Λ_0 of Λ, $\bigvee\{pbclF_\alpha : \alpha \in \Lambda_0\} \neq 1_X$. Then $1_X \setminus \bigvee\{pbclF_\alpha : \alpha \in \Lambda_0\} \neq 0_X$, therefore $\bigwedge\limits_{\alpha\in\Lambda_0} (1_X \setminus pbclF_\alpha) \neq 0_X$, for every finite subset Λ_0 of Λ. Thus $\{1_X \setminus pbclF_\alpha : \alpha \in \Lambda\}$ is a collection of fuzzy p-b-open sets with finite intersection property. By hypothesis, $\bigwedge\limits_{\alpha\in\Lambda} pbcl(1_X \setminus pbclF_\alpha) \neq 0_X$, i.e., $1_X \setminus \bigvee\limits_{\alpha\in\Lambda} pbint(pbclF_\alpha) \neq 0_X$, therefore $\bigvee\limits_{\alpha\in\Lambda} pbint(pbclF_\alpha) \neq 1_X$. Hence $\bigvee\limits_{\alpha\in\Lambda} F_\alpha \neq 1_X$, which is a contradiction as $\mathcal{F}$ is a fuzzy p-b-open cover of X.

Definition 4.12. Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy p-b-open sets in X, i.e., for each member n of a directed set $(D, \geq)$, S_n is a fuzzy p-b-open set in X. A fuzzy point x_α in X is said to be a fuzzy pb-cluster point of the fuzzy net if for every $n \in D$ and every fuzzy p-b-open q-nbd V of x_α, there exists $m \in D$ with $m \geq n$ such that $S_m qV$.

Theorem 4.13. An fts X is fuzzy p-b-almost compact if and only if every fuzzy net of fuzzy p-b-open sets in X has a fuzzy pb-cluster point in X.

Proof. Let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy p-b-open sets in a fuzzy p-b-almost compact space X. For each $n \in D$, let us put $F_n = pbcl[\bigvee\{S_m : m \in D \text{ and } m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$

is a family of fuzzy p-b-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{pbintF : F \in \mathcal{F}_0\} \neq 0_X$. By Theorem 4.10 (a)$\Rightarrow$ (g), $\bigwedge\limits_{n\in D} F_n \neq 0_X$. Let $x_\alpha \in \bigwedge\limits_{n\in D} F_n$. Then $x_\alpha \in F_n$, for all $n \in D$. Thus for any fuzzy p-b-open q-nbd A of x_α and any $n \in D$, $Aq[\bigvee\{S_m : m \geq n\}]$ and so there exists some $m \in D$ with $m \geq n$ and AqS_m, hence x_α is a fuzzy pb-cluster point of $\mathcal{U}$.
Conversely, let $\mathcal{F}$ be a collection of fuzzy p-b-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{pbintF : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '$\gg$' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ if and only if $F_1 \leq F_2$. Let $F^* = pbintF$, for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy p-b-open sets of X. By hypothesis, $\mathcal{U}$ has a fuzzy pb-cluster point, say x_α. We claim that $x_\alpha \in \bigwedge \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy p-b-open q-nbd of x_α. Since $F \in \mathcal{F}^*$ and x_α is a fuzzy pb-cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and G^*qA, therefore GqA, hence FqA. It follows that $x_\alpha \in pbclF = F$, for each $F \in \mathcal{F}$, hence $x_\alpha \in \bigwedge \mathcal{F}$, in particular $\bigwedge \mathcal{F} \neq 0_X$. By Theorem 4.10 (g)$\Rightarrow$ (a), X is fuzzy p-b-almost compact space.

Definition 4.14. A fuzzy cover $\mathcal{U}$ by fuzzy p-b-closed sets of an fts (X, τ) will be called a fuzzy pb-cover of X if for each fuzzy point x_α $(0 < \alpha < 1)$ in X, there exists $U \in \mathcal{U}$ such that U is a fuzzy p-b-open nbd of x_α.

Theorem 4.15. An fts (X, τ) is fuzzy p-b-almost compact if and only if every fuzzy pb-cover of X has a finite subcover.

Proof. Let X be fuzzy p-b-almost compact space and $\mathcal{U}$ be any fuzzy pb-cover of X. Then for each $n \in \mathbb{N}^*$ (the set of all natural numbers) with $n > 1$, there exist $U_x^n \in \mathcal{U}$ and a fuzzy p-b-open set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Then $V_x^n(x) \geq 1-1/n$ for every n, hence $sup\{V_x^n(x) : n \in \mathbb{N}^*\} = 1$, therefore $\mathcal{V} = \{V_x^n : x \in X, n \in \mathbb{N}^*, n > 1\}$ is a fuzzy p-b-open cover of X. As X is fuzzy p-b-almost compact, there exist finitely many points $x_1, x_2, ..., x_m \in X$ and $n_1, n_2, ..., n_m \in N\backslash\{1\}$ such that $1_X = \bigvee\limits_{k=1}^{m} pbclV_{x_k}^{n_k} \leq \bigvee\limits_{k=1}^{m} pbclU_{x_k}^{n_k} = \bigvee\limits_{k=1}^{m} U_{x_k}^{n_k}$.
Conversely, let $\mathcal{U}$ be fuzzy p-b-open cover of X. For any fuzzy point x_α $(0 < \alpha < 1)$ in X, as $\sup\limits_{U\in\mathcal{U}} U(x) = 1$, there exists $U_{x_\alpha} \in \mathcal{U}$ such that $U_{x_\alpha}(x) \geq \alpha$ $(0 < \alpha < 1)$. Then $\mathcal{V} = \{pbclU : U \in \mathcal{U}\}$ is a fuzzy

pb-cover of X and the rest is clear.

The following theorem gives a necessary condition for an fts to be fuzzy p-b-almost compact.

Theorem 4.16. If an fts X is fuzzy p-b-almost compact, then every prefilterbase on X with at most one *pb*-adherent point is *pb*-convergent.

Proof. Let $\mathcal{F}$ be a prefilterbase with at most one *pb*-adherent point in a fuzzy p-b-almost compact fts X. Then by Theorem 4.10, $\mathcal{F}$ has at least one *pb*-adherent point in X. Let x_α be the unique *pb*-adherent point of $\mathcal{F}$ and if possible, let $\mathcal{F}$ do not *pb*-converge to x_α. Then for some fuzzy p-b-open q-nbd U of x_α and for each $F \in \mathcal{F}$, $F \not\leq pbclU$, so that $F \bigwedge \{1_X \setminus pbclU\} \neq 0_X$. Then $\mathcal{G} = \{F \bigwedge (1_X \setminus pbclU) : F \in \mathcal{F}\}$ is a prefilterbase in X and hence has a *pb*-adherent point y_t (say) in X. Now $pbclU \not{q} G$, for all $G \in \mathcal{G}$ so that $x_\alpha \neq y_t$. Again, for each fuzzy p-b-open q-nbd V of y_t and each $F \in \mathcal{F}$, $pbclV q (F \bigwedge (1_X \setminus pbclU))$, hence $pbclV q F$ and it follows that y_t is a fuzzy *pb*-adherent point of $\mathcal{F}$, where $x_\alpha \neq y_t$. This contradicts the fact that x_α is the only fuzzy *pb*-adherent point of $\mathcal{F}$.

Some results on fuzzy p-b-almost compactness of an fts are given by the following theorem.

Theorem 4.17. Let (X, τ) be an fts and $A \in I^X$. Then the following statements are true :
(a) If A is fuzzy p-b-almost compact, then so is $pbclA$,
(b) Union of two fuzzy p-b-almost compact sets is also so,
(c) If X is fuzzy p-b-almost compact, then every fuzzy regularly p-b-closed set A in X is fuzzy p-b-almost compact.

Proof (a). Let $\mathcal{U}$ be a fuzzy p-b-open cover of $pbclA$. Then $\mathcal{U}$ is also a fuzzy p-b-open cover of A. As A is fuzzy p-b-almost compact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \leq \bigvee\{pbclU : U \in \mathcal{U}_0\} = pbcl\{\bigvee U : U \in \mathcal{U}_0\}$, hence

$$pbclA \leq pbcl\{pbcl[\bigvee\{U : U \in \mathcal{U}_0\}]\} = pbcl\{\bigvee U : U \in \mathcal{U}_0\} = \bigvee\{pbclU : U \in \mathcal{U}_0\}.$$

The claim follows.
(b). Obvious.
(c). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy p-b-open cover of a fuzzy regularly p-b-closed set A in X. Then for each $x \notin suppA$, $A(x) = 0 \Rightarrow (1_X \setminus A)(x) = 1 \Rightarrow \mathcal{U} \bigvee \{(1_X \setminus A)\}$ is a fuzzy p-b-open cover of X. Since X is fuzzy p-b-almost compact, there are finitely many members $U_1, U_2, ..., U_n$ in $\mathcal{U}$ such that $1_X = (pbclU_1 \bigvee ... \bigvee pbclU_n) \bigvee pbcl(1_X \setminus A)$. We claim that $pbintA \leq pbclU_1 \bigvee ... \bigvee pbclU_n$. If not, there exists a fuzzy point $x_t \in pbintA$,

but $x_t \notin (pbclU_1 \bigvee ... \bigvee pbclU_n)$, i.e., $t > max\{(pbclU_1)(x), ...,$ $(pbclU_n)(x)\}$. As $1_X = (pbclU_1 \bigvee ... \bigvee pbclU_n) \bigvee pbcl(1_X \setminus A)$, $[pbcl(1_X \setminus A)](x) = 1$, which implies $1 - pbintA(x) = 1$, hence $pbintA(x) = 0$, which implies $x_t \notin pbintA$, a contradiction. Hence $A = pbcl(pbintA) \leq pbcl(pbclU_1 \bigvee .... \bigvee pbclU_n) = pbclU_1 \bigvee ... \bigvee pbclU_n$ (by Result 3.7 and Result 3.8), therefore A is fuzzy p-b-almost compact set.

5. Mutual Relationships between two types of fuzzy almost compactness

Here we establish the mutual relationship between fuzzy almost compactness [11] and fuzzy p-b-almost compactness. Then it is shown that fuzzy p-b-almost compactness implies fuzzy almost compactness, but converse is true in fuzzy p-b-regular space [9]. It is also established that fuzzy p-b-almost compactness remains invariant under fuzzy p-b-irresolute function [9].

Since for any fuzzy set A in an fts X, $pbclA \leq clA$ (as every fuzzy closed set is fuzzy p-b-closed [9]), we can state the following theorem easily.

Theorem 5.1. Every fuzzy p-b-compact space is fuzzy almost compact.

To get the converse we have to recall the following definition and theorem for ready references.

Definition 5.2 [9]. An fts (X, τ) is said to be fuzzy p-b-regular if for each fuzzy p-b-closed set F in X and each fuzzy point x_α in X with $x_\alpha q(1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy p-b-open set V in X such that $x_\alpha qU$, $F \leq V$ and $U \not{q} V$.

Theorem 5.3 [9]. An fts (X, τ) is fuzzy p-b-regular iff every fuzzy p-b-closed set is fuzzy closed.

Theorem 5.4. A fuzzy p-b-regular, fuzzy almost compact space X is fuzzy p-b-almost compact.

Proof. Let $\mathcal{U}$ be a fuzzy p-b-open cover of a fuzzy p-b-regular, fuzzy almost compact space X. Then by Theorem 5.3, $\mathcal{U}$ is a fuzzy open cover of X. As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{clU : U \in \mathcal{U}_0\} = \bigvee\{pbclU : U \in \mathcal{U}_0\}$ (by Theorem 5.3) $= 1_X$, hence X is a fuzzy p-b-almost compact space.
Next we recall the following definition and theorem for ready references.

Definition 5.5 [9]. A function $f : X \to Y$ is said to be fuzzy p-b-irresolute if the inverse image of every fuzzy p-b-open set in Y is fuzzy p-b-open in X.

Theorem 5.6 [9]. For a function $f : X \to Y$, the following statements are equivalent :
(i) f is fuzzy p-b-irresolute,
(ii) $f(pbclA) \leq pbcl(f(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy p-b-open q-nbd V of $f(x_\alpha)$ in Y, there exists a fuzzy p-b-open q-nbd U of x_α in X such that $f(U) \leq V$.

Theorem 5.7. Fuzzy p-b-irresolute image of a fuzzy p-b-almost compact space is fuzzy p-b-almost compact.

Proof. Let $f : X \to Y$ be fuzzy p-b-irresolute surjective function from a fuzzy p-b-almost compact space X onto an fts Y, and let $\mathcal{V}$ be a fuzzy p-b-open cover of Y. Let $x \in X$ and $f(x) = y$. Since $sup\{V(y) : V \in \mathcal{V}\} = 1$, for each $n \in \mathbb{N}^*$ (the set of all natural numbers), there exists some $V_x^n \in \mathcal{V}$ with $V_x^n(y) > 1 - 1/n$ and so $y_{1/n} q V_x^n$. By fuzzy p-b-irresoluteness of f, by Theorem 5.6 (i)$\Rightarrow$ (iii), $f(U_x^n) \leq V_x^n$, for some fuzzy p-b-open set U_x^n in X q-coincident with $x_{1/n}$. Since $U_x^n(x) > 1 - 1/n$, $sup\{U_x^n(x) : n \in \mathbb{N}^*\} = 1$. Then $\mathcal{U} = \{U_x^n : n \in \mathbb{N}^*, x \in X\}$ is a fuzzy p-b-open cover of X. By fuzzy p-b-almost compactness of X, $\bigvee_{i=1}^{k} pbclU_{x_i}^{n_i} = 1_X$, for some finite subcollection $\{U_{x_1}^{n_1}, ..., U_{x_k}^{n_k}\}$ of $\mathcal{U}$. Then $1_Y = f(\bigvee_{i=1}^{k} pbclU_{x_i}^{n_i}) = \bigvee_{i=1}^{k} f(pbclU_{x_i}^{n_i}) \leq \bigvee_{i=1}^{k} pbcl(f(U_{x_i}^{n_i}))$ (by Theorem 5.6 (i) $\Rightarrow$ (ii)) $\leq \bigvee_{i=1}^{k} pbclV_{x_i}^{n_i} \Rightarrow Y$ is fuzzy p-b-almost compact space.

6. Conclusion

This paper is a continuation of [9]. The main goal of this paper is to establish the various results of fuzzy p-b-open sets and fuzzy covering properties. We further want to establish the inter-relations of various types of fuzzy covering properties.

7. Acknowledgement

I express my sincere gratitude to referee for the valuable remarks on this paper.

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